Module 7.4

Static Risk Management
Stocks

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Overview

- Explore static risk measures applications with various stock valuation models
- Review technical details of return calculations
- Identify sample statistics and related R code applied to stock data

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Central Finance Concepts

- Dividend discount models
- Practical consideration for rates of return
- Sample statistics
- SRMs and the LSC model

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Gordon DDM and SRMs

- Gordon dividend discount model
  - Changes in the investor's required rate of return
  - First and second derivatives with respect to the assumed discount rate
  - Estimate the modified stock duration, the Macaulay stock duration, and standard stock convexity
  - Similar analysis with respect to growth rate

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N-Stage DDM and SRMs

- More variables to shock with NDDM
- Generality of the NDDM affords the capacity to incorporate analysts' viewpoints within the SRMs
- Apply SRMs to present value of expected dividends function

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Rates of Return

- GIPS compliance

"2.A.1 TOTAL RETURNS MUST be used.
2.A.2 FIRMS MUST calculate TIME-WEIGHTED RATES OF RETURN that adjust for EXTERNAL CASH FLOWS. Both periodic and sub-period returns MUST be geometrically LINKED. ...
2.A.6 COMPOSITE returns MUST be calculated by asset-weighting the individual PORTFOLIO returns using beginning-of-period values or a method that reflects both beginning-of-period values and EXTERNAL CASH FLOWS."

- Review basic statistics
- SRM and the LSC model

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Quantitative Finance Materials

- SRMs and DDMs
- LSC model applied to PVD
- Technical review of RoRs
- HPR stock statistics with illustrations using rolling data
- SRMs and the LSC model

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Gordon DDM and SRM

- Gordon dividend discount model:

$$V_S = \frac{D_0(1+g)}{k-g}$$

- Modified duration (k):

$$ModDurk_S \equiv -\frac{1}{V_S}\frac{\partial V_S}{\partial k} = \frac{1}{k-g}$$

- Macaulay duration (k):

$$MacDurk_S = ModDurk_S(1+k) = \frac{1+k}{k-g}$$

- Convexity (k): $Convexityk_S \equiv \frac{1}{V_S}\frac{\partial^2 V_S}{\partial k^2} = \frac{2}{(k-g)^2}$

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Influence of Growth

- Modified duration (g):

$$ModDurg_S \equiv -\frac{1}{V_S}\frac{\partial V_S}{\partial g} = \frac{1}{(1+g)(k-g)}$$

- Macaulay duration (g):

$$MacDurg_S = ModDurg_S(1+k) = \frac{1+k}{(1+g)(k-g)}$$

- Convexity (g):

$$Convexityg_S \equiv \frac{1}{V_S}\frac{\partial^2 V_S}{\partial g^2} = \frac{2(1+k)}{(1+g)(k-g)^2}$$

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HPRs and SRMs

- Based on Taylor Series

$$R_{S,dc} \equiv \frac{V_S' - V_S}{V_S}$$

$$\cong \frac{1}{V_S}\left(\frac{\partial V_S}{\partial k}\right)\Delta k + \frac{1}{V_S}\left(\frac{\partial V_S}{\partial g}\right)\Delta g + \frac{1}{2}\frac{1}{V_S}\left(\frac{\partial^2 V_S}{\partial k^2}\right)\Delta k^2 + \frac{1}{2}\frac{1}{V_S}\left(\frac{\partial^2 V_S}{\partial g^2}\right)\Delta g^2$$

$$= -ModDurk_S\Delta k - ModDurg_S\Delta g + \frac{1}{2}Convexityk_S\Delta k^2 + \frac{1}{2}Convexityg_S\Delta g^2$$

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DDM with CAPM

- CAPM: $k = r + \beta\left[E(r_M) - r\right] = r(1-\beta) + \beta E(r_M)$
- Valuation: $V_S = \frac{D_0(1+g)}{r(1-\beta) + \beta E(r_M) - g}$
- Modified duration (r): $ModDurr_S \equiv -\frac{1}{V_S}\frac{\partial V_S}{\partial k}\frac{\partial k}{\partial r} = \frac{1-\beta}{k-g}$
- Convexity (r): $Convexityr_S \equiv \frac{1}{V_S}\frac{\partial^2 V_S}{\partial r^2} = \frac{2(1-\beta)^2}{(k-g)^2}$
- HPR: $R_{S,dc} \equiv \frac{V_S' - V_S}{V_S}$

$$\cong \frac{1}{V_S}\left(\frac{\partial V_S}{\partial r}\right)\Delta r + \frac{1}{V_S}\left(\frac{\partial V_S}{\partial g}\right)\Delta g + \frac{1}{2}\frac{1}{V_S}\left(\frac{\partial^2 V_S}{\partial r^2}\right)\Delta r^2 + \frac{1}{2}\frac{1}{V_S}\left(\frac{\partial^2 V_S}{\partial g^2}\right)\Delta g^2$$

$$= -ModDurr_S\Delta k - ModDurg_S\Delta g + \frac{1}{2}Convexityr_S\Delta r^2 + \frac{1}{2}Convexityg_S\Delta g^2$$

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N-Stage DDM (Review)

- Stub: $Stub = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k}$
- Series:

$$Series = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\sum_{j=1}^{N-1}\left[\prod_{i=1}^{j-1}e^{-(f_i-g_i)m_i}\right]\left[e^{f_j\Delta\tau(3)} + e^{f_j\Delta\tau(2)} + e^{f_j\Delta\tau(1)} + 1\right]\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1}$$

- Final:

$$Final = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[\prod_{i=1}^{N-1}e^{-(f_i-g_i)m_i}\right]\left[e^{f_N\Delta\tau(3)} + e^{f_N\Delta\tau(2)} + e^{f_N\Delta\tau(1)} + 1\right]\frac{1}{e^{(f_N-g_N)}-1}$$

- Value:

$$V_S = Stub + Series + Final$$

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N-Stage DDM and SRM

- Numerical derivatives of underlying parameters
- LSC application of forward rates
- Potential LSC application of growth rates

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LSC Model of PVD

- LSC parameters and numerical derivatives

$$PVD_i = \sum_{j=0}^{N} x_{i,j} f_j$$

$$x_{i,0} = 1,\ x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i/s_1}\right),\ \text{and}\ x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i/s_j}\right) - e^{-\tau_i/s_j};\ j > 1,$$

- Allows time series analysis of changing parameters

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Calculating Rates of Return

- Global Investment Performance Standards (GIPS)

"2.A.1 TOTAL RETURNS MUST be used.
2.A.2 FIRMS MUST calculate TIME-WEIGHTED RATES OF RETURN that adjust for EXTERNAL CASH FLOWS. Both periodic and sub-period returns MUST be geometrically LINKED. ...
2.A.6 COMPOSITE returns MUST be calculated by asset-weighting the individual PORTFOLIO returns using beginning-of-period values or a method that reflects both beginning-of-period values and EXTERNAL CASH FLOWS."

- Interim Rate of Return

$$IRoR = \frac{EMV + Inc - BMV}{BMV}$$

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Averaging Methods

- Arithmetic Average: $\bar{R}_A \equiv \frac{\sum_{t=1}^{n} R_t}{n}$

$$\bar{R}_A \equiv \frac{\sum_{t=1}^{n} R_t}{n} = \frac{1.0 - 0.5}{2} = 0.25$$

- Geometric Average: $\bar{R}_G \equiv \left[\prod_{t=1}^{n}(1+R_t)\right]^{1/n} - 1$

$$\bar{R}_G \equiv \left[\prod_{t=1}^{n}(1+R_t)\right]^{1/n} - 1 = \left[(1+2)(1-0.5)\right]^{1/2} - 1 = 0$$

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Time-Weighted RoR Illustrated

Date	Dollar Dividend Per Share	Other Events	Market Price
January 1			$100
March 15	$1		$105
June 15	$1		$110
September 15	$1		$108
October 1		2-for-1 Stock Split	$52 (post split price)
December 15	$0.5		$51
December 31			$50

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Linking Method

Date	Interim Period	Interim Rate of Return	Time-Weighted Rate of Return
January 1			
March 15	1	0.0600000 (1)	0.0600000
June 15	2	0.0571486 (2)	0.1205775 (7)
September 15	3	-0.0090909 (3)	0.1103905
October 1	4	-0.0370370 (4)	0.0692649
December 15	5	-0.0096154 (5)	0.0589835
December 31	6	-0.0196078 (6)	0.0382192 (8)

(1) (105 + 1 – 100)/100 = 0.06
(2) (110 + 1 – 105)/105 = 0.0571486
(3) (108 + 1 – 110/110 = –0.090909
(4) (2(52) – 108)/108 = –0.0370370
(5) (51 + 0.5 – 52)/52 = –0.0096154
(6) (50 – 51)/51 = –0.0196078
(7) (1 + 0.06)(1 + 0.0571486) – 1 = 0.1205775
(8) (1 + 0.0589835)(1 – 0.0196078) – 1 = 0.0382192

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Index Method

Date	Dividend and Other Events	Additional Shares Purchased	Number of Shares Owned
January 1			100
March 15	$1	0.952381 (1)	100.952381 (2)
June 15	$1	0.917749 (3)	101.870130 (4)
September 15	$1	0.943242 (5)	102.813372 (6)
October 1	2-for-1 Split		205.626744 (7)
December 15	$0.5	2.015948 (8)	207.642692 (9)
December 31			207.642692

(1) 1(100)/105 = 0.952381
(2) 100 + 0.952381 = 100.0952381
(3) 1(100.952381)/110 = 0.917749
(4) 100.952381 + 0.917749 = 101.870130
(5) 1(101.870130)/108 = 0.943242
(6) 101.870130 + 0.943242 = 102.813372
(7) 2(102.813372) = 205.626744
(8) 0.5(205.626744) = 2.015948
(9) 205.626744 + 2.015948 = 207.642692

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Indexes

- General Approach: $I_t = \sum_{i=1}^{n} Q_{i,t} P_{i,t}$
- Price-weighted Index: (DJIA)

$$I_{PW,t} = \sum_{i=1}^{n} Q_{i,t-1} P_{i,t} = \sum_{i=1}^{n} Q_{t-1} P_{i,t} = Q_{t-1} \sum_{i=1}^{n} P_{i,t}$$

- Value-weighted Index: (SP 500)

$$I_{VW,t} = \sum_{i=1}^{n} Q_{i,t-1} P_{i,t} = c_{t-1} \sum_{i=1}^{n} N_{i,t-1} P_{i,t}$$

$$c_0 = 100 \left(\frac{1}{\sum_{i=1}^{n} N_{i,0} P_{i,0}} \right)$$

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Equally-Weighted Indexes

- EW Arithmetic

$$I_{EW,AM,t} = I_{EW,AM,t-1} \left(1 + \frac{\sum_{i=1}^{n} R_{i,t}}{n} \right)$$

- EW Geometric

$$I_{EW,GM,t} = I_{EW,GM,t-1} \left[\prod_{i=1}^{n} (1 + R_{i,t}) \right]^{1/n}$$

- CRSP (both) and Value Line (geometric)

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Index Choice

- Equally-weighted
  - When account is small relative to size of smallest available financial instruments
  - Able to buy as much of any instrument as desired
- Value-weighted
  - When account is large relative to size of smallest available financial instruments
  - Unable to buy as much of smaller instruments as desired

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HPR Stock Statistics

- Discretely compounded, periodic returns

$$R_{dc} \equiv \frac{Profit}{Investment} = \frac{\Delta V}{V} = \frac{V_{t+\Delta t} - V_t}{V_t}$$

- Continuously compounded, periodic

$$R_{cc} \equiv \ln \left(\frac{V_{t+\Delta t}}{V_t} \right)$$

- Dollar profit, periodic

$$\Delta V \equiv Profit = V_{t+\Delta t} - V_t$$

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Using the zoo Package in R

- The zoo package makes time series manipulations easier

```
CDate = mdy.date(Month, Day, Year)
ZDate <- zoo(CDate)
ZPrice <- zoo(Price)
LagPrice <- lag(ZPrice, -1, na.pad = TRUE)
CCReturn <- log(ZPrice) - log(LagPrice) # Continuously compounded rate of Return
DCReturn <- (ZPrice - LagPrice)/LagPrice # Discretely compounded rate of Return
ACPrice <- ZPrice - LagPrice # Actual change in Price (ABM measure)
```

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Sample Statistics

- Mean
  - Sample Average: $\bar{R}_A = \frac{1}{n}\sum_{i=1}^{n} R_i$
  - Geometric Average: $\bar{R}_G = \left[\prod_{i=1}^{n}(1+R_i)\right]^{\frac{1}{n}} - 1$
  - Harmonic Average: $\bar{R}_H = n\left[\sum_{i=1}^{n}\frac{1}{(1+R_i)}\right]^{-1} - 1$

$$P\bar{E}_H = n\left(\sum_{i=1}^{n}\frac{1}{P/E}\right)^{-1} = 5\left(\frac{1}{10}+\frac{2}{50}+\frac{0}{40}+\frac{5}{70}+\frac{4}{30}\right)^{-1}$$

$$= 5(0.34476)^{-1} = 14.5$$

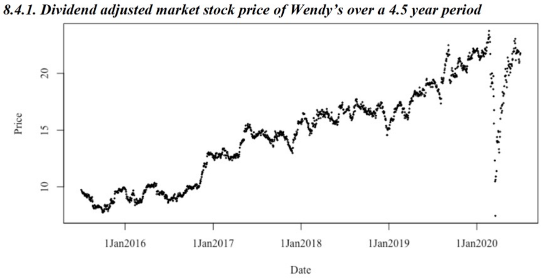
Rolling Statistics

- Remove NAs in first row, use rollapply()

```
dfStockRisk1$CCReturn <- CCReturn
dfStockRisk1$DCReturn <- DCReturn
dfStockRisk1$ACPrice <- ACPrice
dfStockRisk1 <- dfStockRisk1[-1,]      # Remove first line (NAs)
CCMean <- rollapply(dfStockRisk1$CCReturn, RollingWindow,
  function(x)(mean(x)), align = "right")
CCStDev <- rollapply(dfStockRisk1$CCReturn, RollingWindow,
  function(x)(sqrt(var(x))), align = "right")
CCSkewness <- rollapply(dfStockRisk1$CCReturn, RollingWindow,
  function(x)(skewness(x)), align = "right")
CCKurtosis <- rollapply(dfStockRisk1$CCReturn, RollingWindow,
  function(x)(kurtosis(x)), align = "right")
```

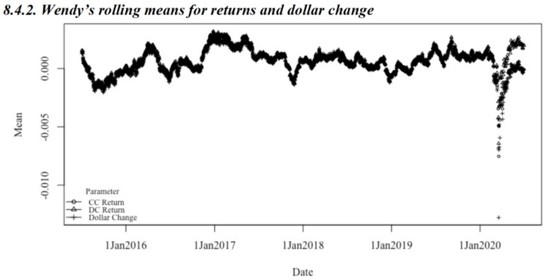

Clearly, the onset of the pandemic resulted in a price movement that defies any sort of predictability based on data analytics, regardless of level of sophistication.

Figure 8.4.1. Dividend adjusted market stock price of Wendy's over a 4.5 year period



Rolling means are very unstable and nearly the same for continuously compounded and discretely compounded returns. Dollar changes also track very closely early but diverge with higher prices.

Figure 8.4.2. Wendy's rolling means for returns and dollar change



Standard Deviation

- Population:

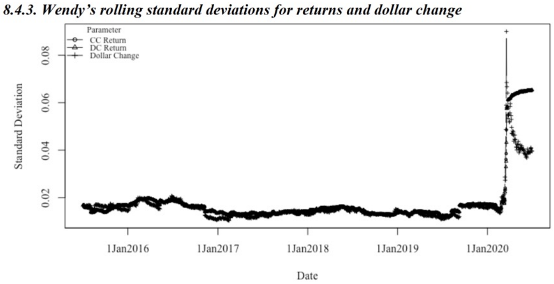
$$\sigma_p = \left[E\left(R-\bar{R}_A\right)^2\right]^{1/2} = \left(\frac{1}{n}\sum_{i=1}^{n}R_i^2 - \bar{R}_A^2\right)^{1/2}$$

- Sample:

$$\sigma_s = \left(\frac{1}{n-1}\sum_{i=1}^{n}R_i^2 - \frac{n}{n-1}\bar{R}_A^2\right)^{1/2}$$

Rolling standard deviations are nearly the same for continuously compounded and discretely compounded returns. Dollar changes also track very closely early but diverge with higher and more volatile prices.

Figure 8.4.3. Wendy's rolling standard deviations for returns and dollar change



Skewness

- Population: $\mu_3 = E\left(R - \bar{R}_A\right)^3$

$$\gamma_{S,p} = \frac{\mu_3}{\sigma_p^3}$$

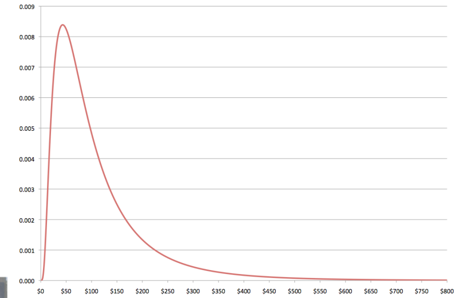
Sample:

$$\gamma_{S,s} = \frac{\sqrt{n(n-1)}}{n-2}\frac{\frac{1}{n}\sum_{i=1}^{n} R_i^3 - \frac{3\bar{R}_A}{n}\sum_{i=1}^{n} R_i^2 - (n-3)\bar{R}_A^3}{\left(\frac{1}{n}\sum_{i=1}^{n} R_i^2 - \bar{R}_A^2\right)^{3/2}}$$

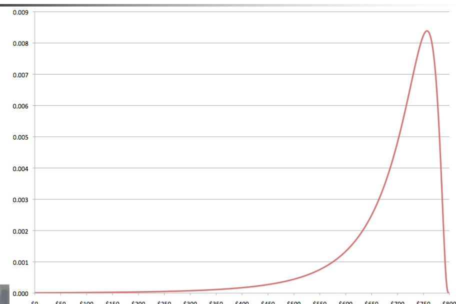
Some have categorized skewness in the following manner:

- Highly skewed: $-1 < \mu_3$ or $\mu_3 > +1$
- Moderately skewed: $-1 < \mu_3 < -1/2$ or $1/2 > \mu_3 > +1$
- Approximately symmetric: $-1/2 < \mu_3 < 1/2$

Positive Skewness

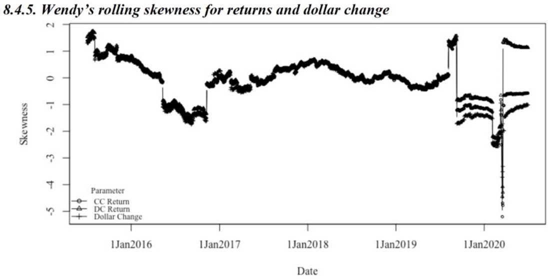


Negative Skewness



Rolling skewness is also very unstable and nearly the same for continuously compounded and discretely compounded returns early. The three measures diverge after the onset of the pandemic.

Figure 8.4.5. Wendy's rolling skewness for returns and dollar change



Excess Kurtosis

- Population: $\mu_4 = E\left(R - \bar{R}_A\right)^4$

$$\gamma_{K,p} = \frac{\mu_4}{\sigma_p^4} - 3$$

- Sample:

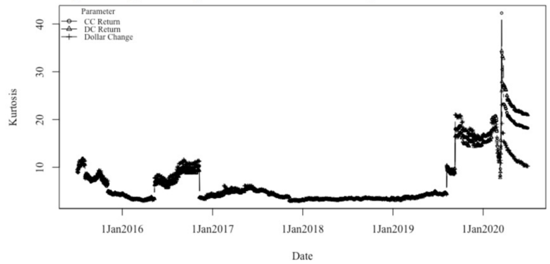
$$\gamma_{K,s} = \frac{n-1}{(n-2)(n-3)}\left[(n+1)\frac{\frac{1}{n}\sum_{i=1}^{n} R_i^4 - \frac{4\bar{R}_A}{n}\sum_{i=1}^{n} R_i^3 + \frac{6\bar{R}_A^2}{n}\sum_{i=1}^{n} R_i^2 + (n-4)\bar{R}_A^4}{\left(\frac{1}{n}\sum_{i=1}^{n} R_i^2 - \bar{R}_A^2\right)^2} + 6\right]$$

Interpreting Kurtosis

- $\gamma_{K,s} = 0$ Mesokurtic (Normal and binomial)
- $\gamma_{K,s} > 0$ Leptokurtic (Sharper peaks and fatter tails, lognormal, Laplace, and logistic)
- $\gamma_{K,s} < 0$ Platykurtic [Flatter peaks and thinner tails, uniform and Bernoulli distribution (p=1/2)]

Rolling kurtosis is nearly the same for all three measures except after shocks. Again, the three measures diverge after the onset of the pandemic.

Figure 8.4.6. Wendy's rolling kurtosis for returns and dollar change



SRMs and the LSC model

- LSC valuation framework

$$V_0 = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[\hat{L}_f + sc_{f,j}\hat{S}_f - \left(\hat{L}_g + sc_{g,j}\hat{S}_g\right)\right]}.$$

- SRMs

$$\frac{\partial V}{\partial L_f} = -CF_0 \sum_{i=1}^{\infty} i e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)},$$

$$\frac{\partial V}{\partial L_g} = CF_0 \sum_{i=1}^{\infty} i e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)},$$

$$\frac{\partial V}{\partial S_f} = -CF_0 \sum_{i=1}^{\infty}\left(\sum_{j=1}^{i} sc_{f,j}\right) e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)}, \text{ and}$$

$$\frac{\partial V}{\partial S_g} = CF_0 \sum_{i=1}^{\infty}\left(\sum_{j=1}^{i} sc_{g,j}\right) e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)}.$$

Note that a positive shock to LSC level growth is the same as a negative shock to LSC level discount rate. The sensitivity of the LSC slope discount rate is greater is size than the LSC slope growth. VCF denotes the value per dollar of cash flow.

Figure 8.4.7. First derivatives (expressed assuming a one percentage point move)

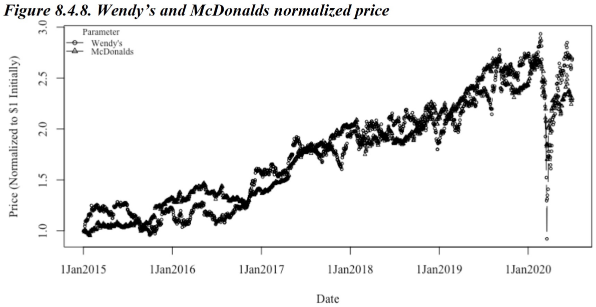
Industry	Ticker	DeltaLg	DeltaLf	DeltaSg	DeltaSf	DeltaVCFLg	DeltaVCFLf	DeltaVCFSg	DeltaVCFSf
Broad Market	SPY	2.480	-2.480	0.311	-0.711	0.420	-0.420	0.053	-0.120
Technology	XLK	0.657	-0.657	0.092	-0.201	0.529	-0.529	0.074	-0.162
Financial	XLF	0.044	-0.044	0.013	-0.023	0.071	-0.071	0.020	-0.038
Industrials	XLI	0.294	-0.294	0.054	-0.113	0.186	-0.186	0.034	-0.072
Consumer Discretionary	XLY	0.905	-0.905	0.118	-0.264	0.566	-0.566	0.074	-0.165
Materials	XLB	0.185	-0.185	0.040	-0.079	0.149	-0.149	0.032	-0.064
Healthcare	XLV	0.473	-0.473	0.083	-0.176	0.201	-0.201	0.035	-0.075
Utilities	XLU	0.199	-0.199	0.043	-0.087	0.098	-0.098	0.021	-0.043
Consumer Stables	XLP	0.233	-0.233	0.045	-0.094	0.139	-0.139	0.027	-0.056
Energy	XLE	0.081	-0.081	0.023	-0.042	0.054	-0.054	0.015	-0.029

R Code Application

- Wendy's and McDonalds
- Detailed analysis of HPR statistics
  - Mean
  - Standard deviation
  - Skewness
  - Excess kurtosis

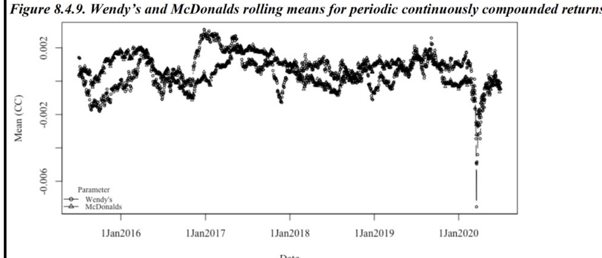
Clearly, Wendy's and McDonalds generally move together but also diverge from time-to-time.

Figure 8.4.8. Wendy's and McDonalds normalized price



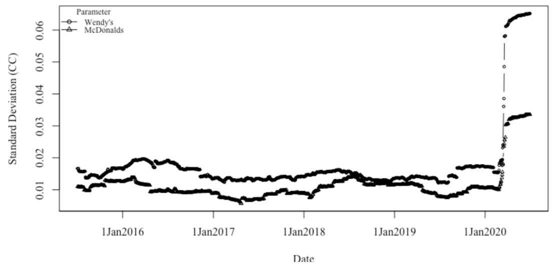
Again, Wendy's and McDonalds rolling means are unstable. Sometimes one is higher than the other. It would be very difficult to predict.

Figure 8.4.9. Wendy's and McDonalds rolling means for periodic continuously compounded returns



Wendy's and McDonalds rolling standard deviations are much more predictable in terms of ordinal ranking. Wendy's simply has a higher standard deviation.

Figure 8.4.10. Wendy's and McDonalds rolling standard deviations for periodic continuously compounded returns



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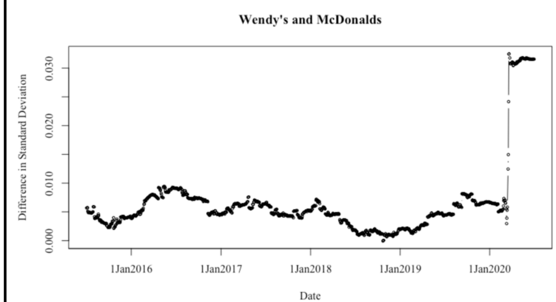
Again, Wendy's rolling standard deviations is higher than McDonalds. The magnitude of this difference varies, however.

Figure 8.4.11. Wendy's less McDonalds rolling standard deviations for periodic continuously compounded returns



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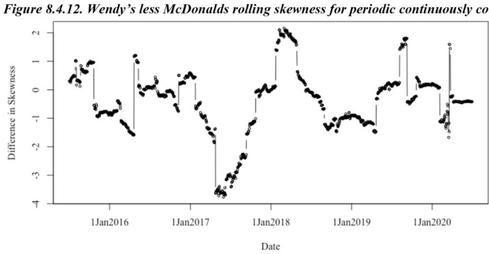
Wendy's less McDonalds rolling skewness is quite erratic defying predictability.

Figure 8.4.12. Wendy's less McDonalds rolling skewness for periodic continuously compounded returns



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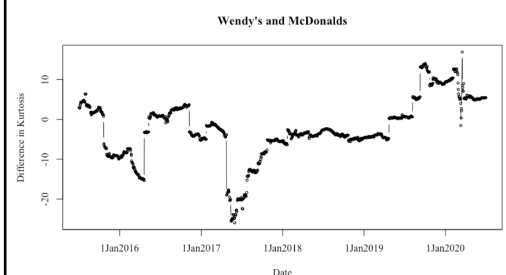
Wendy's less McDonalds rolling kurtosis is also somewhat erratic defying predictability.

Figure 8.4.13. Wendy's less McDonalds rolling kurtosis for periodic continuously compounded returns



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Summary

- Explored static risk measures applications with various stock valuation models
- Reviewed technical details of return calculations
- Identified sample statistics and related R code applied to stock data

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