Module 8.2

Static Risk Measures

ABM-Based Binomial Models

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Overview

- Review ABM-Based binomial OVMs
- Addresses both European-style and American-style
- Explore five option Greeks
- Variety of plots generated in R

2

Central Finance Concepts

- SRMs of ABM BOVM is primarily quantitative
- Review standard ABM-based BOVM objectives
- Address standard Greeks based on ABM-based BOVM

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ABM-Based BOVM Objectives

- Additive (constant absolute volatility)
- Recombining (no exploding lattice)
- Incorporate dividends (escrow method)
- Address early exercise, if required

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Standard OVM Greeks

- Delta sensitivity to changes in underlying
- Gamma delta's sensitivity to changes in underlying
- Vega sensitivity to changes in volatility
- Theta sensitivity to changes in time
- Rho sensitivity to changes in the interest rate.

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Quantitative Finance Materials

- Review coherence conditions
- Option Greeks within ABM BOVM
  - Delta
  - Gamma
  - Theta
  - Vega
  - Rho

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Coherence Conditions

- No arbitrage boundary condition

$$d < S_0\left(e^{r\Delta t}-1\right) < u$$

- Probability condition

$$0 << \pi << 1$$

- No arbitrage condition

$$\pi = \frac{S_0\left(e^{r\Delta t}-1\right)-d}{u-d}$$

- Variance condition

$$Var_\pi\left(\Delta S_T\right) = \left(u-d\right)^2 \pi\left(1-\pi\right)$$

u and d Conditions

- Condition for u:

$$u = S_0\left(e^{r\Delta t}-1\right)+\left(1-\pi\right)\frac{\sigma\sqrt{\Delta t}}{\sqrt{\pi\left(1-\pi\right)}}$$

- Condition for d:

$$d = S_0\left(e^{r\Delta t}-1\right)-\pi\frac{\sigma\sqrt{\Delta t}}{\sqrt{\pi\left(1-\pi\right)}}$$

GBM- and ABM-Based BOVM

- GBM-based BOVM

$$O_0 = PV_r\left[E_0\left(O_T\right)\right]$$
$$= PV_r\left[\sum_{j=0}^{n}\Pr\left(n,j\right)Payoff\left(\iota,n,j\right)\right]$$
$$= PV_r\left\{\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)\pi^j\left(1-\pi\right)^{n-j}\max\left[0,\iota\left(S_0u^jd^{n-j}-X\right)\right]\right\}$$

- ABM-based BOVM

$$O_0 = PV_r\left[E_0\left(O_T\right)\right]$$
$$= PV_r\left[\sum_{j=0}^{n}\Pr\left(n,j\right)Payoff\left(\iota,n,j\right)\right]$$
$$= PV_r\left[\sum_{j=0}^{n}\Pr\left(n,j\right)\max\left\{0,\iota_u\left[S_0+ju+\left(n-j\right)d-X\right]\right\}\right]$$

Option Greeks

- Static risk measure
- Mathematical derivatives
- Coherent
  - Delta (first derivative, underlying price)
  - Gamma (second derivative, underlying price)
  - Theta (first derivative, calendar time)
- Incoherent
  - Vega (first derivative, volatility)
  - Rho (first derivative, interest rate)

Delta (Same as GBM)

- Formal definition: $\Delta_O \equiv \frac{\partial O}{\partial S}$
- Standard binomial method

$$\Delta_{O,i,j} = \frac{O_{i+1,j+1}-O_{i+1,j}}{S_{i+1,j+1}-S_{i+1,j}}$$

- Enhanced binomial method

$$\Delta_{O,i,j} = \frac{O_{i,j+1}-O_{i,j-1}}{S_{i,j+1}-S_{i,j-1}}$$

- Numerical method

$$\Delta_{O,i,j} = \frac{O\left(S+h\right)-O\left(S-h\right)}{2h}$$

Figure 9.2.1. Illustration of standard and enhanced delta within ABM-based binomial models

Standard Binomial Delta ***Enhanced Binomial Delta***

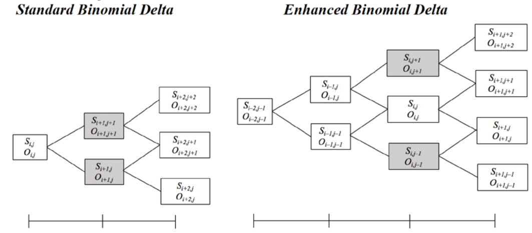
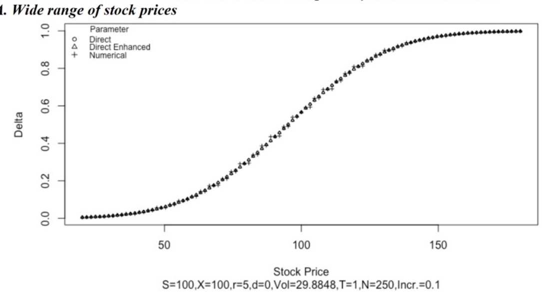


Figure 9.2.2. Three methods to estimate ABM-based European-style binomial call delta
Panel A. Wide range of stock prices



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Panel B. Narrow range of stock prices

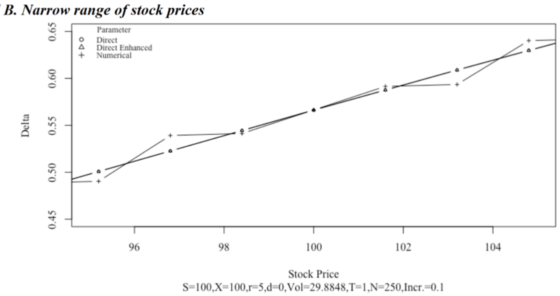

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Figure 9.2.3. ABM-based European-style binomial model for calls and puts

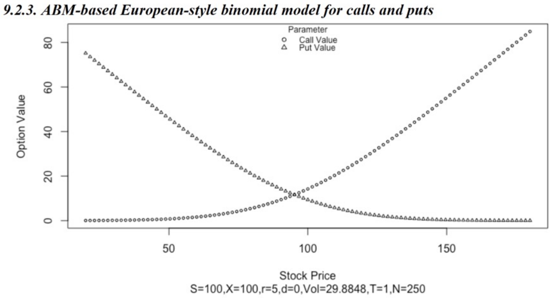

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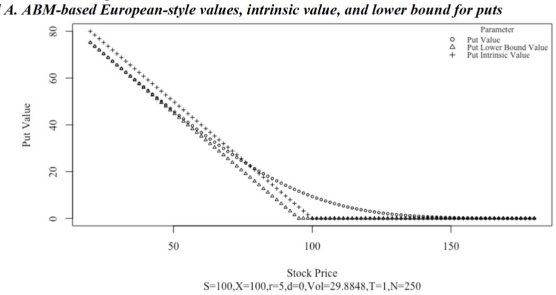
Figure 9.2.4. Call and put values based on the ABM-based binomial model
Panel A. ABM-based European-style values, intrinsic value, and lower bound for puts



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Panel B. ABM-based European-style values, intrinsic value, and lower bound for calls

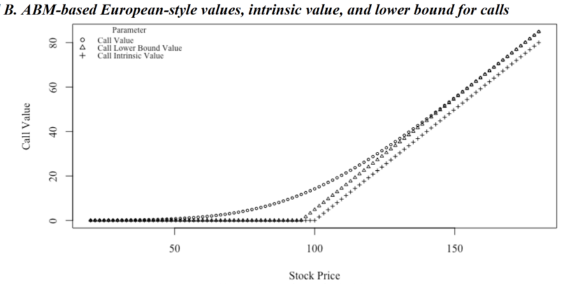

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Figure 9.2.5. ABM-based call and put deltas based on binomial model without dividends
Enhanced Method ***Numerical Method***

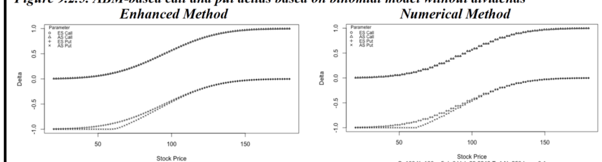

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Figure 9.1.6. Call and put deltas based on GBM-based binomial model without dividends

Enhanced Method ***Numerical Method***

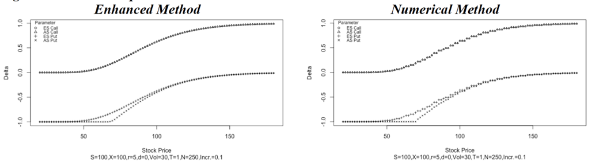


Figure 9.2.6. ABM-based call and put deltas based on binomial model with dividends

Enhanced Method ***Numerical Method***

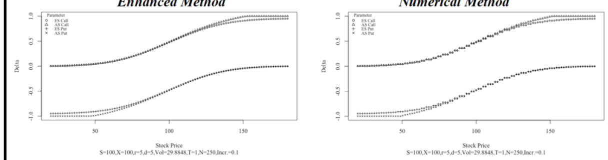
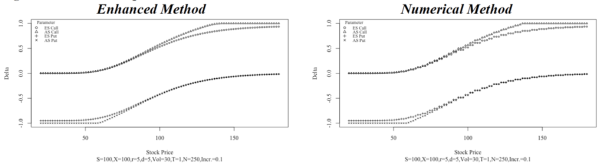


Figure 9.1.7. Call and put deltas based on GBM-based binomial model with dividends

Enhanced Method ***Numerical Method***



Gamma

- Formal definition: $\Gamma_O \equiv \dfrac{\partial^2 O}{\partial S^2}$
- Standard binomial method

$$\Gamma_{O,i,j} = \frac{\dfrac{O_{i+2,j+2} - O_{i+2,j+1}}{S_{i+2,j+2} - S_{i+2,j+1}} - \dfrac{O_{i+2,j+1} - O_{i+2,j}}{S_{i+2,j+1} - S_{i+2,j}}}{\dfrac{S_{i+2,j+2} - S_{i+2,j}}{2}}$$

- Enhanced binomial method

$$\Gamma_{O,i,j} = \frac{\dfrac{O_{i,j+1} - O_{i,j}}{S_{i,j+1} - S_{i,j}} - \dfrac{O_{i,j} - O_{i,j-1}}{S_{i,j} - S_{i,j-1}}}{\dfrac{S_{i,j+1} - S_{i,j-1}}{2}}$$

- Numerical method $\Gamma_{O,i,j} = \dfrac{\left[O(S+h) - O(S)\right] - \left[O(S) - O(S-h)\right]}{h^2}$

Figure 9.2.7. Illustration of standard and enhanced gamma within binomial models

Standard Binomial Gamma ***Enhanced Binomial Gamma***

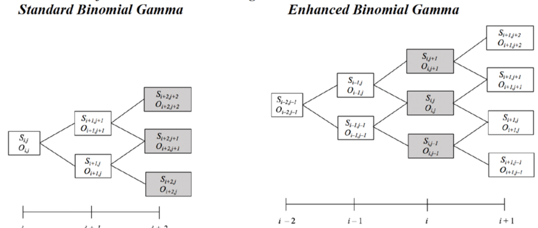


Figure 9.2.8. Three methods to estimate ABM-based European-style option gamma with no dividends

Call Option ***Put Options***

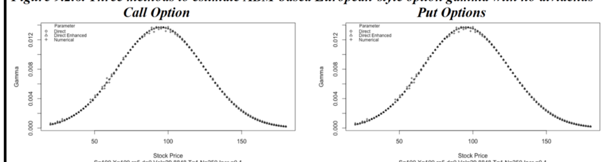


Figure 9.2.9. Three methods to estimate ABM-based European-style gamma with dividends

Call Option ***Put Options***

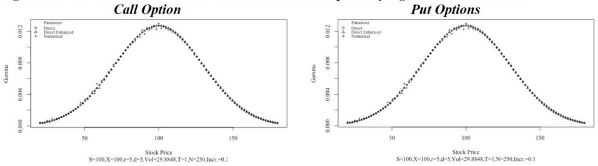


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Figure 9.2.10. ABM-based call and put gammas based on binomial model with no dividends

Enhanced Method ***Numerical Method***

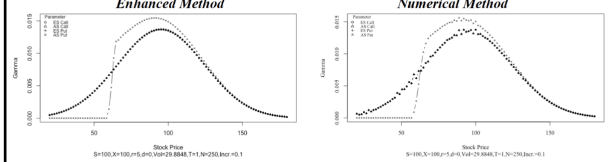

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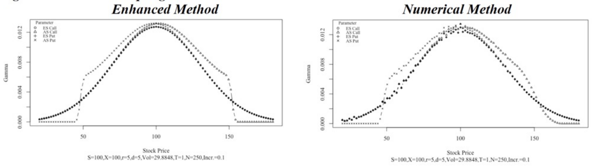
Figure 9.2.11. Call and put gammas based on binomial model with dividends

Enhanced Method ***Numerical Method***



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Theta

- Formal definition: $\theta_O \equiv \dfrac{\partial O}{\partial t}$
- Standard binomial method

$$\theta_{O,i,j} = \frac{O_{i+2,j+1} - O_{i,j}}{2\Delta t}$$

- Enhanced binomial method

$$\theta_{O,i,j} = \frac{O_{i+2,j+1} - O_{i-2,j-1}}{4\Delta t}$$

- Numerical method $\theta_{O,i,j} = \dfrac{O(t+h) - O(t-h)}{2h}$

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Figure 9.2.12. Illustration of standard and enhanced theta within binomial models

Standard binomial theta ***Enhanced binomial theta***

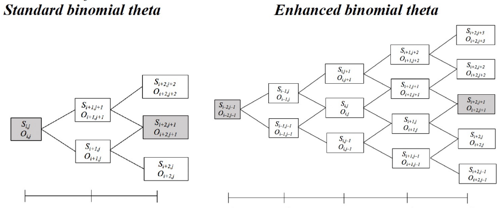

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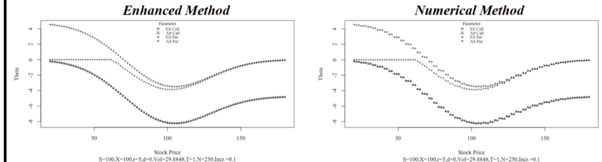
Figure 9.2.13. Illustration of theta within ABM-based binomial model

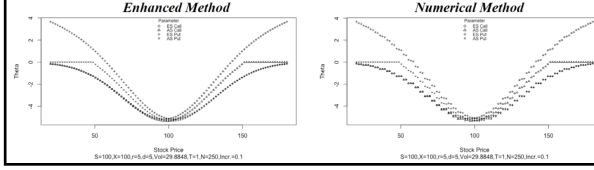
Panel A. Without dividends

Enhanced Method ***Numerical Method***



Panel B. With dividends

Enhanced Method ***Numerical Method***



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Vega

- Formal definition: $v_O \equiv \frac{\partial O}{\partial \sigma}$
- Numerical method

$$v_{O,i,j} = \frac{O_{\sigma+h,i,j} - O_{\sigma-h,i,j}}{2h}$$

- Other methods are not possible as vega is incoherent (but very important and useful)

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Figure 9.2.14. Illustration of vega within binomial model without dividends

European-style *American-style*

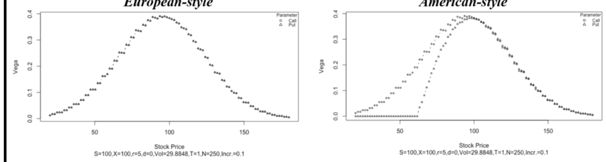

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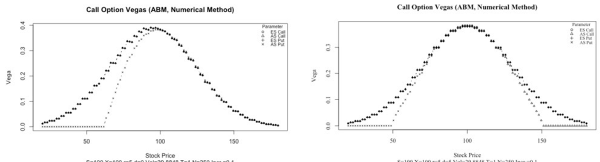
Figure 9.2.15. Vega with respect to stock price within binomial model with and without dividends

No dividends *Dividend yield (5%)*



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Rho

- Formal definition: $\rho_O \equiv \frac{\partial O}{\partial r}$
- Numerical method

$$\rho_{O,i,j} = \frac{O_{r+h,i,j} - O_{r-h,i,j}}{2h}$$

- Other methods are not possible as rho is incoherent (and not that important in most cases)

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Figure 9.2.16. Illustration of rho within binomial model without dividends

European-style *American-style*

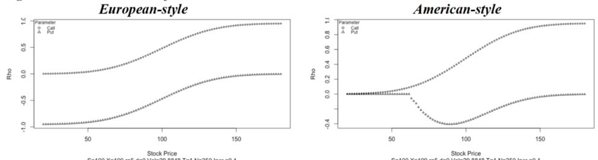

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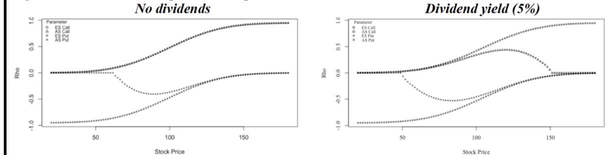
Figure 9.2.17. Rho with respect to stock price within binomial model with and without dividends

No dividends *Dividend yield (5%)*



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Summary

- Reviewed ABM-Based binomial OVMs
- Addressed both European-style and American-style
- Explored five option Greeks
  - Coherent: Delta, Gamma, Theta
  - Incoherent: Vega, Rho
- Variety of plots generated in R reviewed