Chapter 5. Valuation Options

"*The price of an article is charged according to difference in location, time or risk to which one is exposed in carrying it from one place to another or in causing it to be carried. Neither purchase nor sale according to this principle is unjust.*" St. Thomas Aquinas (1264)[1]

Introduction

In this chapter, we explore various valuation issues related to option contracts. The key to understanding this chapter and why these insights have drastically changed modern finance is to understand the thought process of arbitrageurs. Module 1 introduces static arbitrage involving initial transactions and potentially transactions at the option expiration. Modules 2-5 and 7 are based on dynamic arbitrage involving transactions potentially over the option life.

Module 1 explores option boundary conditions along with put-call parities and examines these relationships with selected option data. The key insight is that observed U.S. stock option prices generally behave within the proposed boundaries as well as consistent with put-call parity. The driving economic force for these relationships is static arbitrage.

Module 2 focuses on applying the traditional binomial framework that converges to the lognormal distribution consistent with geometric Brownian motion. This model is identified as the GBM-based binomial option valuation model or GBM-BOVM. Based on the theoretical work presented as well as R code reviewed, several graphical results are illustrated and selected insights identified. The driving economic force for this valuation model is dynamic arbitrage that occurs at each discrete time step.

Module 3 focuses on applying a brand new binomial framework that converges to the normal distribution consistent with arithmetic Brownian motion. This model is identified as the ABM-based binomial option valuation model or ABM-BOVM. Again, based on the new theoretical results presented as well as R code introduced, several graphical results are illustrated and selected insights identified. Further, we provide a detailed look at the contrast between GBM-BOVM and ABM-BOVM. Again, the driving economic force for this valuation model is dynamic arbitrage that occurs at each discrete time step. The technical specifications for the dynamic arbitrage are different between GBM- and ABM-based models, but the philosophical perspective is the same.

Module 4 focuses on the continuous time version of GBM-BOVM identified simply as GBMOVM. This framework is consistent with the traditional Black–Scholes–Merton option valuation model (BSMOVM). Detailed derivations are provided as well as graphical analysis based on provided R code. The driving economic force for this valuation model is dynamic arbitrage that theoretically occurs continuously.

Module 5 focuses on the continuous time version of ABM-BOVM identifies simply as ABMOVM. This framework can be viewed as pre-dating the 1973 BSMOVM. That is, ABMOVM is within the same model family as the historic Bachelier 1900 model. Detailed derivations are provided as well as graphical analysis

[1]For this and many other great quotes, see Don Chance's website: http://www.bus.lsu.edu/academics/finance/faculty/dchance/MiscProf/DerivaQuote/Qt19.htm.

based on provided R code. Again, the driving economic force for this valuation model is dynamic arbitrage that theoretically occurs continuously. Like the binomial models, the technical specifications for the dynamic arbitrage are different between GBM- and ABM-based models, but the philosophical perspective remains the same.

Module 6 addresses techniques for solving implied parameters within option models. Here we do not derive new valuation models, rather we illustrate using these models to impute implied parameter values. Often, the quantitative analyst is more interested in implied parameters, such as implied volatility, than they are interested in the theoretical option value itself.

Module 7 extends option valuation to the compound option framework with GBM identified simply as COVM. This framework is consistent with the assumptions of the traditional BSMOVM except the option is assumed to be compound in nature. For example, rather than viewing a stock option as simply a plain vanilla call or put on a stock, the stock option is viewed as an option on stock that itself is an option on the firm. Detailed derivations are provided as well as graphical analysis based on provided R code. Again, the driving economic force for this valuation model as with all the others in this chapter is arbitrage. In this case, it is dynamic arbitrage that theoretically occurs continuously.