Chapter 7. Static Risk Management Bonds and Stocks

"*… I would have walked head on into the deep end of the river clinging to your stocks and bonds …* " Elton John, "Someone Saved My Life Tonight." (1975)[1]

Introduction

This is the first of several chapters focused on static risk management. We define static risk management as involving assessing the impact of a change in a salient parameter. Comparative static analysis is one example of static risk management. For example, assessing how a financial instrument's value changes as some relevant underlying parameter changes—essentially taking the mathematical derivative of the valuation function with respect to some variable. The goal is to improve one's understanding of the sensitivity of a financial instrument's value with respect to specific potential changes, not necessarily actual changes that occur in the future.

Alternatively, assessing the historical statistics of one stock when compared to another stock. The goal is to understand the structural differences between financial instruments. Further, the financial analysis is interested in the stability of historical statistics.

In this chapter, we present different ways to compute a variety of financial instrument static risk measures (SRM). In the first module, we introduce centered differencing as an efficient way to estimate numerically various sensitivities. The objective here is to introduce these methods within the R framework. We explore here various SRM related to U.S. Treasury bonds, corporate bonds, and stocks.

Before diving into this material, we first review some elementary concepts related to mathematical derivatives.

Mathematical derivatives review

Recall if we have some variable that is a function of several other variables, say

$$y = f\left(x_1, x_2, \ldots, x_n\right).$$

then the derivative with respect to say x_1 is denoted

$$\frac{dy}{dx_1} = \frac{df\left(x_1, x_2, \ldots, x_n\right)}{dx_1}.$$

Technically, we are taking the total derivative of y with respect to x_1. Thus, if any other variables, say x_2, is a function of x_1, then the total derivative must incorporate this embedded functionality.

If, however, we take the partial derivative, then we assume all other embedded functionality is ignored. That is,

[1]See http://www.bus.lsu.edu/academics/finance/faculty/dchance/MiscProf/DerivaQuote/Qt1.htm.

$$\frac{\partial y}{\partial x_1} = \frac{\partial f\left(x_1, x_2, \ldots, x_n\right)}{\partial x_1}.$$

Although similar in appearance, the values could be different. Again, consider the case where x_2 is a function of x_1.

$$y = f\left[x_1, x_2\left(x_1\right), \ldots, x_n\right].$$

That is, the total derivative is

$$\frac{dy}{dx_1} = \frac{\partial f\left[x_1, x_2\left(x_1\right), \ldots, x_n\right]}{\partial x_1} + \frac{\partial f\left[x_1, x_2\left(x_1\right), \ldots, x_n\right]}{\partial x_2}\frac{dx_2}{dx_1}.$$

The partial derivative is

$$\frac{\partial y}{\partial x_1} = \frac{\partial f\left[x_1, x_2\left(x_1\right), \ldots, x_n\right]}{\partial x_1}.$$

In the case of the partial derivative, the embedded functionality is assumed constant.

Many finance-related functions could potentially have embedded functionality. For example, you may assume that a stock price is a function of volatility. In many cases, if volatility rises, then a particular stock price will decline. We know option valuation models depend on both the stock price as well as volatility. When taking derivatives, we will assume embedded functionality is ignored. Often the partial and total derivative symbols are used interchangeably.