Module 5.6

Valuation GBM-Based
Implied Option Parameters

1

Overview

- Introduce method to solve for implied parameters within the GBMOVM
- Illustrate R code snippets
- Explore equity as a call option

2

Central Finance Concepts

- Setting bounds
- Computing option implied parameters often requires the minimum and maximum bounds for the parameters
  - User-defined (*)
  - Hard coded
- Tolerance level (hard coded)

3

Quantitative Finance Materials

- Implied volatility
- Implied stock price
  - Equity as call option on firm
  - Underlying is firm value
  - Volatility is firm volatility
- Other implied parameters: Strike price, interest rate, dividend yield and time to maturity

4

Implied Parameters (GBMOVM)

- Solve for embedded parameters

$$C_{Model} = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$$

$$P_{Model} = Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

$$N(d) = \int_{-\infty}^{d} \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

5

Implied Volatility of Model (IVM)

- Given option market prices, solve for model implied volatility

$$C_{Market} = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$$

$$P_{Market} = Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma_{Model}^2}{2}\right)T}{\sigma_{Model}\sqrt{T}}$$

$$d_2 = d_1 - \sigma_{Model}\sqrt{T}$$

6

IVM Approximations

$$\sigma_{Model} \cong \frac{C_{Market}\sqrt{2\pi}}{S\sqrt{T}} \text{ for S=X (Brenner and Subrahmanyam, 1988)} \quad (5.6.10)$$

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}\left(C_{Market} - 0.5(S-X)\right)}{\sqrt{T}(S+X)} + \sqrt{2\pi\left(\frac{C_{Market} - 0.5(S-X)}{S+X}\right)^2 - \alpha\left(\frac{S-X}{S+X}\right)^2} \text{ for } S \neq X$$

(α=4, Corrado and Miller, 1996) (5.6.11)

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}}{\sqrt{T}(S+X)}\left[C_{Market} - 0.5(S-X) + \sqrt{\left(C_{Market} - 0.5(S-X)\right)^2 - \left(\frac{S-X}{\pi}\right)^2}\right] \text{ for } S \neq X$$

(α=2, Corrado and Miller, 1996 "improved quadratic formula") (5.6.12)

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}}{2\sqrt{T}(S+X)}\left[2C_{Market} + X - S + \sqrt{\left(2C_{Market} + X - S\right)^2 - 1.85\frac{(S+X)(X-S)^2}{\pi\sqrt{XS}}}\right]$$

(Hallerbach, 2004) (5.6.13)

7

IVM Approx. Inaccurate

Table 5.6.1. Implied Volatility Estimates Based on Various Methods (Correct σ = 30%)

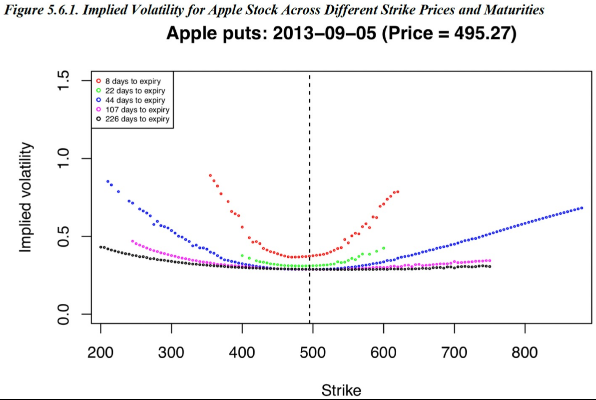
Strike	C(Model)	BS, 1988	CM4, 1996	CM2, 1996	H, 2004
\$85.12	\$22.833	57.25%	34.13%	40.69%	38.60%
\$90.12	\$19.617	49.19%	35.69%	38.27%	37.38%
\$95.12	\$16.728	41.95%	36.03%	36.62%	36.41%
\$100.12	\$14.166	35.52%	35.65%	35.65%	35.65%
\$105.12	\$11.919	29.89%	29.89%	29.89%	29.89%
\$110.12	\$9.970	25.00%	33.06%	35.46%	34.63%
\$115.12	\$8.294	20.80%	30.47%	36.09%	34.29%
\$120.12	\$6.866	17.22%	25.39%	37.13%	34.00%
\$125.12	\$5.659	14.19%	#NUM!	38.53%	33.71%

S = \$100, r = 5%, δ = 0%, σ = 30%, T = 1 year, and initial $X = Se^{rT}$ =105.12 (increment by 5 above and below).

 8

8

Figure 5.6.1. Implied Volatility for Apple Stock Across Different Strike Prices and Maturities



9

Implied Parameters

```
GBMDYOptionImpliedVolatility <- function(B, inputOptionValue){
  TestFunctionGBMDYOptionImpliedVolatility<-function(testImpliedVolatility,B,
    inputOptionValue){
      B$Volatility = testImpliedVolatility
      return( abs(inputOptionValue - GBMOptionValue(B))^2 )
  }
  solution = optimize(TestFunctionGBMDYOptionImpliedVolatility, B,
    inputOptionValue, interval = c(B$ImpliedLowerBound, B$ImpliedUpperBound),
    tol = .Machine$double.eps^0.25)
  ImpliedVolatility = solution$minimum
  B$Volatility = ImpliedVolatility
  Difference = inputOptionValue - GBMOptionValue(B)
  if (abs(Difference) < 0.01)return(ImpliedVolatility)
  else return(NA)
}
```

10

Simple Example

- S=\$100; X=\$100; r=5%; T=1; σ=30%
- C=14.23125; LB=0; UB=1,000

```
> # Test implied volatility
> ImpliedCallVolatility = GBMDYOptionImpliedVolatility(GBMInputData,
+   inputOptionValue)
> CallValue = GBMOptionValue(GBMInputData)
> ImpliedCallVolatility; CallValue
[1] 29.99999
[1] 14.23125
```

 11

11

Implied Stock Price

- Given option market prices, solve for model implied stock price

$$C_{Market} = S_{Model}e^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$$

$$P_{Market} = Xe^{-rT}N(-d_2) - S_{Model}e^{-\delta T}N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_{Model}}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

 12

12

Equity as Call on Firm Value

- *FV* – firm value
- *T* – duration of the firm debt is the time to maturity
- *DPV* – debt par value equivalent to the strike price
- *σ* – volatility of firm assets
- Solve for firm value (*FV*)

 13

13

Implied Firm Value

- GBMOVM approach to equity value

$$EV_0 = FV_0 N(d_1) - DPVe^{-rT} N(d_2)$$

- Where

$$d_1 = \frac{\ln\left(\frac{FV_0}{DPV}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

 14

14

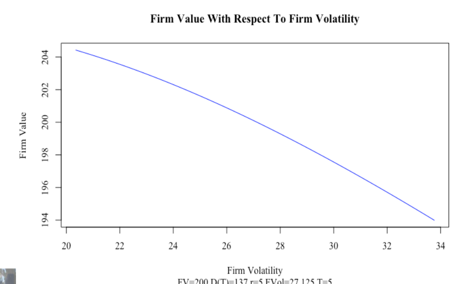
Simple Example

- EV=$100; X=$137; r=5%; T=5; ⊠=27.125%
- LB=0; UB=1,000; FV = ?

```
> # Example of solving for implied firm value
> GBMInputData$StockPrice <- -99    # Firm value
> GBMInputData$StrikePrice <- 137.0 # Debt par value
> GBMInputData$InterestRate <- 5.0  # Risk free rate
> GBMInputData$DividendYield <- 0.0
> GBMInputData$TimeToMaturity <- 5.0  # Years to debt maturity
> GBMInputData$Volatility <- 27.125   # Volatility of firm
> GBMInputData$Type <- 1 # Call option
> inputEquityValue = 100.0  # Equity value
> ImpliedFirmValue = GBMDYOptionImpliedStockPrice(GBMInputData, inputEquityValue)
> ImpliedFirmValue
[1] 200.0061
```

15

Firm Value and Volatility



 16

16

Implied Strike Price

- Given option market prices, solve for model implied strike price

$$C_{Market} = Se^{-\delta T} N(d_1) - X_{Model} e^{-rT} N(d_2)$$

$$P_{Market} = X_{Model} e^{-rT} N(-d_2) - Se^{-\delta T} N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{X_{Model}}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

 17

17

Implied Interest Rate

- Given option market prices, solve for model implied interest rate

$$C_{Market} = Se^{-\delta T} N(d_1) - Xe^{-r_{Model} T} N(d_2)$$

$$P_{Market} = Xe^{-r_{Model} T} N(-d_2) - Se^{-\delta T} N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_{Model} - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

 18

18

Implied Dividend Yield

- Given option market prices, solve for model implied dividend yield

$$C_{Market} = Se^{-\delta_{Model}T}N(d_1) - Xe^{-rT}N(d_2)$$
$$P_{Market} = Xe^{-rT}N(-d_2) - Se^{-\delta_{Model}T}N(-d_1)$$
$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta_{Model} + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

 19

19

Implied Time to Maturity

- Given option market prices, solve for model implied time to maturity

$$C_{Market} = Se^{-\delta T_{Model}}N(d_1) - Xe^{-rT_{Model}}N(d_2)$$
$$P_{Market} = Xe^{-rT_{Model}}N(-d_2) - Se^{-\delta T_{Model}}N(-d_1)$$
$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T_{Model}}{\sigma\sqrt{T_{Model}}}$$
$$d_2 = d_1 - \sigma\sqrt{T_{Model}}$$

 20

20

Puts and Time to Maturity

- Lower Bound: max[0,*PV*(*X*) – *S*]
  - Longer time to maturity, lower bound declines
  - Because European-style, at some point more time is less valuable (two solutions)

```
> GBMInputData$ImpliedLowerBound <- 0.0
> GBMInputData$ImpliedUpperBound <- 1000
> GBMInputData$Type = -1        # Put
> PutValue = GBMInputData$OptionValue = GBMOptionValue(GBMInputData)
> GBMInputData$TimeToMaturity = -99
> ImpliedPutTimeToMaturity = GBMDYOptionImpliedTimeToMaturity(GBMInputData,
+   inputOptionValue)
> ImpliedPutTimeToMaturity; PutValue
[1] 20.29694
[1] 9.354197
```

21

Constrain Boundaries

- Lower the upper bound

```
> GBMInputData$ImpliedLowerBound <- 0.0
> GBMInputData$ImpliedUpperBound <- 10
> GBMInputData$Type = -1        # Put
> GBMInputData$TimeToMaturity = 1.0
> PutValue = GBMInputData$OptionValue = GBMOptionValue(GBMInputData)
> GBMInputData$TimeToMaturity = -99
> ImpliedPutTimeToMaturity = GBMDYOptionImpliedTimeToMaturity(GBMInputData,
+   inputOptionValue)
> ImpliedPutTimeToMaturity; PutValue
[1] 0.9999998
[1] 9.354197
```

 22

22

Summary

- Introduced method to solve for implied parameters within the GBMOVM
- Illustrated R code snipets
- Explored equity as a call option
  - Compute implied firm value
  - Explored relation between firm value and volatility when equity value is fixed

 23

23