Module 5.6: Implied Option Parameters

Learning objectives

- Explain how to estimate implied option parameters, based on the Black, Scholes, Merton option valuation model
- Illustrate various applications of this procedure

See *5.6 Implied GBM Option Parameters Test.R*. The functions are provided in a separate file, *GBMOVM and Extended Functions.R*.

Executive summary

This module brings together two modules previously covered, 3.7 Embedded Parameters and 5.4 GBM-Based OVM. The value of the modular approach is illustrated here by the flexibility to compute any implied parameter, even those not typically needing estimation. In finance, however, we often encounter unusual quantitative needs and having the capacity to compute any embedded parameter is helpful.

Central finance concepts

The key task addressed in this module is computing implied parameters. We use the GBMOVM for illustration. The task of computing implied parameters often requires setting bounds which we now discuss.

Setting Bounds

When developing software, often there are trade-offs between providing user flexibility and hiding complexity. For example, solving for embedded parameters often requires establishing lower and upper bounds. The exercise price cannot be negative, but interest rates can be negative. With put options, implied time to maturity may have multiple solutions. Recall that the maximum payoff is the exercise price; hence, as time to maturity increases there is a tradeoff between the time value effect and the discounting effect.

In the R code presented here, we give the user the flexibility to change the lower and upper bounds when necessary. We do not give the user discretion on setting the tolerance level when deciding whether to return the solution or *NA*. At times however it may be better to hide various decisions from the end user.

We now illustrate computing implied parameters of the GBMOVM.

Quantitative finance materials

Although atypical, we document how to compute all the implied parameters of the GBMOVM. We address these computations one at a time.

Implied volatility

The GBMOVM can be represented as (where "Model" denotes the theoretical model price)

$$C_{Model} = Se^{-\delta T} N\left(d_1\right) - Xe^{-rT} N\left(d_2\right) \tag{5.6.1}$$

$$P_{Model} = Xe^{-rT} N\left(-d_2\right) - Se^{-\delta T} N\left(-d_1\right) \tag{5.6.2}$$

where N(d) is the area under the standard cumulative normal distribution up to d (see the table nearby), or

$$N\left(d\right) = \int_{-\infty}^{d} \frac{e^{-x^2/2}}{\sqrt{2\pi}} \tag{5.6.3}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{5.6.4}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{5.6.5}$$

Implied volatility is the model stock volatility that results in the market option price ("Market" denotes observed market option price and not the theoretical "Model" option price). Thus we have

$$C_{Market} = Se^{-\delta T} N\left(d_1\right) - Xe^{-rT} N\left(d_2\right) \tag{5.6.6}$$

$$P_{Market} = Xe^{-rT} N\left(-d_2\right) - Se^{-\delta T} N\left(-d_1\right) \tag{5.6.7}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \delta + \frac{\sigma^2_{Model}}{2}\right)T}{\sigma_{Model}\sqrt{T}} \tag{5.6.8}$$

$$d_2 = d_1 - \sigma_{Model}\sqrt{T} \tag{5.6.9}$$

There are no known methods to derive an exact equation for model or implied volatility. There are, however, innumerable methods to solve for implied volatility. Methods include bisection, false position, secant, Ridders, Brent, and Newton-Rhapson (when first derivative is available). See, for example, *Numerical Recipes in C++*.

Many option traders when discussing market conditions refer to the implied volatility rather than the price. Different option pricing models will result in different implied volatilities; hence, it is important to know what model the trader is using. Implied volatility on the S&P 500 index has both futures and options contracts trading (see VIX and related materials provided by the Chicago Board Options Exchange).

Observed implied volatilities are not the same across strike prices, maturities, and type (put or call) as suggested by the BSMOVM. These observations have led to volatility sneers, skews, and surfaces.

When estimating implied volatility, it is often helpful to have a quality first guess. The following are approximations in increasing order of accuracy:

$$\sigma_{Model} \cong \frac{C_{Market}\sqrt{2\pi}}{S\sqrt{T}} \text{ for S=X (Brenner and Subrahmanyam, 1988)} \tag{5.6.10}$$

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}\left(C_{Market} - 0.5\left(S - X\right)\right)}{\sqrt{T}\left(S + X\right)} + \sqrt{2\pi\left(\frac{C_{Market} - 0.5\left(S - X\right)}{S + X}\right)^2 - \alpha\left(\frac{S - X}{S + X}\right)^2} \text{ for S}\neq\text{X}$$

$$(\alpha\text{=4, Corrado and Miller, 1996}) \tag{5.6.11}$$

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}}{\sqrt{T}\left(S + X\right)}\left[C_{Market} - 0.5\left(S - X\right) + \sqrt{\left(C_{Market} - 0.5\left(S - X\right)\right)^2 - \left(\frac{S - X}{\pi}\right)^2}\right] \text{ for S}\neq\text{X}$$

$$(\alpha\text{=2, Corrado and Miller, 1996 "improved quadratic formula"}) \tag{5.6.12}$$

$$\sigma_{Model} \cong \frac{\sqrt{2\pi}}{2\sqrt{T}\left(S + X\right)}\left[2C_{Market} + X - S + \sqrt{\left(2C_{Market} + X - S\right)^2 - 1.85\frac{\left(S + X\right)\left(X - S\right)^2}{\pi\sqrt{XS}}}\right]$$

$$\text{(Hallerbach, 2004)} \tag{5.6.13}$$

We illustrate the accuracy of these approximation techniques assuming the market prices are generated with the following parameters in the GBMOVM: S = \$100, r = 5%, δ = 0%, σ = 30%, and T = 1 year. The strike price is initially set at $X = Se^{rT}$ and then increment by \$5 four times above and below this initial value. Table 5.6.1 provides the results. In theory, the estimates should all be around 30%.

Table 5.6.1. Implied Volatility Estimates Based on Various Methods (Correct σ = 30%)

Strike	C(Model)	BS, 1988	CM4, 1996	CM2, 1996	H, 2004
$85.12	$22.833	57.25%	34.13%	40.69%	38.60%
$90.12	$19.617	49.19%	35.69%	38.27%	37.38%
$95.12	$16.728	41.95%	36.03%	36.62%	36.41%
$100.12	$14.166	35.52%	35.65%	35.65%	35.65%
$105.12	$11.919	29.89%	29.89%	29.89%	29.89%
$110.12	$9.970	25.00%	33.06%	35.46%	34.63%
$115.12	$8.294	20.80%	30.47%	36.09%	34.29%
$120.12	$6.866	17.22%	25.39%	37.13%	34.00%
$125.12	$5.659	14.19%	#NUM!	38.53%	33.71%

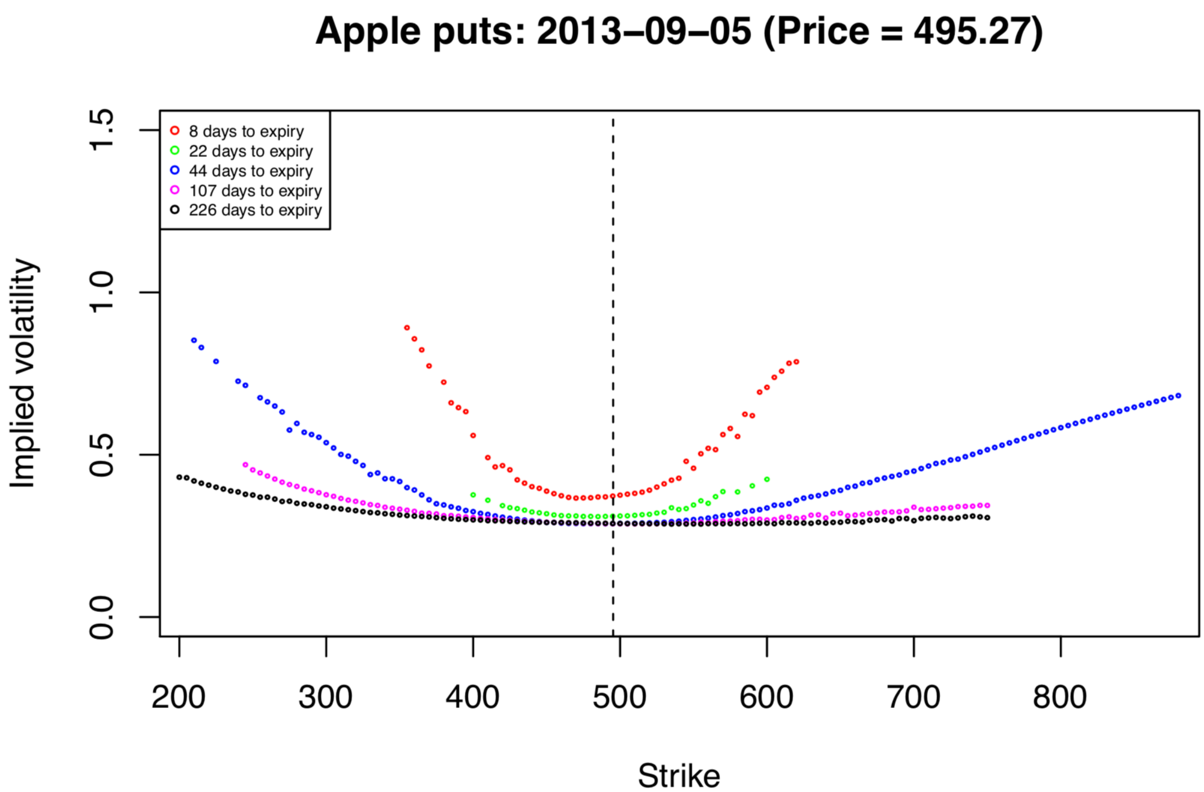
S = $100, r = 5%, δ = 0%, σ = 30%, T = 1 year, and initial $X = Se^{rT}$ =105.12 (increment by 5 above and below).

Conclusion: Approximation methods are not accurate enough to be useful. Thus, we pursue iterative techniques.

We apply the `optimize()` method within the `stats` package as it does not require taking the derivative of the function and converges rapidly

Before moving to reviewing other implied parameters, we reproduce an implied volatility graph in Figure 5.6.1 generated by Robert McDonald's R code applied to Apple stock options data taken from OptionMetrics®.[1]

Figure 5.6.1. Implied Volatility for Apple Stock Across Different Strike Prices and Maturities



[1]Source code is available at https://www.dropbox.com/s/9qwj4s62bacqsyl/plotallimpvol.Rmd?dl=0.

Unlike other modules, we integrate our discussion of different implied parameters within the R code comments.

Summary

We introduced the idea of computing implied option parameters using the GBMOVM as an illustration. We discussed the challenges related to setting bounds and methods to compute various implied parameters, including implied volatility.

References

Brenner, Menachem and Marti G. Subrahmanyam, "A Simple Formula to Compute the Implied Standard Deviation," *Financial Analysts Journal* (September-October 1988), 80-83.

Corrado, Charles J. and Thomas W. Miller, Jr., "Efficient Option-Implied Volatility Estimators," *Journal of Futures Markets* (May 1996), 247-272 1996

Hallerbach, Winfried George, An Improved Estimator for Black-Scholes-Merton Implied Volatility (July 5, 2004). ERIM Report Series No. ERS-2004-054-F&A, Available at SSRN: https://ssrn.com/abstract=567721 or http://dx.doi.org/10.2139/ssrn.567721