Module 8.4

Static Risk Measures

ABM-Based Option Valuation Models

1

Overview

- Review static risk measures related to the ABMOVM
- Contrast European-style with American-style (binomial) results
- Understand role of dividend yield
- Identify measurement error with binomial compared with ABMOVM (European-style)

2

Central Finance Concepts

- ABMOVM option Greeks
- Measurement error
- Graphical analysis of option Greeks
  - Delta
  - Gamma
  - Theta
  - Vega
  - Rho

3

Option Greeks

- Static risk measure
- Mathematical derivatives
- Coherent
  - Delta (first derivative, underlying price)
  - Gamma (second derivative, underlying price)
  - Theta (first derivative, calendar time)
- Incoherent
  - Vega (first derivative, volatility)
  - Rho (first derivative, interest rate)

4

Measurement Error

- Contrast binomial OVM and ABMOVM
- European-style and w/ and w/o dividends
- Error with respect to:
  - Stock price
  - Volatility
  - Time to maturity
- Graphs illustrate N = 250

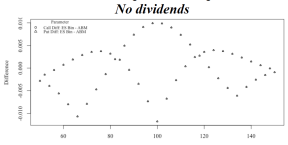
5

Binomial measurement error is relatively small when compared to GBMOVM when N = 250.

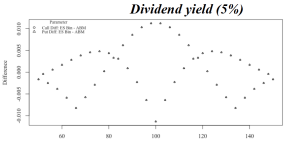
Figure 9.4.1. Measurement error between binomial and ABMOVM

Panel A. Error with respect to stock price

No dividends

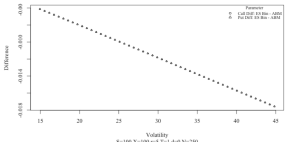


Dividend yield (5%)

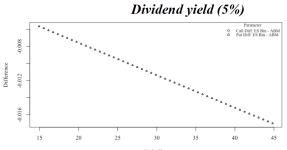


Panel B. Error with respect to volatility

No dividends



Dividend yield (5%)

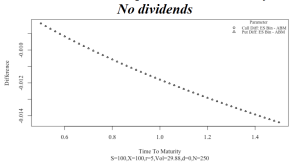


6

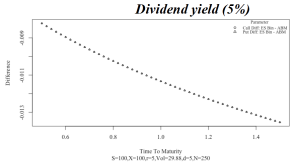
Binomial measurement error is relatively small when compared to GBMOVM when N = 250.

Panel C. Error with respect to time to maturity

No dividends



Dividend yield (5%)



7

Delta

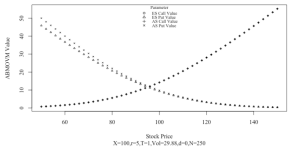
- Panel A: The slopes of the lines is delta as illustrated in Panel B
- Panel B illustrates the influence of the early exercise feature on the delta for the American-style options
- Panel C and D show the influence of volatility and time to maturity

8

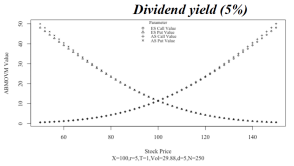
Figure 9.4.2. Call and put deltas based on ABMOVM with and without dividends

Panel A. Option value with respect to stock price

No dividends

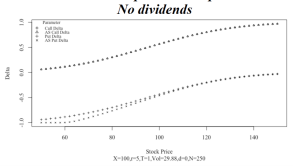


Dividend yield (5%)

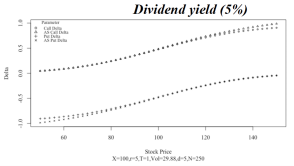


Panel B. Delta with respect to stock price

No dividends



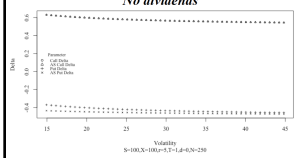
Dividend yield (5%)



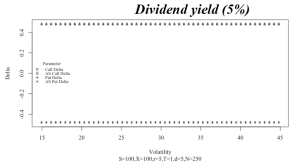
9

Panel C. Delta with respect to volatility

No dividends

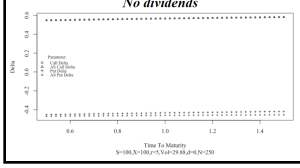


Dividend yield (5%)

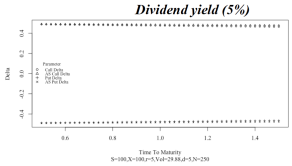


Panel D. Delta with respect to time to maturity

No dividends



Dividend yield (5%)



10

Gamma

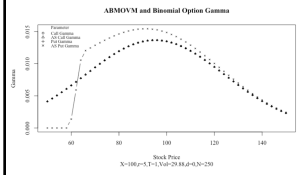
- The *normal* distribution assumption is clear in Panel A except when the early exercise feature impacts option gammas
- Panels B and C show that the gamma increases with declining volatility and with declining time to maturity

11

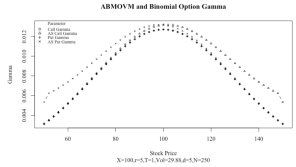
Figure 9.4.3. Call and put gamma based on ABMOVM with and without dividends

Panel A. Gamma with respect to stock price

No dividends

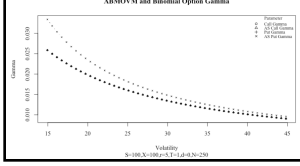


Dividend yield (5%)

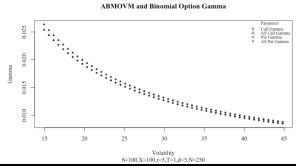


Panel B. Gamma with respect to volatility

No dividends



Dividend yield (5%)

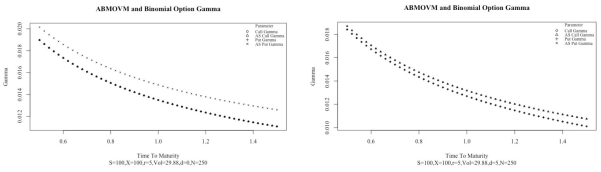


12

Panel C. Gamma with respect to time to maturity

No dividends

Dividend yield (5%)



13

Theta

- Panel A: Well-known time value decay
- Panel B: Non-monotonic relationship with the stock price. At-the-money options have the highest time value
- Panel C shows that theta generally declines with higher volatility
- Panel D shows that theta generally increases with time to maturity

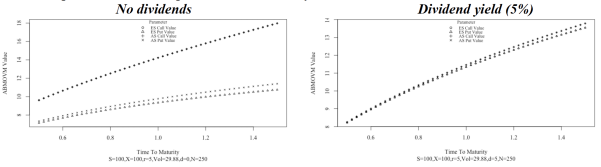
14

Figure 9.4.4. Call and put thetas based on ABMOVM with and without dividends

Panel A. Option value with respect to time to maturity

No dividends

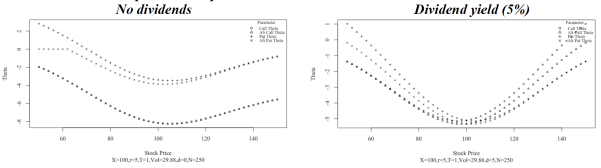
Dividend yield (5%)



Panel B. Theta with respect to stock price

No dividends

Dividend yield (5%)

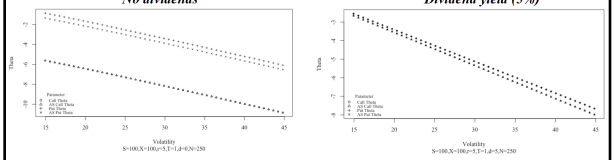


15

Panel C. Theta with respect to volatility

No dividends

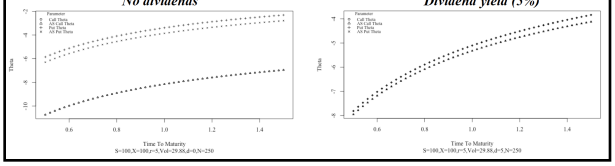
Dividend yield (5%)



Panel D. Theta with time to maturity

No dividends

Dividend yield (5%)



16

Vega

- Deep out-of-the-money: Vega is near zero
- Deep in-the-money: Vega is near zero
- Panel A: Option values increase with volatility
- Panel B: Vega mirrors the *normal* distribution
- Panel C: Vega has complex link with volatility
- Panel D: Vega generally increases with time to maturity.

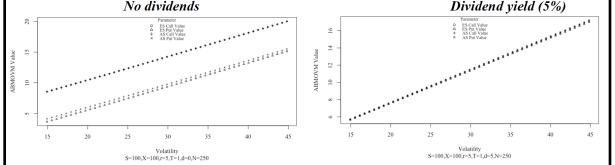
17

Figure 9.4.5. Call and put vegas based on ABMOVM with and without dividends

Panel A. Option value with respect to volatility

No dividends

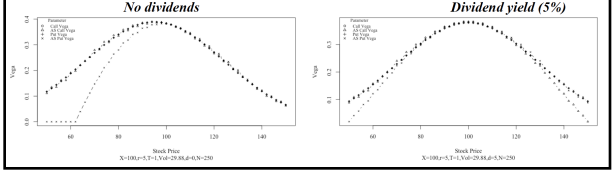
Dividend yield (5%)



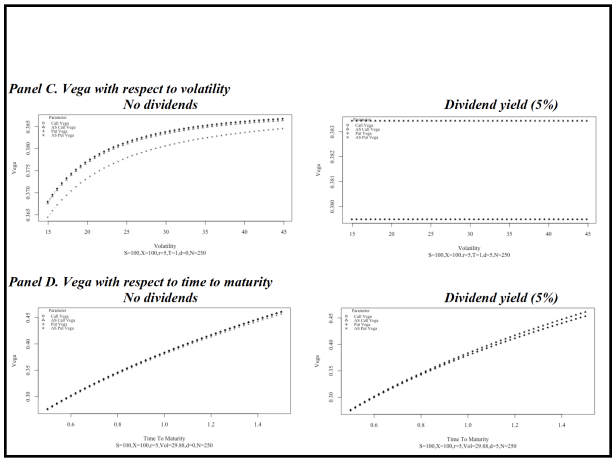
Panel B. Vega with respect to stock price

No dividends

Dividend yield (5%)



18

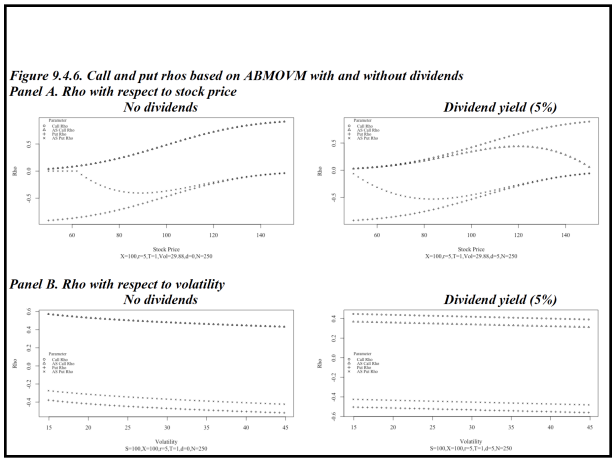


19

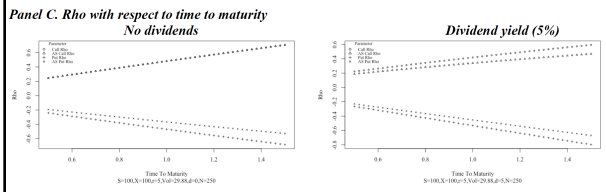
Rho

- Panel A shows that the rho generally increases with the stock price, except for American-style puts
- Panel B shows that rho generally declines with volatility.
- Panel C shows that rho generally increases with time to maturity for calls and decreases for puts.

20



21



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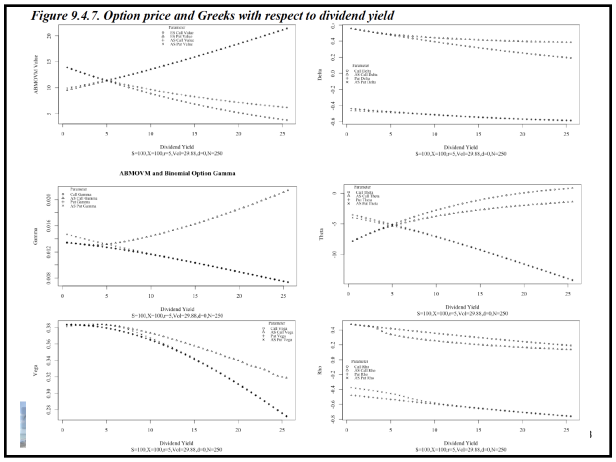

23

Quantitative Finance Materials

- Review ABMOVM
- Define option Greeks with selected insights
- Review sensitivity to dividend yield
- Introduce extended Greeks
- Estimating option price changes with Taylor series

24

Dividend Yield Only

$$c_t = \left(S_t e^{-\delta(T-t)} - Xe^{-r(T-t)}\right) N(d_n) + \sigma_A e^{-r(T-t)} n(d_n)$$

$$p_0 = \left(Xe^{-r(T-t)} - S_0 e^{-\delta(T-t)}\right) N(-d_n) + \sigma_A e^{-r(T-t)} n(d_n)$$

$n(d) = \frac{e^{-d^2/2}}{\sqrt{2\pi}}$, (standard normal probability density function)

$A = \frac{B_{-2(r-\delta)} - 1}{2(r-\delta)}$, (periodic adjustment to volatility)

$\sigma_A^2 = \sigma^2 A = \sigma^2 \frac{B_{-2(r-\delta)} - 1}{2(r-\delta)}$, and (periodic adjusted volatility)

$d_n = \frac{S_t B_{-(r-\delta)} - X}{\sigma_A}$. (quasi "z" score)

25

Option Greeks

- Static risk measure
- Mathematical derivatives
- Coherent
  - Delta (first derivative, underlying price)
  - Gamma (second derivative, underlying price)
  - Theta (first derivative, calendar time)
- Incoherent
  - Vega (first derivative, volatility)
  - Rho (first derivative, interest rate)

26

Measurement Error

- Contrast binomial OVM and ABMOVM
- European-style and w/ and w/o dividends
- Error wrt
  - Stock price
  - Volatility
  - Time to maturity

27

Delta

- Based on ABMOVM, generic delta

$$\Delta_O \equiv \frac{\partial O}{\partial S} = \iota_U e^{-\delta T} N(\iota_U d_n).$$

- Based on GBMOVM, generic delta

$$\Delta_O \equiv \frac{\partial O}{\partial S} = \iota_U e^{-\delta T} N(\iota_U d_1)$$

- Put-call parity yield same expression of put result $c = Se^{-\delta T} - Xe^{-rT} + p$

$$\Delta_c = e^{-\delta T} + \Delta_p$$

28

Gamma

- Based on ABMOVM, generic gamma

$$\Gamma_O \equiv \frac{\partial^2 O}{\partial S^2} = \frac{B_{-(r-2\delta)} n(d_n)}{A\sigma}.$$

- Based on GBMOVM, generic delta

$$\Gamma_O \equiv \frac{\partial^2 O}{\partial S^2} = \frac{e^{-\delta T} n(d_1)}{S\sigma\sqrt{T}}$$

- Put-call parity yields same expression of put result

29

Theta

- Based on ABMOVM, generic theta

$$\Theta_O = \frac{\partial O}{\partial t} = \iota_U \left(\delta B_\delta S_t - rB_r X\right) N(\iota_U d_n) + \sigma r B_r A n(d_n) - \left(\frac{\sigma B_{-(r-2\delta)}}{2A}\right) n(d_n).$$

- Based on GBMOVM, generic theta

$$\theta_O \equiv \frac{\partial O}{\partial t} = -\frac{\partial O}{\partial T} = -\frac{Se^{-\delta T} n(d_1)\sigma}{2\sqrt{T}} - \iota_U rXe^{-rT} N(\iota_U d_2) + \iota_U \delta Se^{-\delta T} N(\iota_U d_1)$$

- Put-call parity: $c = Se^{-\delta T} - Xe^{-rT} + p$

$$\frac{\partial p}{\partial T} = \frac{\partial c}{\partial T} + \delta Se^{-\delta T} - rXe^{-rT}$$

30

Vega

- Based on ABMOVM, generic vega

$$\upsilon_O = \frac{\partial O}{\partial \sigma} = B_r An(d_n).$$

- Based on GBMOVM, generic vega

$$v_O \equiv \frac{\partial O}{\partial \sigma} = Se^{-\delta T} n(d_1)\sqrt{T} = Xe^{-rT} n(d_2)\sqrt{T}$$

- Put-call parity: $c = Se^{-\delta T} - Xe^{-rT} + p$

$$\frac{\partial c}{\partial \sigma} = \frac{\partial p}{\partial \sigma}$$

31

Rho

- Based on ABMOVM, generic rho

$$\rho_O = \frac{\partial O}{\partial r} = (T-t)B_r \iota_U XN(\iota_U d_n) + B_r \sigma A\left\{\left[\frac{(T-t)B_{\frac{2(r-\delta)}{\sigma^2}}}{2(r-\delta)A^2} - 1\right] - (T-t)\right\} n(d_n)$$

- Based on GBMOVM, generic rho

$$\rho_O \equiv \frac{\partial O}{\partial r} = \iota_U XTe^{-rT} N(\iota_U d_2)$$

- Put-call parity: $c = Se^{-\delta T} - Xe^{-rT} + p$

$$\frac{\partial p}{\partial r} = \frac{\partial c}{\partial r} - rXe^{-rT} = rXe^{-rT} N(d_2) - rXe^{-rT} = -rXe^{-rT} N(-d_2)$$

32

Summary

- Reviewed static risk measures related to the ABMOVM
- Contrasted European-style with American-style (binomial) results
- Examined role of dividend yield
- Identified measurement error with binomial compared with ABMOVM (European-style)

33