Module 7.3

Static Risk Measures

Corporate Bonds

1

Overview

- Corporate bonds have additional risks
- Modeled as an additional LSC model curve
  - Develop static risk measures related to spreads
  - Illustrate with numerous graphs

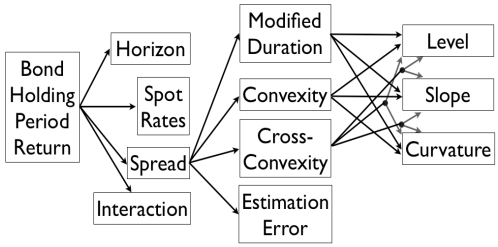
2

Central Finance Concepts

- Holding period return decomposition
  - Horizon, spot rate, spread, interaction
  - Mod. Duration, convexity, cross-convexity, and estimation error
  - Level, slope, and curvatures
- Extended graphical analysis

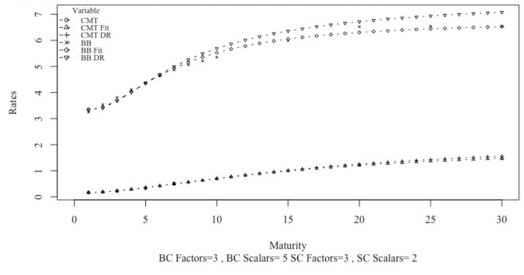
3

HPR Decomposition



4

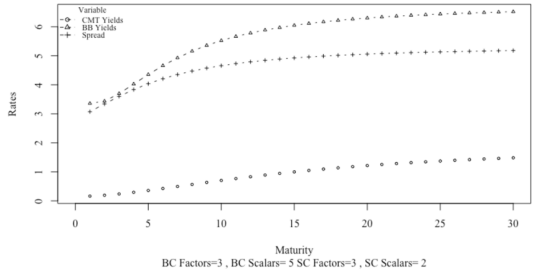
Figure 8.3.2. CMT yields and BB yields along with four factor LSC fit and discount rates



Three factor models of CMT and BB yields fit well.

5

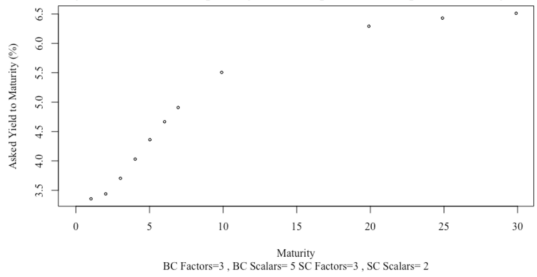
Figure 8.3.3. Fitted CMT yields and BB yields along with the implied spread



Credit spread is the dominant driver of BB yield curve.

6

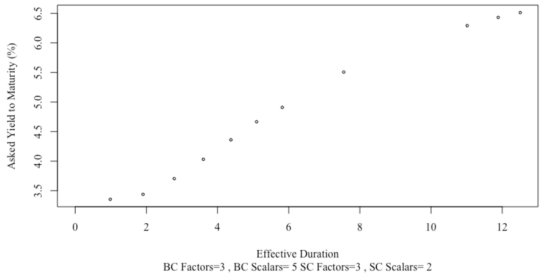
Figure 8.3.4. Asked yield to maturities implied by the asked prices with respect to maturity



There is no way to appraise which bond is trading for a lower price relative to the current market conditions.

7

Figure 8.3.5. Asked yield to maturities implied by the asked prices with respect to duration

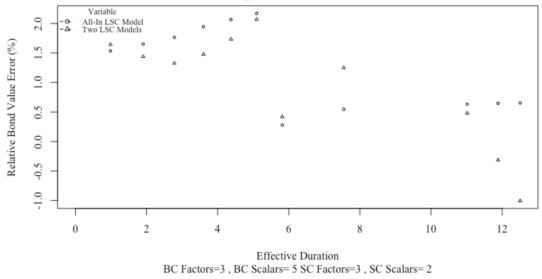


With effective duration, the relationship is now closer to linear, it remains unclear which bonds are expensive or cheap.

8

The lower the relative bond value error the more valuable the bond. Note that in most cases the bonds are trading over the all-in BB curve indicating a mark-up due to trading costs. Further, fitting two LSC curves results in larger errors compared to fitting just the all-in curve.

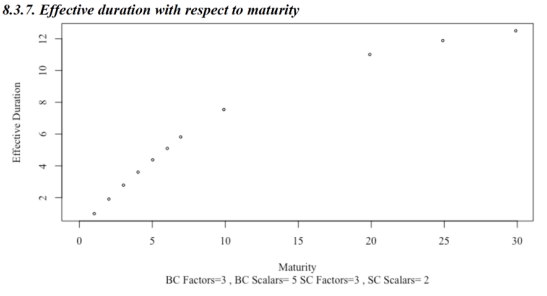
Figure 8.3.6. Relative bond value error with respect to duration



9

The longer the maturity the lower the marginal increase in effective duration, especially for higher coupon bonds. Thus, from an effective duration perspective there is not much difference between 20-, 25-, and 30-year bonds.

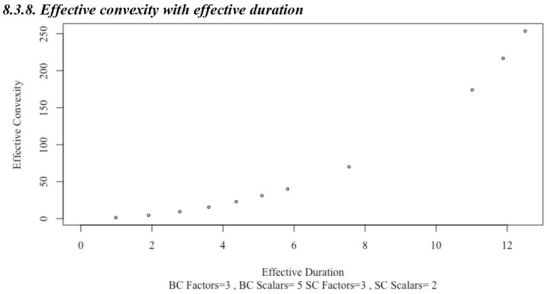
Figure 8.3.7. Effective duration with respect to maturity



10

Effective convexity has a convex relationship to effective duration. The higher the effective duration the higher the marginal increase in effective convexity.

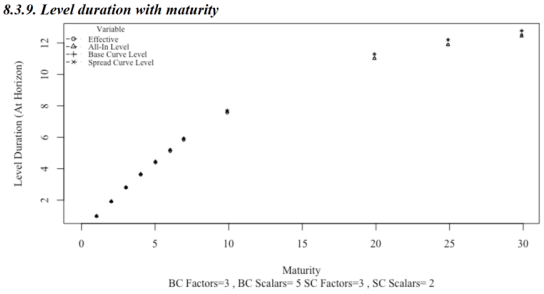
Figure 8.3.8. Effective convexity with effective duration



11

The all-in level duration and effective durations are essentially equivalent. Recall that effective duration assumes a parallel shift in the yield curve and the all-in level duration essentially has the same effect. The base curve and spread curve level duration are essentially equivalent as it results in a parallel shift in the net of these two curves.

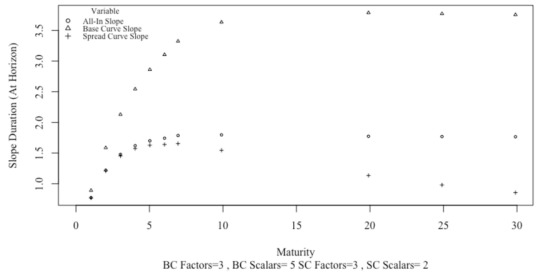
Figure 8.3.9. Level duration with maturity



12

The base curve slope duration has the same pattern as level duration but plateaus at a much lower value. The spread curve slope duration peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar.

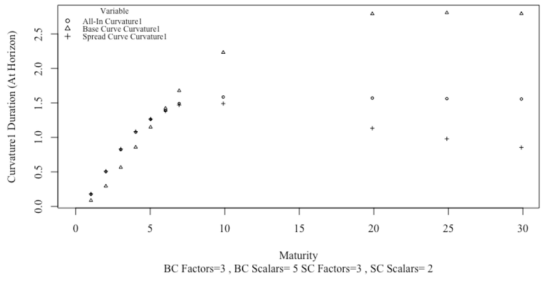
Figure 8.3.10. Slope duration with maturity



13

The all-in curvature1 duration rises steeply and then plateaus around 1.5. The base curve curvature1 duration has the same pattern but plateaus at a much higher value. The spread curve curvature1 duration peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar. Note the pattern is like the spread curve slope duration

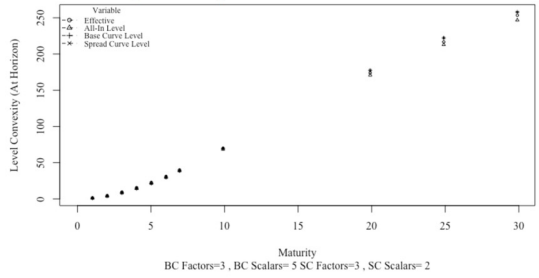
Figure 8.3.11. Curvature1 duration with maturity



14

The base curve and spread curve level convexities are essentially equivalent. The effective convexity is slightly below the base and spread curve versions and the all-in level is the lowest.

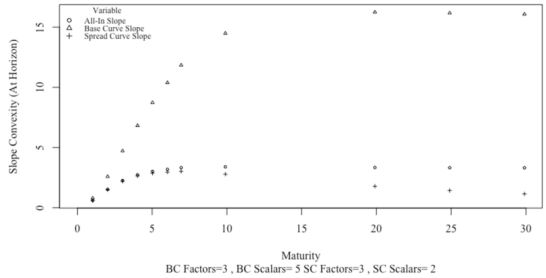
Figure 8.3.12. Level convexity with maturity



15

The all-in slope convexity has the same pattern as base curve slope convexity but plateaus at a much lower value. The spread curve slope convexity peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar.

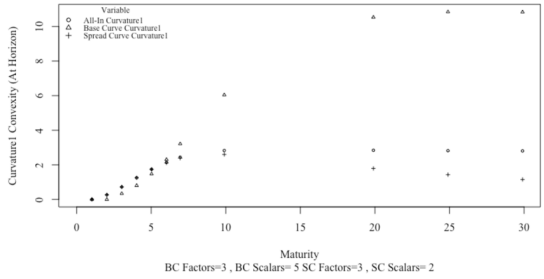
Figure 8.3.13. Slope convexity with maturity



16

The all-in curvature1 convexity has the same pattern as base curve curvature1 but plateaus at a much lower value. The spread curve curvature1 convexity peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar. Note the pattern is like the spread curve slope duration.

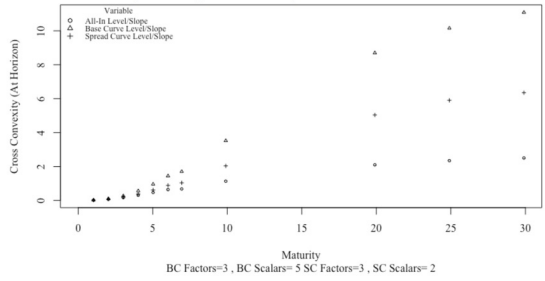
Figure 8.3.14. Curvature1 convexity with maturity



17

Cross convexity level/slope is sensitive to interest rate levels. The higher the interest rate, the lower the cross convexity measure.

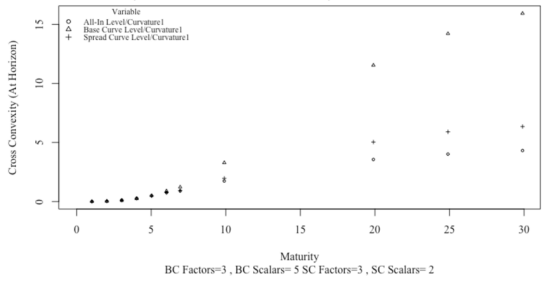
Figure 8.3.15. Cross convexity level/slope with maturity



18

Cross convexity level/curvature1 is sensitive to interest rate levels. The higher the interest rate, the lower the cross convexity measure.

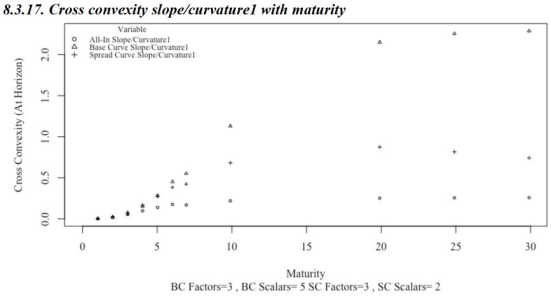
Figure 8.3.16. Cross convexity level/curvature1 with maturity



19

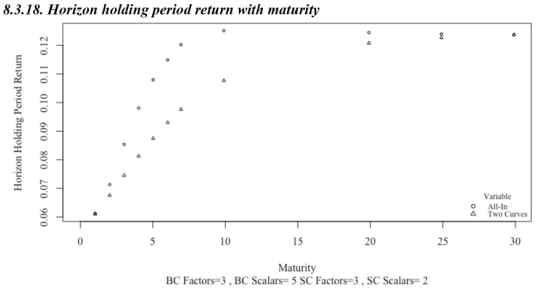
Cross convexity slope/curvature1 is sensitive to interest rate levels. The higher the interest rate, the lower the cross convexity measure.

Figure 8.3.17. Cross convexity slope/curvature1 with maturity



20

Figure 8.3.18. Horizon holding period return with maturity



21

Quantitative Finance Materials

- Advanced bond static risk measures
- Return contributions
- Factor durations, convexities, and cross-convexities

22

LSC Model of Spreads

- Spreads: $sp_{i,t}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}$
- Value with spread curve values at $t + \Delta$

$$\tilde{V}_{t+\Delta}^{sp} \equiv \sum_{i=1}^{N_{t+\Delta}} CF_{i,t+\Delta} D\tilde{F}_{i,t+\Delta}^{sp} = \sum_{i=0}^{N_{t+\Delta}} CF_i e^{-\left(r_{i,t+\Delta}^{LSC} + s\tilde{p}_{i,t+\Delta}^{LSC}\right)(\tau_i - \Delta)}$$

$$D\tilde{F}_{i,t+\Delta}^{sp} \equiv e^{-\left(r_{i,t+\Delta}^{LSC} + s\tilde{p}_{i,t+\Delta}^{LSC}\right)(\tau_i - \Delta)} \qquad sp_{i,t} = \hat{r}_{i,t} - r_{i,t}^{LSC}$$

- Bond spread HPR: $\tilde{R}_{\Delta}^{sp} \equiv \dfrac{\tilde{V}_{t+\Delta}^{sp} - V_{t+\Delta}^{LSC}}{V_{t+\Delta}^{LSC}}$

23

Base Rate Estimation (Review)

$$r_{i,t}^{LSC} \equiv \sum_{j=0}^{N^r} x_{i,j,t} f_{j,t}^{r}, \text{ (Fit at time } t\text{, analyzed at time } t\text{)}$$

$$r_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N^r} x_{i,j,t+\Delta} f_{j,t}^{r}, \text{ (Fit at time } t\text{, analyzed at time } t + \Delta\text{)}$$

$$\tilde{r}_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N^r} x_{i,j,t+\Delta} \tilde{f}_{j,t+\Delta}^{r}. \text{ (Fit at time } t + \Delta\text{, analyzed at time } t + \Delta\text{)}$$

24

Spread Estimation

$$sp_{i,t}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}, \text{ (Fit at time } t\text{, analyzed at time } t\text{)}$$

$$sp_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}, \text{ (Fit at time } t\text{, analyzed at time } t+\Delta\text{)}$$

$$\tilde{sp}_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} \tilde{f}_{j,t+\Delta}^{sp}. \text{ (Fit at time } t+\Delta\text{, analyzed at time } t+\Delta\text{)}$$

$$\Delta\tilde{f}_{j,t+\Delta}^{sp} = \tilde{f}_{j,t+\Delta}^{sp} - f_{j,t}^{sp} \text{ or (Spread)}$$

25

LSC Factor Static Risk Measures

- Unknown HPR decomposition:

$$\tilde{R}_{\Delta}^{Unknown} = \tilde{R}_{\Delta}^{LSC} = \frac{\tilde{V}_{t+\Delta}^{LSC} - V_{t+\Delta}^{LSC}}{V_{t+\Delta}^{LSC}} \equiv R\tilde{C}_{FD}^{r} + R\tilde{C}_{FC}^{r} + R\tilde{C}_{FCC}^{r} + R\tilde{C}_{FD}^{sp} + R\tilde{C}_{FC}^{sp} + R\tilde{C}_{FCC}^{r,sp} + R\tilde{C}_{FCC}^{r,sp}$$

$$R\tilde{C}_{FD}^{sp} = R\tilde{C}_{FD}^{sp,L} + R\tilde{C}_{FD}^{sp,S} + R\tilde{C}_{FD}^{sp,C},$$

$$R\tilde{C}_{FC}^{sp} = R\tilde{C}_{FC}^{sp,L} + R\tilde{C}_{FC}^{sp,S} + R\tilde{C}_{FC}^{sp,C},$$

$$R\tilde{C}_{FCC}^{sp} = R\tilde{C}_{FCC}^{sp,L,S} + R\tilde{C}_{FCC}^{sp,S,C} + R\tilde{C}_{FCC}^{sp,L,C}, \text{ and}$$

$$R\tilde{C}_{FCC}^{r,sp} = R\tilde{C}_{FCC}^{r(L),sp(L)} + R\tilde{C}_{FCC}^{r(L),sp(S)} + R\tilde{C}_{FCC}^{r(L),sp(C)} + R\tilde{C}_{FCC}^{r(S),sp(L)} + R\tilde{C}_{FCC}^{r(S),sp(S)} + R\tilde{C}_{FCC}^{r(S),sp(C)} + R\tilde{C}_{FCC}^{r(C),sp(L)} + R\tilde{C}_{FCC}^{r(C),sp(S)} + R\tilde{C}_{FCC}^{r(C),sp(C)}.$$

26

Return Contribution for Spreads

$$R\tilde{C}_{FD}^{sp,L} \equiv -FD_{L}^{sp}\Delta\tilde{f}_{L}^{sp},$$

$$R\tilde{C}_{FD}^{sp,S} \equiv -FD_{S}^{sp}\Delta\tilde{f}_{S}^{sp},$$

$$R\tilde{C}_{FD}^{sp,C} \equiv -FD_{C}^{sp}\Delta\tilde{f}_{C}^{sp},$$

$$R\tilde{C}_{FC}^{sp,L} \equiv \frac{1}{2}FC_{FC}^{sp,L}\left(\Delta\tilde{f}_{L}^{sp}\right)^2,$$

$$R\tilde{C}_{FC}^{sp,S} \equiv \frac{1}{2}FC_{FC}^{sp,S}\left(\Delta\tilde{f}_{S}^{sp}\right)^2,$$

$$R\tilde{C}_{FC}^{sp,C} \equiv \frac{1}{2}FC_{FC}^{sp,C}\left(\Delta\tilde{f}_{C}^{sp}\right)^2,$$

$$R\tilde{C}_{FCC}^{sp,L,S} \equiv FCC_{FCC}^{sp,L,S}\Delta\tilde{f}_{L}^{sp}\Delta\tilde{f}_{S}^{sp},$$

$$R\tilde{C}_{FCC}^{sp,L,C} \equiv FCC_{FCC}^{sp,L,C}\Delta\tilde{f}_{L}^{sp}\Delta\tilde{f}_{C}^{sp}, \text{ and}$$

$$R\tilde{C}_{FCC}^{sp,S,C} \equiv FCC_{FCC}^{sp,S,C}\Delta\tilde{f}_{S}^{sp}\Delta\tilde{f}_{C}^{sp}.$$

27

Factor Durations and Convexities

$$FD_{L}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)w_{i,t+\Delta}^{LSC}, \qquad FC_{L}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 w_{i,t+\Delta}^{LSC},$$

$$FD_{S}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)x_{i,S}w_{i,t+\Delta}^{LSC}, \qquad FC_{S}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 x_{i,S}{}^2 w_{i,t+\Delta}^{LSC},$$

$$FD_{C}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)x_{i,C}w_{i,t+\Delta}^{LSC}, \qquad FC_{C}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 x_{i,C}{}^2 w_{i,t+\Delta}^{LSC},$$

$$FCC_{L,S}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 x_{i,S}w_{i,t+\Delta}^{LSC},$$

$$FCC_{S,C}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 x_{i,S}x_{i,C}w_{i,t+\Delta}^{LSC}, \text{ and}$$

$$FCC_{L,C}^{sp} = \sum_{i=0}^{N_t}\left(\tau_i - \Delta\right)^2 x_{i,C}w_{i,t+\Delta}^{LSC}.$$

28

Selected R Code Illustrations

- Fitted LSC CMT and BB-rated curve
- Implied spread
- Selected asked yields for BB bonds
- Estimated bond yield error wrt duration
- Numerous LSC-based graphs

29

Summary

- Corporate bonds have additional risks
- Modeled as an additional LSC model curve
  - Developed static risk measures related to spreads
  - Illustrated with numerous graphs

30