Module 7.3: Static Risk Management Corporate Bonds

Learning objectives

- Explores bond static risk management with defaultable bonds
- Illustrate estimation of spread curve layered on top of the base curve
- Identify factor sensitivities related to spread risk

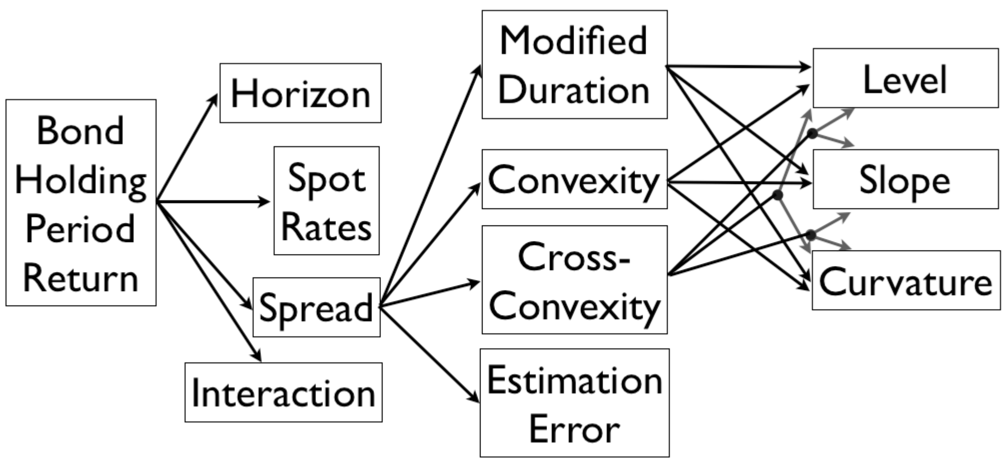
Executive summary

We extend various aspects of traditional bond static risk measures to credit risky bonds. With this foundation, we then moved to advanced bond static risk measures of spreads based on an application of the LSC model. Within a detailed bond holding period return decomposition, we reviewed numerous new measures of related to spreads. The module concludes with selected explanations of selected R code.

Central finance concepts

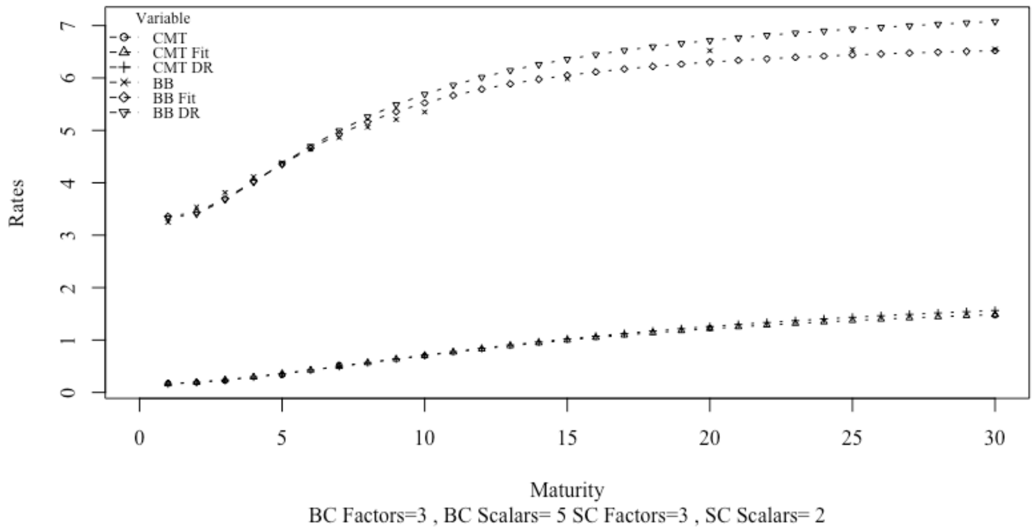
Recall in the last module, we provided a detailed decomposition of interest rate and spread risk. We elaborate more fully here on the spread risk. Figure 7.3.1 once again provides the bond holding period return decomposition related solely to changes in the spread component.

Figure 7.3.1. Bond holding period return decomposition of spread



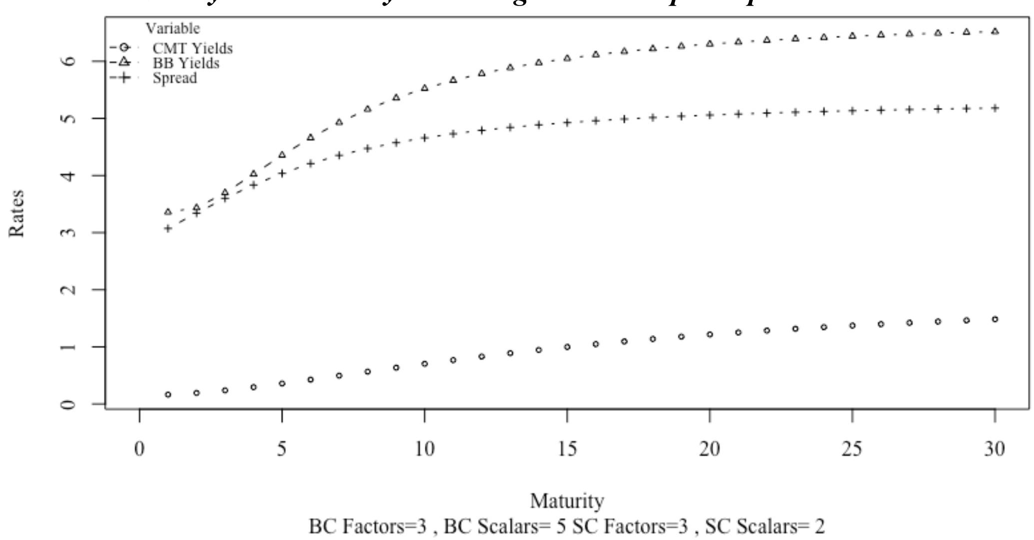
Setting aside the technical complexities, we now illustrate the results of the R code provided in this module. With both the CMT yields and BB yields at a particular point in time, we fit a three factor LSC model illustrated in Figure 7.3.2.

Figure 7.3.2. CMT yields and BB yields along with four factor LSC fit and discount rates



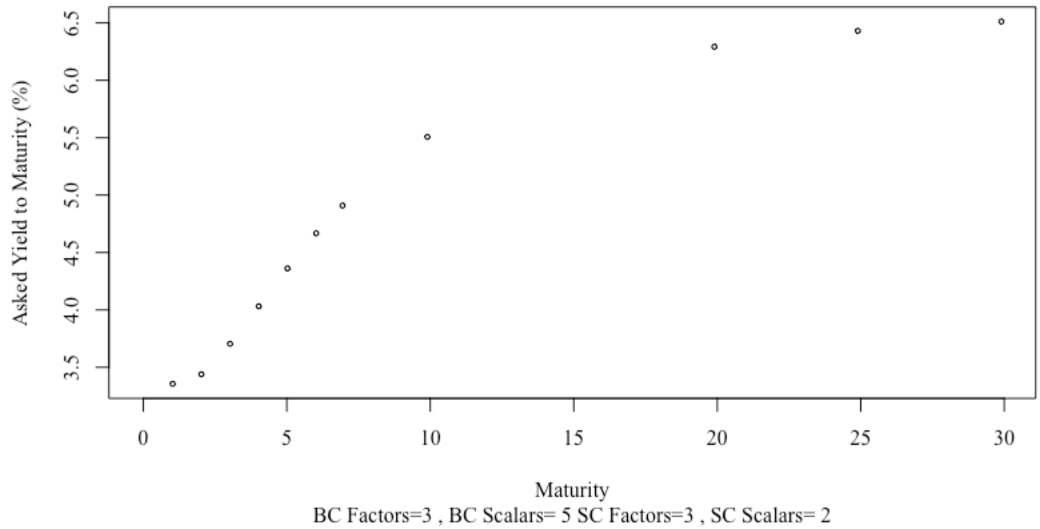
Once the LSC parameters are estimated, we can easily compute several values. Figure 7.3.3 presents the fitted yields as well as the spread. Clearly, the spread is the dominant driver of the all-in BB yield.

Figure 7.3.3. Fitted CMT yields and BB yields along with the implied spread



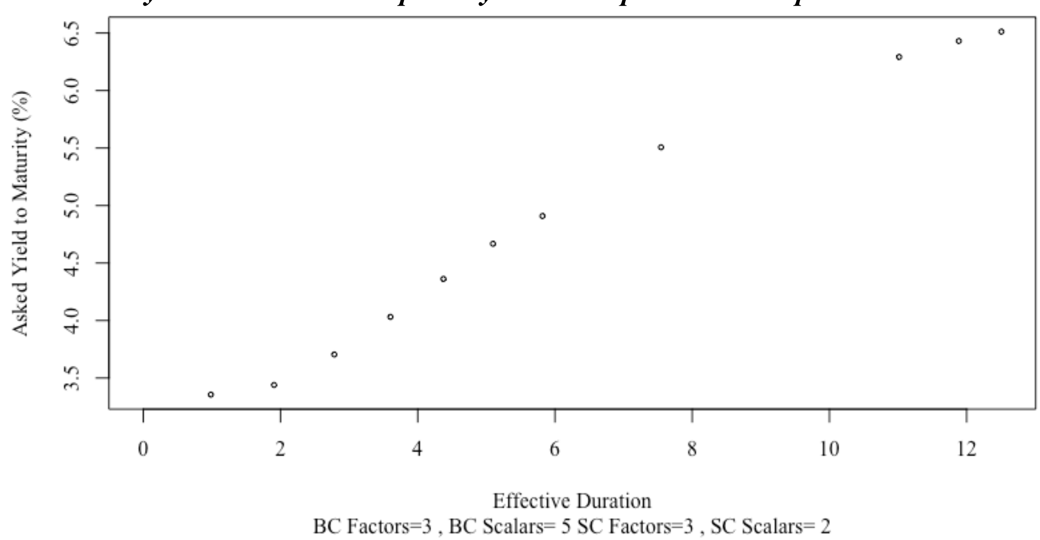
Based on the quantitative model and R code discussed below, we have both the base curve and spread curve estimated with the LSC model. Within this framework, we are ready to explore a host of static risk measures. The all-in implied asked yield to maturity for the BB bonds is reported in Figure 7.3.4.

Figure 7.3.4. Asked yield to maturities implied by the asked prices with respect to maturity



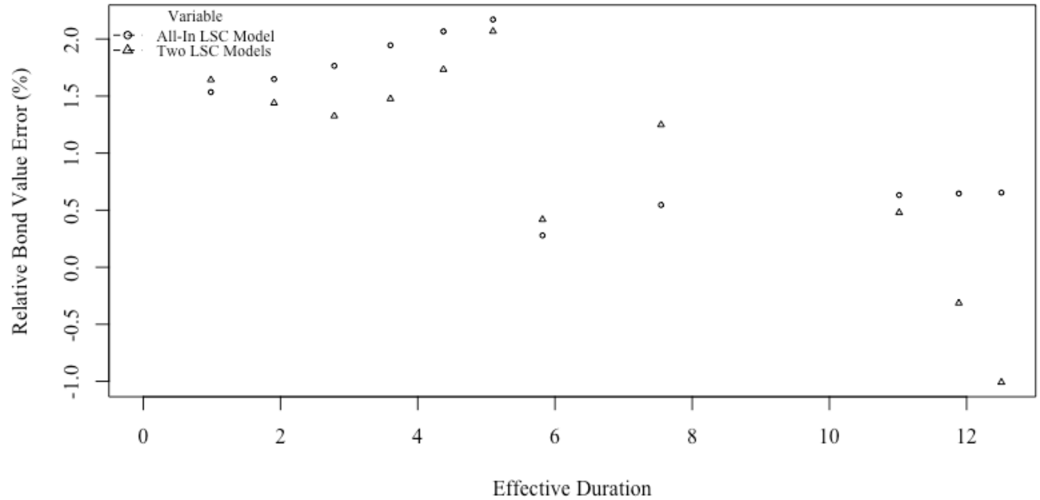
Note that there is no way to appraise which bond is trading for a lower price relative to the current market conditions. Figure 7.3.5 provide the implied asked yield to maturities with respect to duration.

Figure 7.3.5. Asked yield to maturities implied by the asked prices with respect to duration



With effective duration, the relationship is now closer to linear, it remains unclear which bonds are expensive or cheap. Figure 7.3.6 illustrates the relative bond value error in two ways. The all-in LSC model is defined as the natural log of the bond value based on the all-in BB curve divided by the actual ask price. The two LSC models is defined as the natural log of the bond value based on the fitting the base curve and then the spread curve divided by the actual ask price. Thus, the lower the relative bond value error the more valuable the bond relative to the curves. Note that in most cases the bonds are trading over the all-in BB curve indicating a mark-up due to trading costs. Further, fitting two LSC curves results in larger errors compared to fitting just the all-in curve.

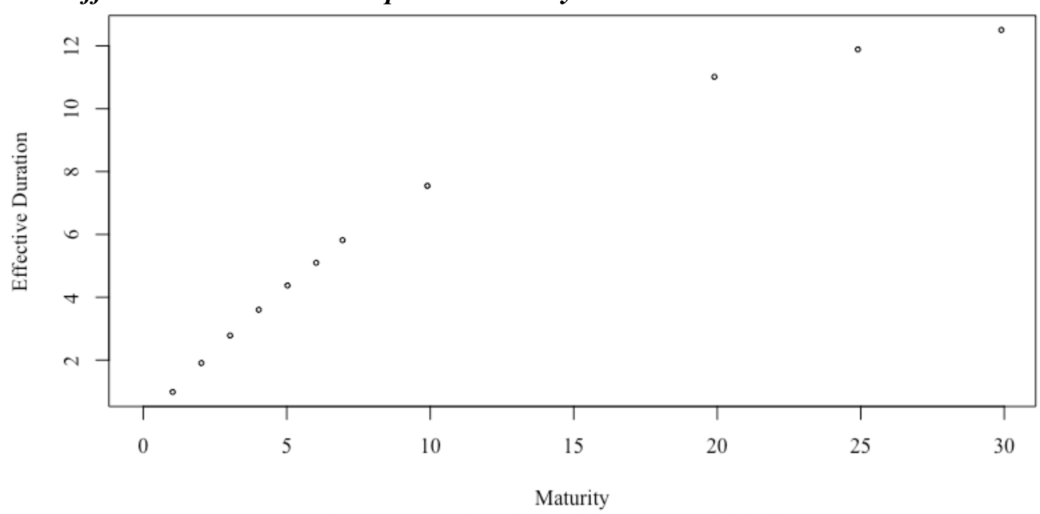
Figure 7.3.6. Relative bond value error with respect to duration



BC Factors=3 , BC Scalars= 5 SC Factors=3 , SC Scalars= 2

Figure 7.3.7 illustrates the well-known relationship between effective duration and maturity. The longer the maturity the lower the marginal increase in effective duration, especially for higher coupon bonds. Thus, from an effective duration perspective there is not much difference between 20-, 25-, and 30-year bonds.

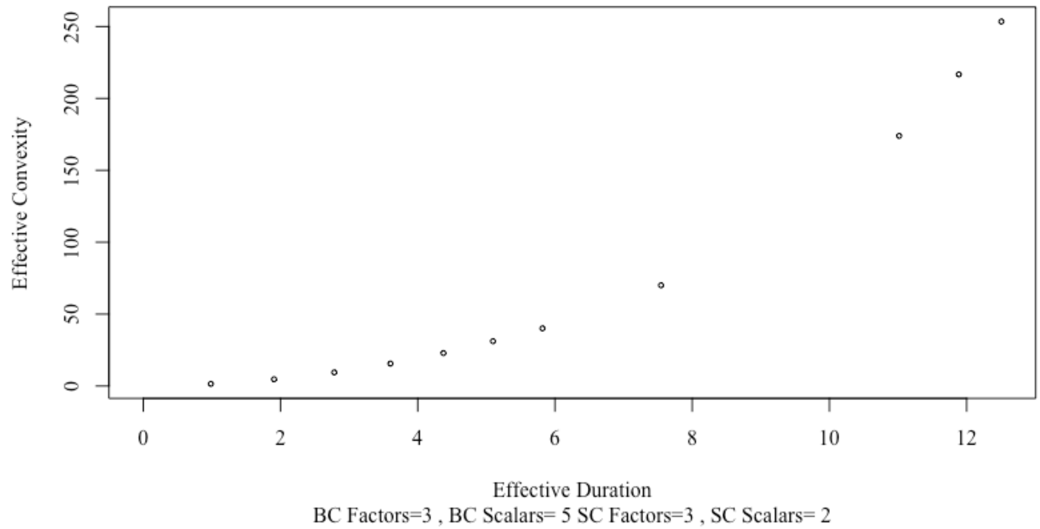
Figure 7.3.7. Effective duration with respect to maturity



BC Factors=3 , BC Scalars= 5 SC Factors=3 , SC Scalars= 2

Figure 7.3.8 shows that effective convexity has a convex relationship to effective duration. The higher the effective duration the higher the marginal increase in effective convexity.

Figure 7.3.8. Effective convexity with effective duration



We are now ready to turn to static risk measures based on decomposition of the LSC factors. While the number of different graphs that could be generated is vast, we focus on factor static risk measures based on a three factor LSC model with all-in scalar of 2, base curve scalar of 5, and spread curve scalar of 2. We purposely chose different scalars for the base curve and spread curve to illustration the flexibility of the LSC model approach.

Figure 7.3.9 illustrates four versions of duration. The traditional effective duration as well as the all-in, base curve, and spread curve level durations. The all-in level duration and effective durations are essentially equivalent. Recall that effective duration assumes a parallel shift in the yield curve and the all-in level duration essentially has the same effect. The base curve and spread curve level duration are essentially equivalent as it results in a parallel shift in the net of these two curves.

Figure 7.3.9. Level duration with maturity

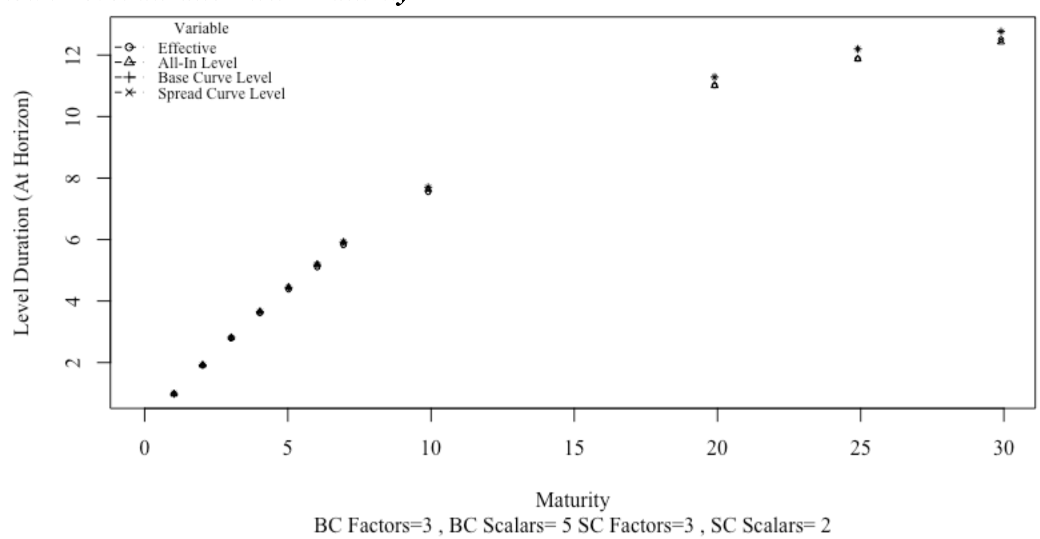


Figure 7.3.10 illustrates three versions of slope duration. Recall the traditional effective duration can only handle parallel shifts thus there is no comparable slope or curvature measures available. The all-in slope duration rises steeply and then plateaus. The base curve slope duration has the same pattern as level duration, but plateaus at a much lower value. The spread curve slope duration peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar.

Figure 7.3.10. Slope duration with maturity

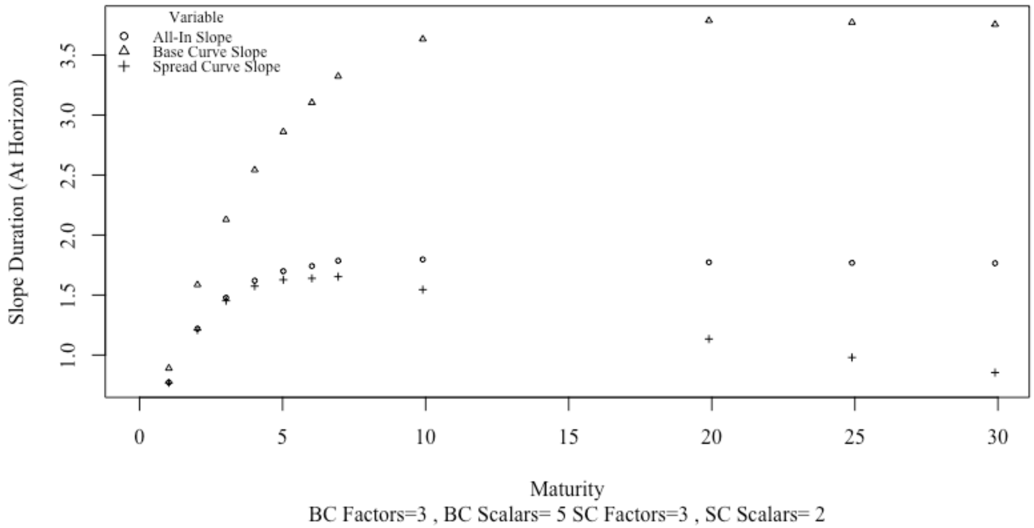


Figure 7.3.11 illustrates three versions of curvature1 duration. The all-in curvature1 duration rises steeply and then plateaus around 1.5. The base curve curvature1 duration has the same pattern but plateaus at a much higher value. The spread curve curvature1 duration peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar. Note the pattern is like the spread curve slope duration.

Figure 7.3.11. Curvature1 duration with maturity

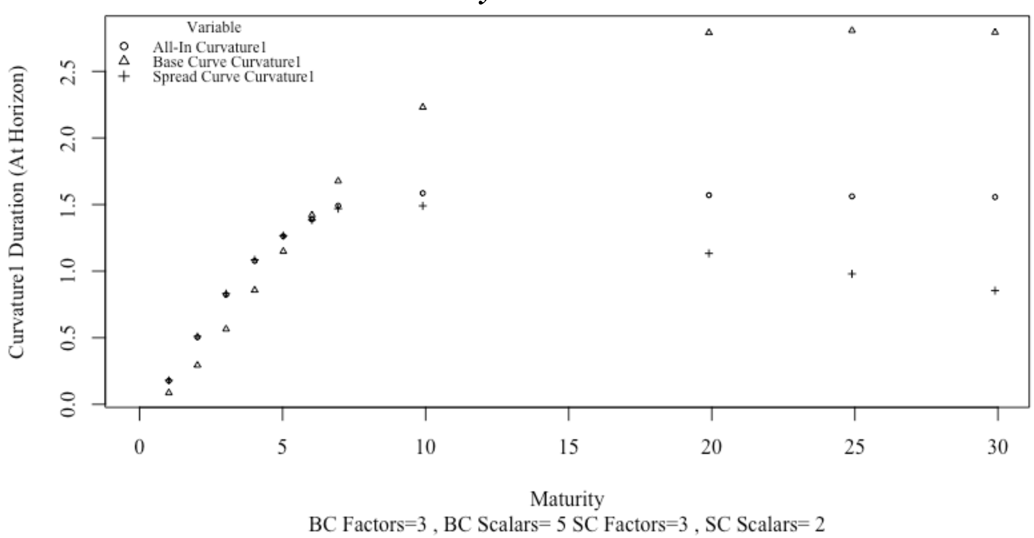


Figure 7.3.12 illustrates four versions of convexity. The traditional effective convexity as well as the all-in, base curve, and spread curve level convexities. The base curve and spread curve level convexities are essentially equivalent. The effective convexity is slightly below the base and spread curve versions and the all-in level is the lowest.

Figure 7.3.12. Level convexity with maturity

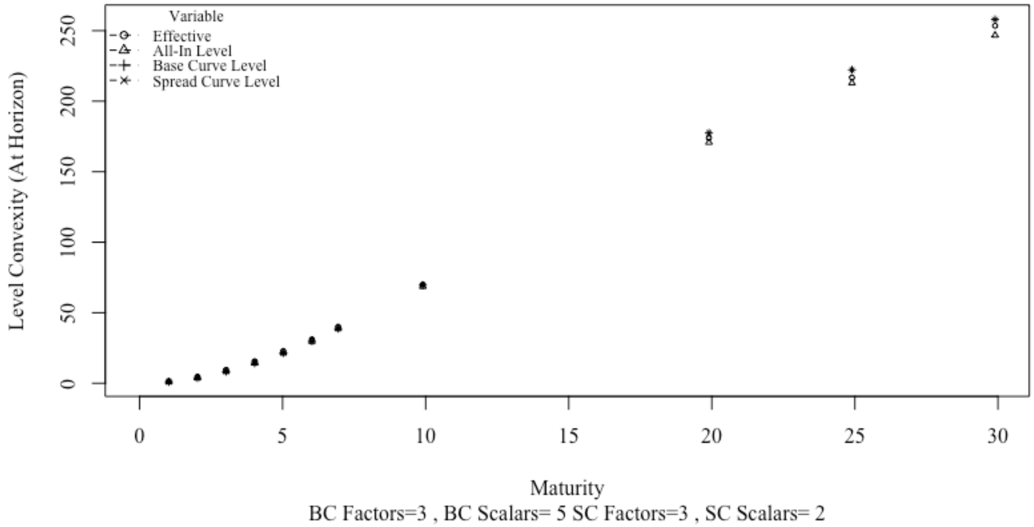


Figure 7.3.13 illustrates three versions of slope convexity. As with duration, the traditional effective convexity can only handle parallel shifts thus there is no comparable slope or curvature measures available. The base curve slope convexity rises steeply and then plateaus. The all-in slope convexity has the same pattern as base curve slope convexity but plateaus at a much lower value. The spread curve slope convexity peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar.

Figure 7.3.13. Slope convexity with maturity

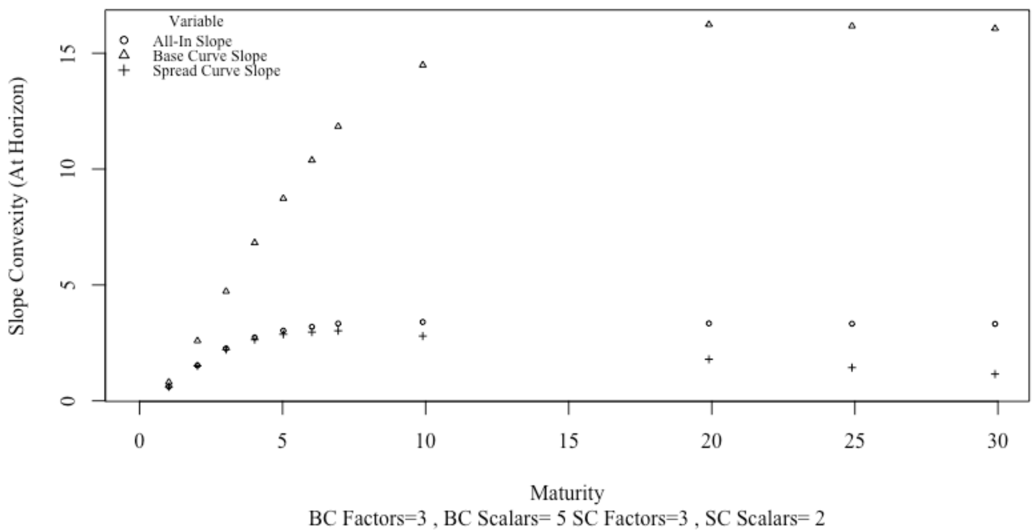
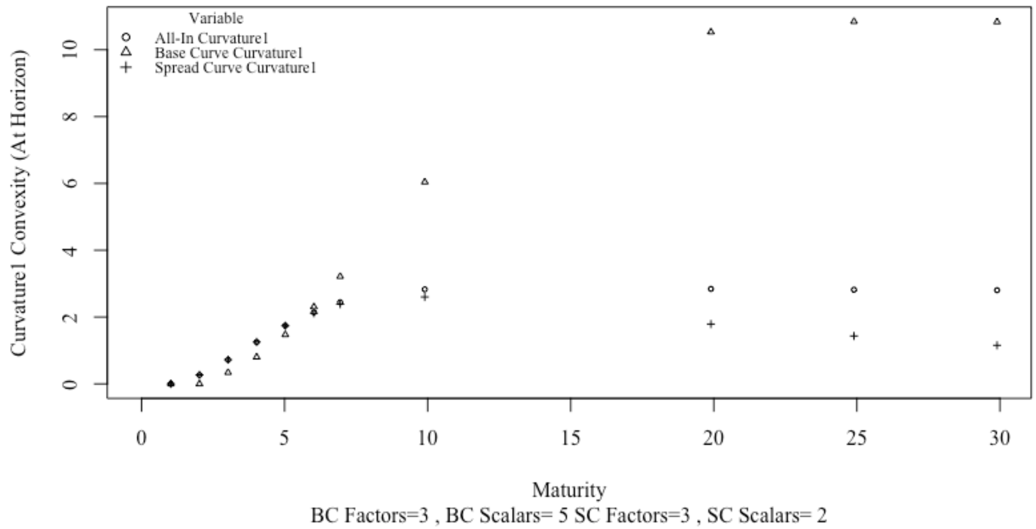


Figure 7.3.14 illustrates three versions of curvature1 convexity. The base curve curvature1 convexity rises steeply and then plateaus. The all-in curvature1 convexity has the same pattern as base curve curvature1 but plateaus at a much lower value. The spread curve curvature1 convexity peaks due to being layered on top of the base curve that already is being influenced by the 5.0 scalar. Note the pattern is like the spread curve slope duration.

Figure 7.3.14. Curvature1 convexity with maturity



The next three figures illustrate cross convexity results.

Figure 7.3.15. Cross convexity level/slope with maturity

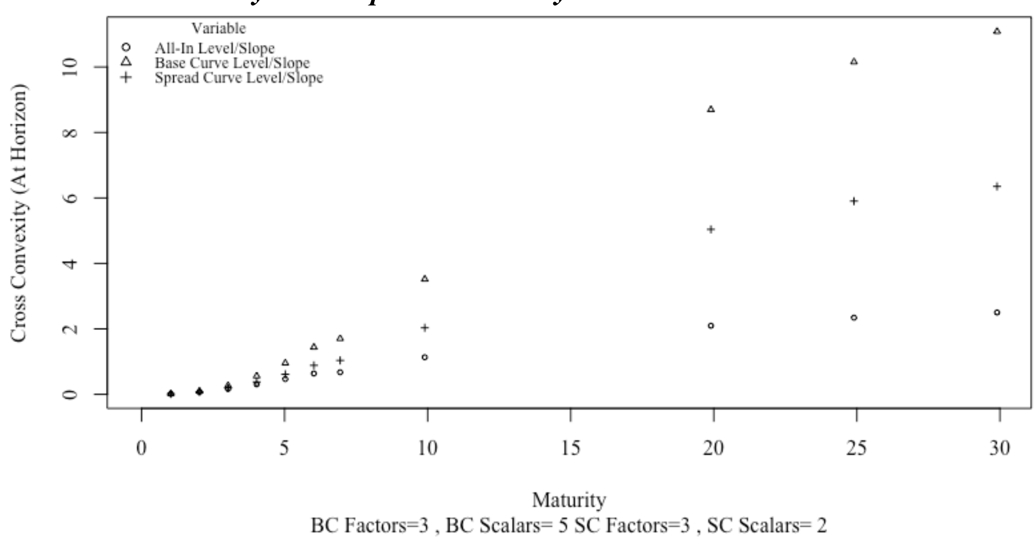


__Figure 7.3.16. Cross convexity level/curvature1 with maturity__

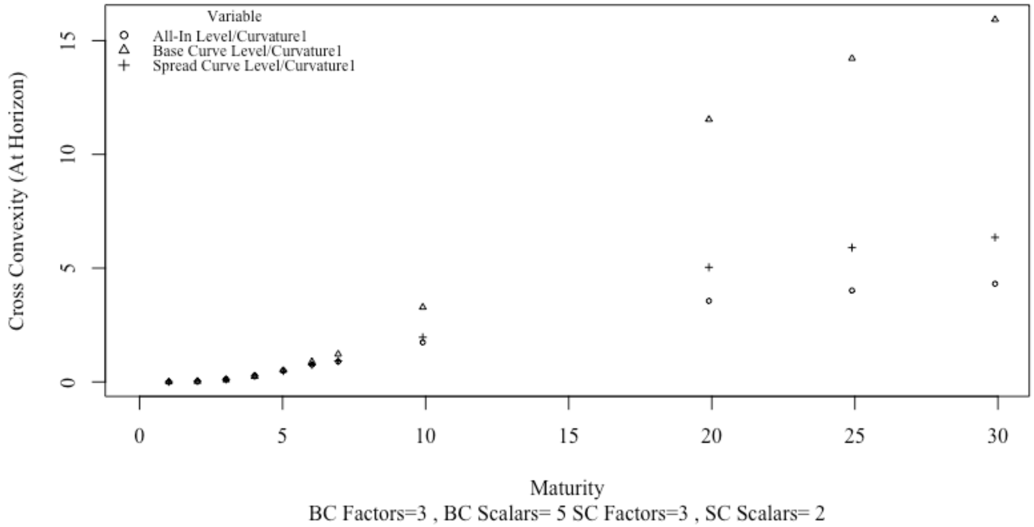


__Figure 7.3.17. Cross convexity slope/curvature1 with maturity__

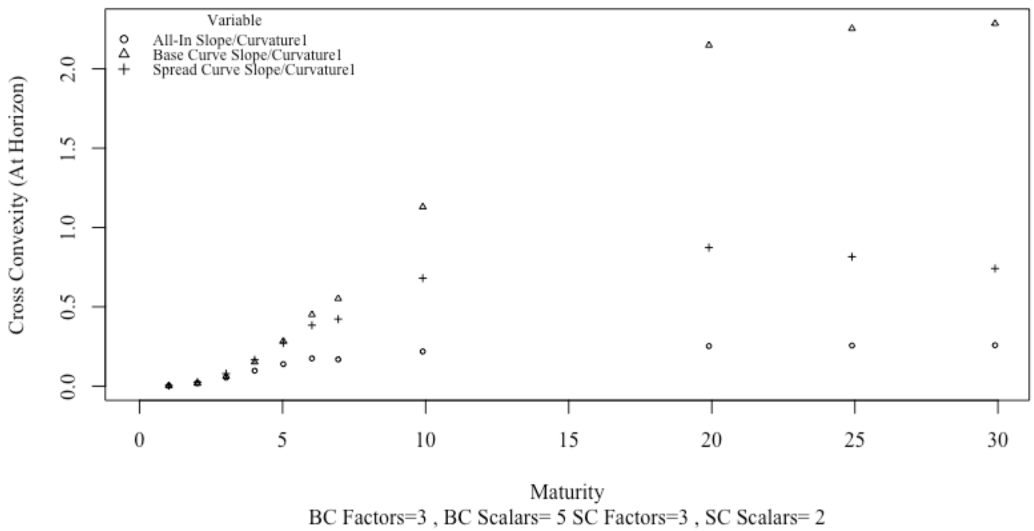
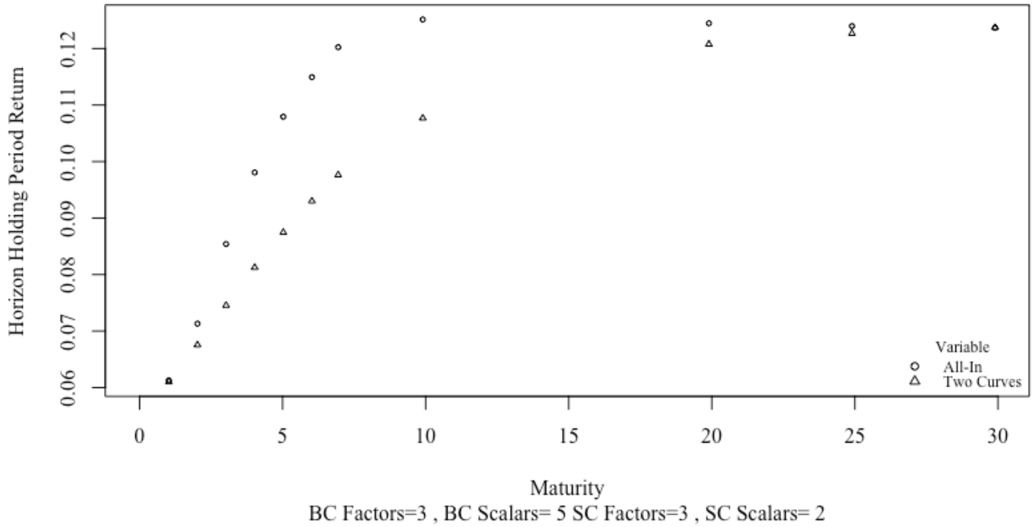


Figure 7.3.18 provides estimates of the horizon holding period return under the two curve fitting methodologies. The all-in curve is more sensitive to maturity than the two curves approach.

Figure 7.3.18. Horizon holding period return with maturity



We turn now to briefly review the technical aspects of spreads.

Quantitative finance materials

With credit risky bonds, we can apply the same LSC model approach to credit spreads. We now explore the technical details that a similar in construct to rates.

Advanced bond static risk measures: Applying the LSC model with default risk[1]

Based on the notation provided in Module 7.1, recall

$$sp_{i,t}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}. \tag{7.3.1}$$

Thus, the framework allows for multiple spread curves with as many individual factors as desired. Our focus here will be just one spread curve (BB yields over CMT) for illustration purposes.

Recall the bond value can be estimated at t with the LSC model as

$$V_t \equiv \sum_{i=1}^{N_t} CF_{i,t} DF_{i,t} = \sum_{i=0}^{N_t} CF_i e^{-\left(r_{i,t}^{LSC} + sp_{i,t}^{LSC}\right)\tau_i}. \tag{7.3.2}$$

At time $t + \Delta$ assuming only the spread changes, we have

$$\tilde{V}_{t+\Delta}^{sp} \equiv \sum_{i=1}^{N_{t+\Delta}} CF_{i,t+\Delta} D\tilde{F}_{i,t+\Delta}^{sp} = \sum_{i=0}^{N_{t+\Delta}} CF_i e^{-\left(r_{i,t+\Delta}^{LSC} + s\tilde{p}_{i,t+\Delta}^{LSC}\right)(\tau_i - \Delta)}. \text{ (LSC Spread Curve at } t+\Delta) \tag{7.3.3}$$

where

$$D\tilde{F}_{i,t+\Delta}^{sp} \equiv e^{-\left(r_{i,t+\Delta}^{LSC} + s\tilde{p}_{i,t+\Delta}^{LSC}\right)(\tau_i - \Delta)}. \tag{7.3.4}$$

Most data services that provide credit spread information report the all-in rate for different credit ratings, such as BB. Thus, the input is not the actual spread, rather the spread must be computed. There are numerous approaches to make these estimates. The approach we take is to first estimate the base curve via the LSC model. Thus, we have the functional form for the base curve, $r_{i,t+\Delta}^{LSC}$, as well as the spread adjusted all-in rate denoted here as $\hat{r}_{i,t+\Delta}$ for selected maturities. We therefore can estimate the inputted spreads based on the following relationship.

$$sp_{i,t} = \hat{r}_{i,t} - r_{i,t}^{LSC}. \tag{7.3.5}$$

[1]The next several sections are based on Brooks and Upton (2017) and Brooks (2017).

With these estimated spreads, we can compute the functional form for the spread curve, $sp_{i,t}^{LSC}$.

Recall the bond spread holding period return can be expressed as

$$\tilde{R}_{\Delta}^{sp} \equiv \frac{\tilde{V}_{t+\Delta}^{sp} - V_{t+\Delta}^{LSC}}{V_{t+\Delta}^{LSC}}. \tag{7.3.6}$$

Also, the spreads are computed in three ways,

$$sp_{i,t}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}, \text{ (Fit at time } t\text{, analyzed at time } t) \tag{7.3.7}$$

$$sp_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} f_{j,t}^{sp}, \text{ (Fit at time } t\text{, analyzed at time } t+\Delta) \tag{7.3.8}$$

$$\tilde{sp}_{i,t+\Delta}^{LSC} \equiv \sum_{j=0}^{N_F^{sp}} x_{i,j}^{sp} \tilde{f}_{j,t+\Delta}^{sp}. \text{ (Fit at time } t+\Delta\text{, analyzed at time } t+\Delta) \tag{7.3.9}$$

Thus,

$$\Delta\tilde{f}_{j,t+\Delta}^{sp} = \tilde{f}_{j,t+\Delta}^{sp} - f_{j,t}^{sp} \text{ or (Spread)} \tag{7.3.10}$$

Interpreting LSC factor static risk measures

We focus now on interpreting the static risk measures defined above within the context of bond HPRs. From the definition of bond HPR based on the LSC framework, we have

$$\tilde{R}_{\Delta}^{Unknown} = \tilde{R}_{\Delta}^{LSC} = \frac{\tilde{V}_{t+\Delta}^{LSC} - V_{t+\Delta}^{LSC}}{V_{t+\Delta}^{LSC}} \cong R\tilde{C}_{FD}^{r} + R\tilde{C}_{FC}^{r} + R\tilde{C}_{FCC}^{r} + R\tilde{C}_{FD}^{sp} + R\tilde{C}_{FC}^{sp} + R\tilde{C}_{FCC}^{sp} + R\tilde{C}_{FCC}^{r,sp}, \tag{7.3.11}$$

where *RC* denotes the return contribution, *r* denotes the base rate, *sp* denotes the spread, *FD* denotes the LSC factor durations, *FC* denotes the LSC factor convexities, and *FCC* denotes the LSC factor cross-convexities.

Each of the above return contributions can be further decomposed in the following manner,

$$R\tilde{C}_{FD}^{sp} = R\tilde{C}_{FD}^{sp,L} + R\tilde{C}_{FD}^{sp,S} + R\tilde{C}_{FD}^{sp,C}, \tag{7.3.12}$$

$$R\tilde{C}_{FC}^{sp} = R\tilde{C}_{FC}^{sp,L} + R\tilde{C}_{FC}^{sp,S} + R\tilde{C}_{FC}^{sp,C}, \tag{7.3.13}$$

$$R\tilde{C}_{FCC}^{sp} = R\tilde{C}_{FCC}^{sp,L,S} + R\tilde{C}_{FCC}^{sp,S,C} + R\tilde{C}_{FCC}^{sp,L,C}, \text{ and} \tag{7.3.14}$$

$$\begin{aligned} R\tilde{C}_{FCC}^{r,sp} &= R\tilde{C}_{FCC}^{r(L),sp(L)} + R\tilde{C}_{FCC}^{r(L),sp(S)} + R\tilde{C}_{FCC}^{r(L),sp(C)} \\ &+ R\tilde{C}_{FCC}^{r(S),sp(L)} + R\tilde{C}_{FCC}^{r(S),sp(S)} + R\tilde{C}_{FCC}^{r(S),sp(C)} \\ &+ R\tilde{C}_{FCC}^{r(C),sp(L)} + R\tilde{C}_{FCC}^{r(C),sp(S)} + R\tilde{C}_{FCC}^{r(C),sp(C)} \end{aligned}. \tag{7.3.15}$$

Note that for most applications, most return contributions will be negligible. Thus, the actual analysis will be more straightforward. When developing software solutions however, it is better to have a thorough design that can be simplified by the user.

Recall the return contributions for spreads are as follows,

$$R\tilde{C}_{FD}^{sp,L} \equiv -FD_L^{sp} \Delta\tilde{f}_L^{sp}, \tag{7.3.16}$$

$$R\tilde{C}_{FD}^{sp,S} \equiv -FD_S^{sp} \Delta\tilde{f}_S^{sp}, \tag{7.3.17}$$

$$R\tilde{C}_{FD}^{sp,C} \equiv -FD_C^{sp} \Delta\tilde{f}_C^{sp}, \tag{7.3.18}$$

$$R\tilde{C}_{FC}^{sp,L} \equiv \frac{1}{2} FC_{FC}^{sp,L} \left(\Delta\tilde{f}_L^{sp}\right)^2, \tag{7.3.19}$$

$$R\tilde{C}_{FC}^{sp,S} \equiv \frac{1}{2} FC_{FC}^{sp,S} \left(\Delta\tilde{f}_S^{sp}\right)^2, \tag{7.3.20}$$

$$R\tilde{C}_{FC}^{sp,C} \equiv \frac{1}{2} FC_{FC}^{sp,C} \left(\Delta \tilde{f}_C^{sp}\right)^2, \tag{7.3.21}$$

$$R\tilde{C}_{FCC}^{sp,L,S} \equiv FCC_{FCC}^{sp,L,S} \Delta \tilde{f}_L^{sp} \Delta \tilde{f}_S^{sp}, \tag{7.3.22}$$

$$R\tilde{C}_{FCC}^{sp,L,C} \equiv FCC_{FCC}^{sp,L,C} \Delta \tilde{f}_L^{sp} \Delta \tilde{f}_C^{sp}, \text{ and} \tag{7.3.23}$$

$$R\tilde{C}_{FCC}^{sp,S,C} \equiv FCC_{FCC}^{sp,S,C} \Delta \tilde{f}_S^{sp} \Delta \tilde{f}_C^{sp}. \tag{7.3.24}$$

For completeness, we also provide the explicit factor durations, factor convexities, and factor cross convexities for spreads.

$$FD_L^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right) w_{i,t+\Delta}^{LSC}, \tag{7.3.25}$$

$$FD_S^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right) x_{i,S} w_{i,t+\Delta}^{LSC}, \tag{7.3.26}$$

$$FD_C^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right) x_{i,C} w_{i,t+\Delta}^{LSC}, \tag{7.3.27}$$

$$FC_L^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 w_{i,t+\Delta}^{LSC}, \tag{7.3.28}$$

$$FC_S^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 x_{i,S}{}^2 w_{i,t+\Delta}^{LSC}, \tag{7.3.29}$$

$$FC_C^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 x_{i,C}{}^2 w_{i,t+\Delta}^{LSC}, \tag{7.3.30}$$

$$FCC_{L,S}^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 x_{i,S} w_{i,t+\Delta}^{LSC}, \tag{7.3.31}$$

$$FCC_{S,C}^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 x_{i,S} x_{i,C} w_{i,t+\Delta}^{LSC}, \text{ and} \tag{7.3.32}$$

$$FCC_{L,C}^{sp} = \sum_{i=0}^{N_t} \left(\tau_i - \Delta\right)^2 x_{i,C} w_{i,t+\Delta}^{LSC}. \tag{7.3.33}$$

Again, for most applications, most return contributions will be negligible. Thus, the actual analysis will be more straightforward. When developing software solutions however, it is better to have a thorough design that can be simplified by the user.

We are now ready to extend the analysis of Module 7.2 to include spreads in R code.

Summary

We extended various aspects of traditional bond static risk measures to credit risky bonds. With this foundation, we then moved to advanced bond static risk measures of spreads based on an application of the LSC model. Within a detailed bond holding period return decomposition, we reviewed numerous new measures of related to spreads. The module concludes with selected explanations of selected R code.

References

See Module 7.2.