Module 4.2: Valuation Corporate Bonds

Learning objectives

- Review corporate bond valuation
- Compute yield to maturity
- Illustrate sourcing separate files for inputs and functions
- Build upon Module 4.1 by incorporating credit spreads
- Corporate bond valuation based on applying the LSC model to the Constant Maturity Treasury rates as well as to credit spreads

Executive summary

In this chapter, we review several unique aspects of corporate bonds, including the influence of term, credit rating, and holding period returns. With this foundation, we extend our work to illustrate valuing corporate bonds with R code from a somewhat independent source—the BB curve.

Central finance concepts

As with U. S. Treasuries (USTs), the standard approach to valuing corporate bonds (CBs) is simply the present value of future cash flows. Most CBs pay semi-annual coupons and are quoted without including accrued interest. Again, as with USTs, the yield to maturity is typically solved implicitly. The semi-annual coupon amount is typically computed based on a 30/360 day count basis. Thus, every semi-annual period has 180 days based on a 360 day year. Hence, the semi-annual coupon amount will always be the stated annual amount divided by two.

There are again several nuances related to bond calculations consistent with those related to USTs. The bond credit rating is important for corporate bonds that we review after first addressing the alternatives for the base rate.

UST, Libor or Something Else

At the time of the writing, Libor has been terminated. Thus, the appropriate base rate for all of financial analysis is a bit uncertain. Historically, Libor has served as a better proxy for short term interest rates than UST-based constant maturity treasuries or CMT.

We prefer Libor-based interest rate swap rates (Libor) over U.S. Treasury yields (USTs) for a host of reasons. We briefly enumerate several reasons here.

- USTs are exempt from state and local taxes resulting in a biased low yield.
- Many USTs holders enjoy additional income from the repurchase agreement market, especially when the USTs are on special (in high demand in the repo market).
- USTs enjoy higher levels of liquidity when compared with Libor deposits as these deposits are not always easily transferable.
- USTs enjoy higher prices during global stress and crisis events due to flight to quality behavior of traders.
- USTs holding period returns are uniquely negatively correlated with broad equity market indexes during crises making them particularly attractive diversification instruments.
- USTs have a unique default status as the U.S. Treasury can print money.
- USTs have unique benefits to commercial banks as they have no or small capital requirements and often fulfill a variety of regulatory requirements.
- USTs are suitable instruments for advanced refunding escrow for municipal bonds. Municipalities can issue federally tax exempt debt securities than can be advanced refunded by issuing new bonds, investing the proceeds in USTs, and then placing these USTs in a bankruptcy proof trust for the purpose of defeasing the old bonds.

Our primary reason for preferring Libor-based rates is that it is good proxy for the *marginal dealer's* cost of funds or the short-term opportunity cost of capital for financial institutions prior to the Libor fraud. The marginal dealer's credit rating is usually assumed to be AA, hence default risk is a possibility. Although there was evidence that the entire debt market was moving to benchmark to some variation of Libor (swap curve or zero-coupon curve), the Libor fraud and the regulatory response to it resulted in Libor's termination. Some have suggested that the overnight index swap (OIS) or the secured overnight financing rate (SOFR) rate is an improvement. These rates are based on some overnight rate, a rate inaccessible to many market participants. Due to these enumerated reasons, we preferred Libor-based base curve as the appropriate foundational curve for investment analysis, but that is no longer an option. Obviously, now that Libor has ceased to exist, one will have to appraise the currently available alternatives and proceed to build the corresponding base curve.

Corporate Bond Credit Ratings

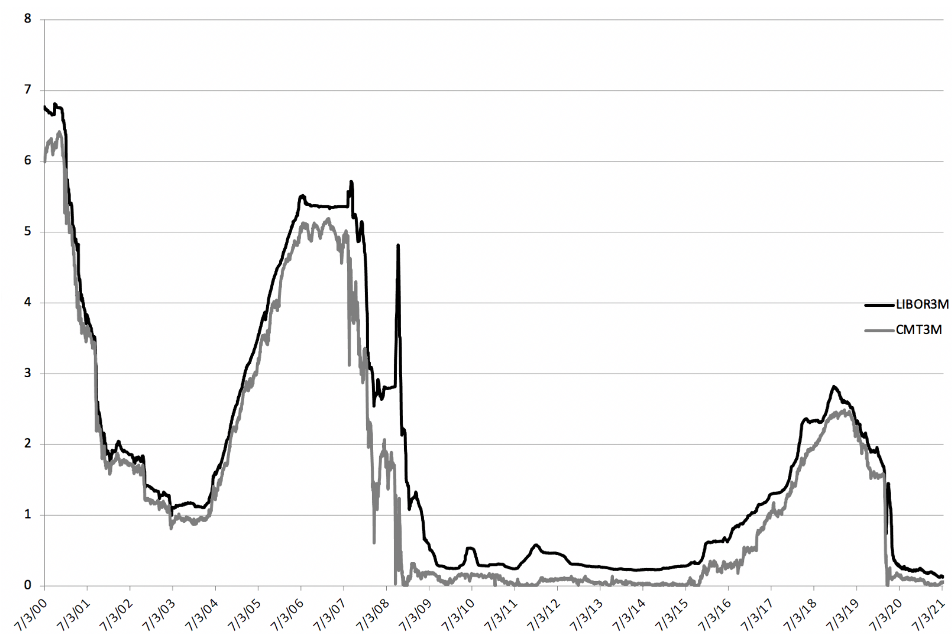
Corporate bonds have default risk as well as interest rate risk. When a bond defaults, the company no longer makes coupon payment or principal payments. Historically, credit rating agencies have attempted to assign various scales to communicate their beliefs regarding the likelihood and damage related to a default event. Three large rating agencies are Moody's, S&P, and Fitch. We very briefly review the Moody's rating scale. The highest rating is Aaa and is sometimes referred to as prime. High grade rating included Aa1, Aa2, and Aa3, where Aa1 is better than Aa2 and so forth. Upper medium grade bonds are rated A1, A2, and A3. Lower medium grade bonds are rated Baa1, Baa2, and Baa3. Bond rated Ba1 and below are no longer investment grade and sometimes referred to as junk bonds. The remining scales are Ba1, Ba2, Ba3 (speculative), B1, B2, B3 (highly speculative), Caa1, Caa2, Caa3, Ca, C. The last rating, C, implies the bond is already in default. The S&P scale is similar but it is denoted with all caps, such as AAA, AA, and so forth.

History of Various Yields and Spreads

Corporate bonds offers different yields depending on maturity, credit rating, liquidity, embedded options, as well as other factors. We briefly review selected history of various relationships.

First, we illustrate the relationship between 3-month CMT and 3-month Libor. As expected, 3-month Libor typically trades at a positive spread over 3-month CMT. Libor is based on financing between typically AA-rated banks whereas CMT is based on U. S. government backed instruments.

Figure 4.2.1 Three Month CMT and Three Month Libor



Second, we illustrate the term spread. Although measured in various ways, the goal is to show the additional yield offered for holding longer term debt instruments. Figure 4.2.2 illustrates the term spread with five year interest rate swap rates compared to 3-month Libor rates. Because the underlying swap floating rate is 3-month Libor, the only difference is term—five years in this case. Notice that the term spread is usually positive, but not always. When negative, it is often referred to as an inverted curve and some economists believe that it is a leading indicator of recession.

Figure 4.2.2 Term Spread, Five Year Interest Rate Swap Rate Minus 3-Month Libor

Due to centralized clearing or cross-collateralizations, interest rate swaps tend to be viewed as representing highly rated bonds. Figure 4.2.3 illustrates the spread between AA-rated yields and five-year swap rates. Although variable over time, it has remained significantly positive. Therefore, on average, AA-rate bonds offer a higher yield than a comparable bond represented by five-year swap rates.

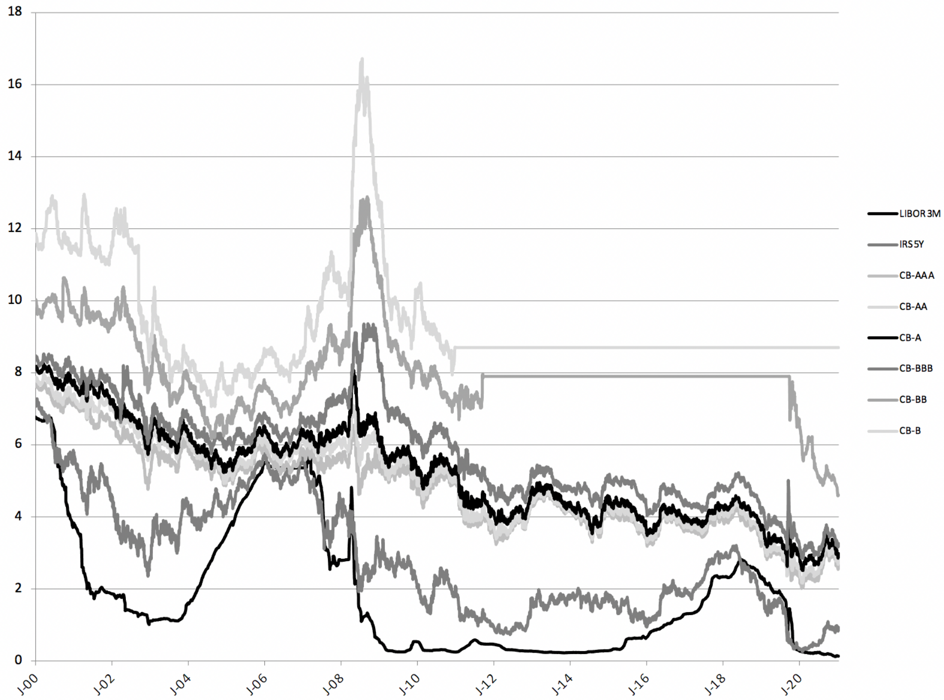
Figure 4.2.3 Investment Grade Spread, AA-Rated Yields Minus Five-Year Interest Rate Swap Rate

High yield bonds are expected to offer a higher yield to maturity due to higher default risk. Figure 4.2.4 confirms this expectation. The high yield spread is a barometer of market participants' views related to potential future default risk. Note that during the great recession of 2008-2009 when fears of numerous defaults ran high, the high yield spread was at historic highs. As those fears mitigated so did the high yield spread. Again, as the fears related to the COVID-19 pandemic began to ameliorate and the U. S. economy began showing signs of recovering, the high yield spread also declined.

Figure 4.2.4 High Yield Spread, BB-Rated Yields Minus AA-Rated Yields

Figure 4.2.5 illustrates the rationality of the corporate bond market with respect to credit rating. The lower the credit rating, the higher the offered yield. After the financial crisis of 2008-2009, it has become much more difficult to find proxies for speculative grade ratings which explains the flat lines for BB and B (for a season) yields.

Figure 4.2.5 Corporate Bond Yields



Higher yielding bonds do not necessarily tranlate into higher actual returns due to the potential for bond investor loss when there are defaults. Thus, it is important to examine the holding period returns from various bond investments. We explore two bond exchange traded funds (ETFs), a investment grade bond ETF (AA/A rated fund with ticker: LQD) and a high yield bond ETF (BB/B rated fund with ticker: HYG).

Figure 4.2.6 shows the value of $1 invested in LQD on July 30, 2002. Any coupons paid to the fund and then paid out to the bond ETF investors are assumed to be reinvested into the fund. Hence, we are reporting the total return. During the 2008-2009 time frame, we see this bond fund suffered loss, but not as dramatic as most stock indexes. Also, with the onset of the COVID-19 pandemic, the LQD ETF also suffered a signicant loss but then it quickly recovered.

Figure 4.2.6 Long History of AA/A Rated Bond Fund (LQD)

Figure 4.2.7 tracks the total return for both the AA/A rated fund and the BB/B rated fund during the financial crisis. As expected, the lower credit rated fund suffered more loss during the crisis. But does the BB/B rate fund actuall perform better in the long run?

Figure 4.2.7 Crisis History of AA/A Rated Bond Fund (LQD) Versus BB/B Rated Bond Fund (HYG)

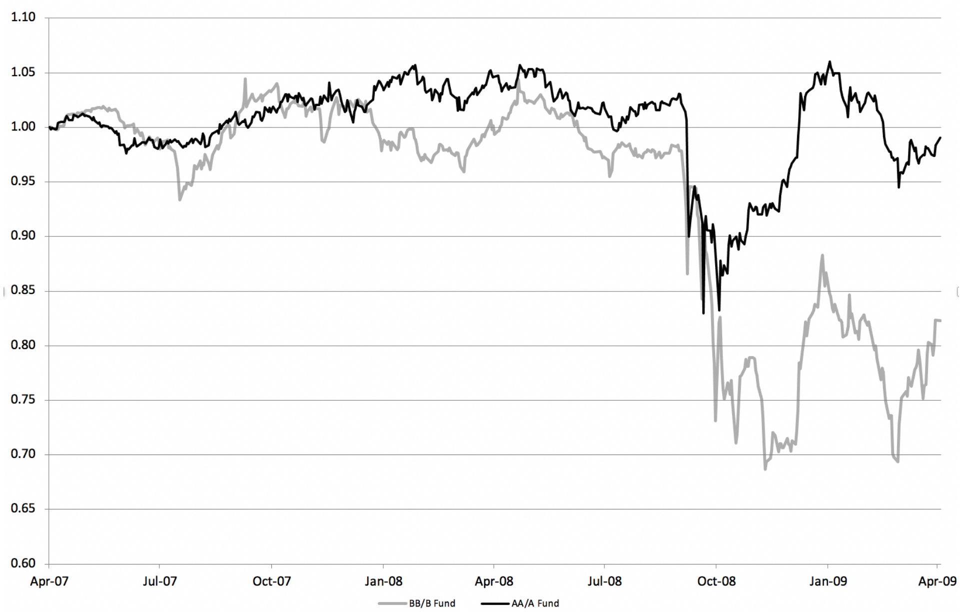
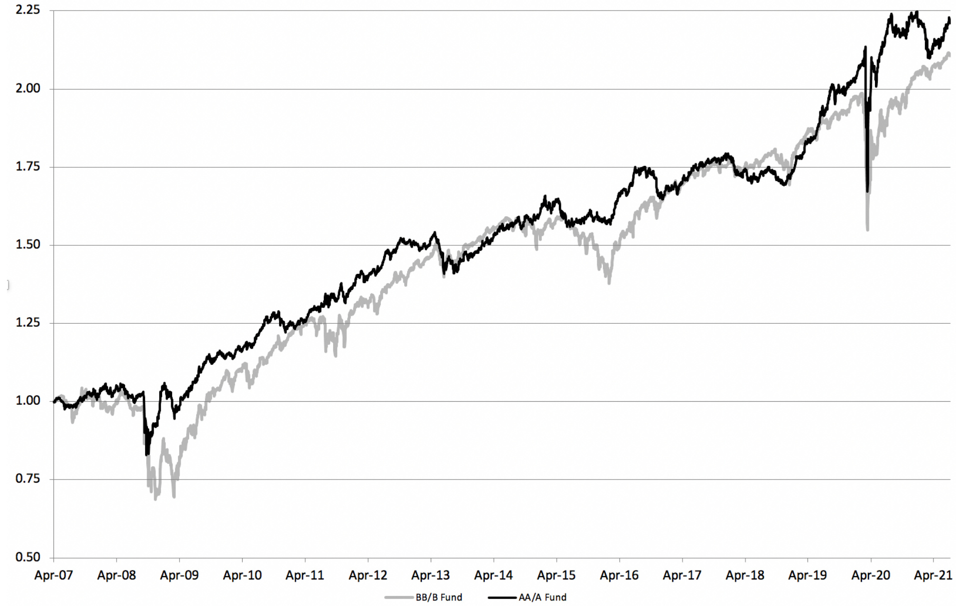


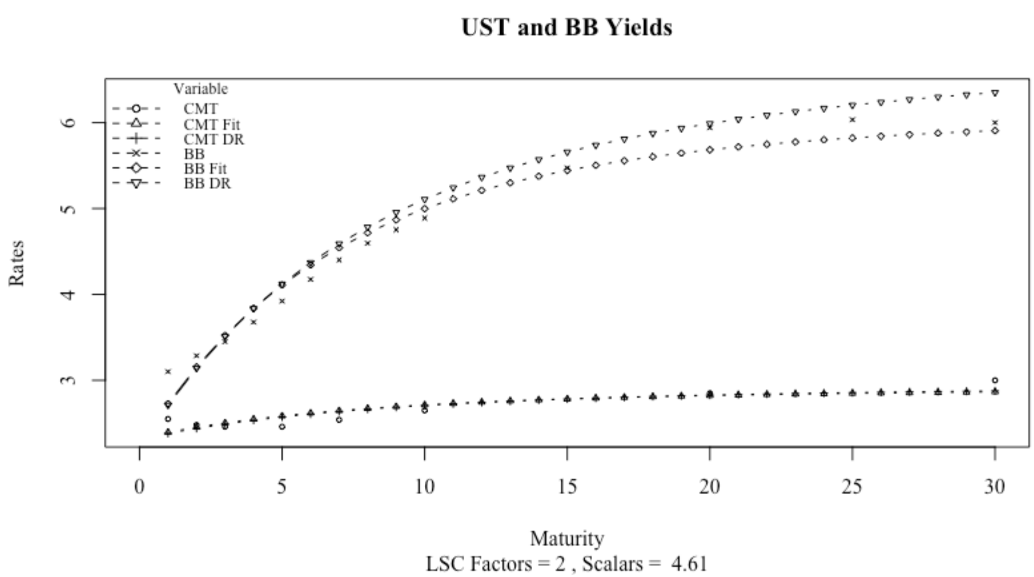
Figure 4.2.8 illustrates the long history of these two funds. As we saw in the previous figure, HYG suffered more during both the financial crisis as well as the COVID-19 pandemic onset. We see here that HYG was able to recoup the losses from the financial crisis in 2008-2009 and essentially caught up to LQD. Once again, however, with the COVID-19 pandemic HYG suffered more and is gradually catching back up. Interestingly, although HYG always has a higher weighted average yield to maturity when compared to LQD, the default losses incurred have more than offset these higher initial yields.

__Figure 4.2.8 Long History of AA/A Rated Bond Fund (LQD) Versus BB/B Rated Bond Fund (HYG)__



We now apply the 2-factor LSC model application first for the CMT rates and then for BB yields to produce Figure 4.2.9 (see R program). Note that the 2-factor LSC fit has some estimation error in the near-term.

__Figure 4.2.9 CMT and BB Yields With Fitted CMT and BB Yields and Fitted Discount Factors__



Further, we can now estimate the credit spread with the 2-factor LSC model as illustrated in Figure 4.2.10.

Figure 4.2.10 CMT and BB Yields Less CMT Spread with Fitted Spot Rates and Fitted Discount Factors

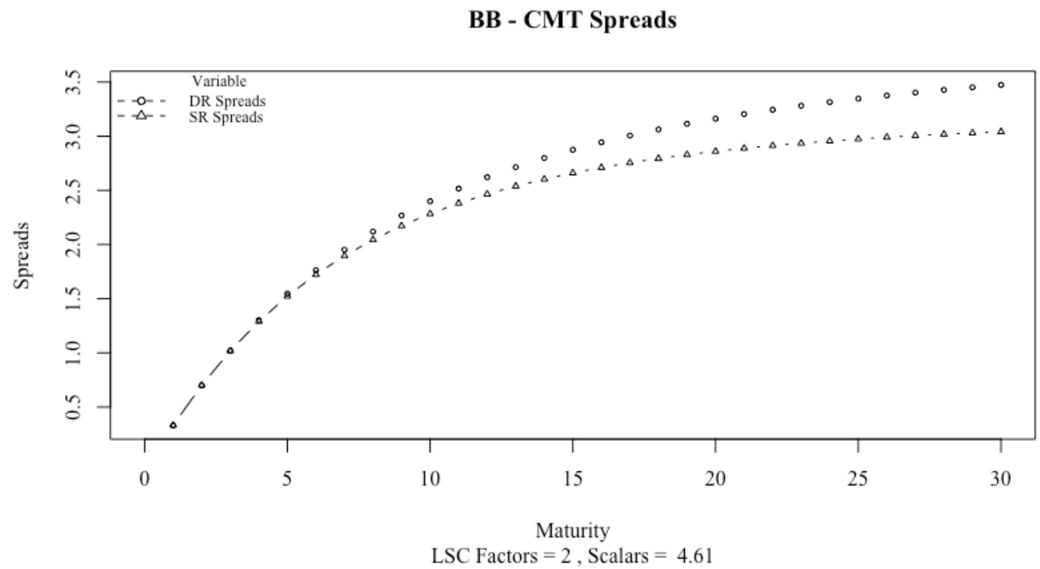
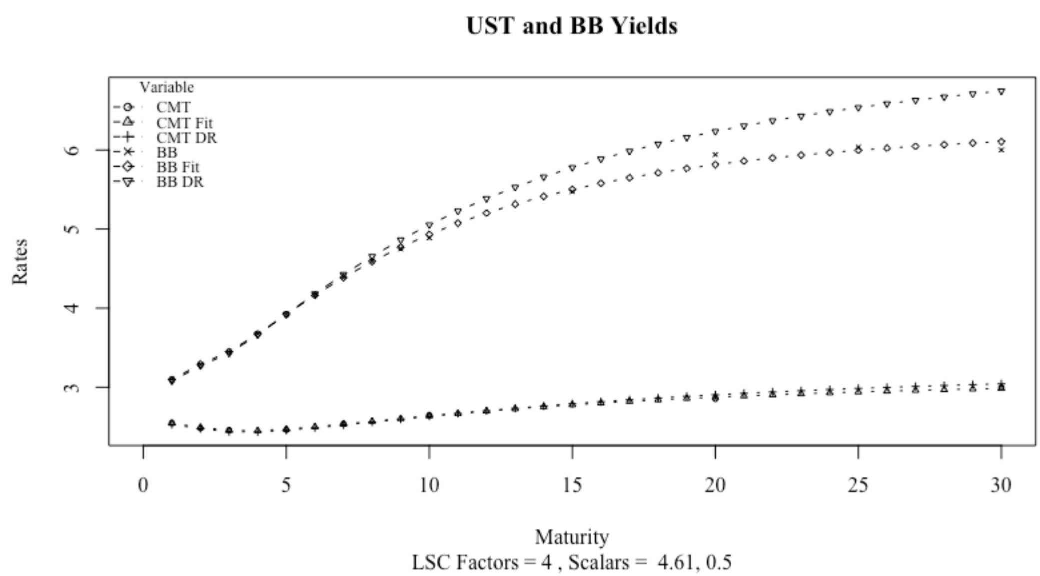


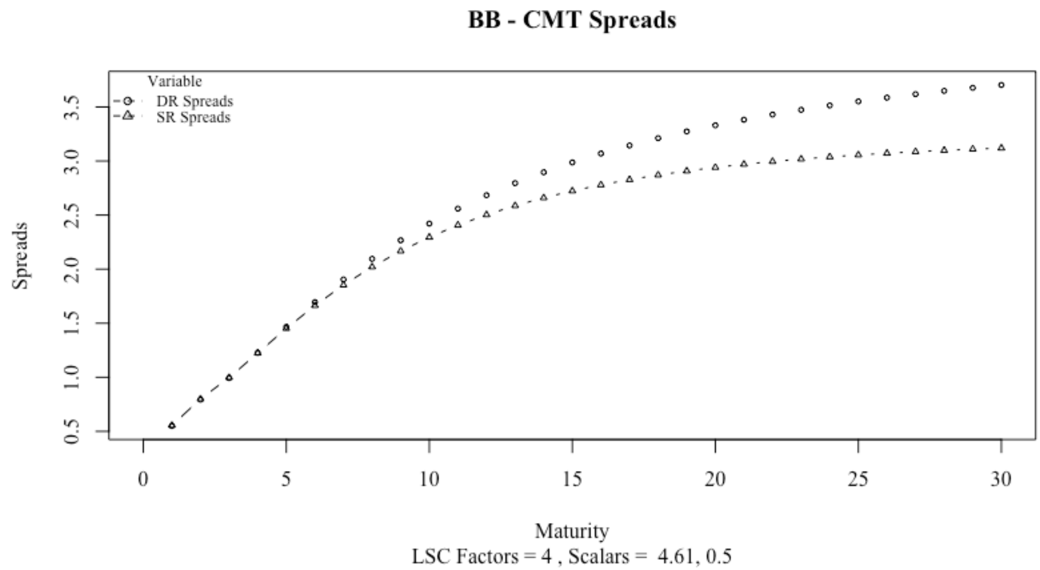
Figure 4.2.11 presents the 4-factor LSC fit, and we observe an improved fit.

Figure 4.2.11 CMT and BB Yields With Fitted CMT and BB Yields and Fitted Discount Factors



We again estimate the credit spread with the 4-factor LSC model as illustrated in Figure 4.2.12.

Figure 4.2.12 CMT and BB Yields Less CMT Spread with Fitted Spot Rates and Fitted Discount Factors



With the unique features of corporate bonds identified, we now turn to address some of these challenges with quantitative techniques.

Quantitative finance materials

The corporate bond value today, V_{CB}, depends on the number of payments per year, $m = 2$, the par value, *Par*, fraction of payment period elapsed already, f, the number of remaining coupon payments, N, the stated annual dollar coupon, *Coupon*, and the yield to maturity, y. Mathematically, the CB value can be expressed as

$$V_{CB} = \sum_{i=1}^{N} \frac{\left(\frac{Coupon}{m}\right) Par}{\left(1+\frac{y}{m}\right)^{i-f}} + \frac{Par}{\left(1+\frac{y}{m}\right)^{N-f}}. \tag{4.2.1}$$

Thus, all the analytics covered in the last module apply here and will be briefly reviewed in the R code comments below.

Applying the LSC model to CMTs and Spreads

From Module 4.1, we learned the mechanics of the LSC model applied to CMT yields. We now apply the LSC model to the BB-rated yield curve. The CMT rates will again form the basis for estimating a base spot rate curve. We will first fit an LSC model to the CMT rates. Once we obtain the LSC spot rate factors, we can estimate the entire CMT curve. With this complete dataset, as before we can easily compute the annualized, continuously compounded discount rates for this base curve. We term this component of the analysis the base rate function.

For our purposes, we now decompose the bond value into two major components, the base rate function and the spread function. With the approach taken here, we can easily layer several spread functions rather than just one. For our purposes, we assume only one spread, for example, the spread of BB-yields over UST CMT.

We denote the spread over the fitted LSC spot rate as sp_i. Thus, we can decompose the spot yield, y_i, into LSC spot rates, r_i^{LSC}, the LSC spreads sp_i^{LSC}, and an error term ε_i. Note that with this approach any error in estimating both LSC models is completely captured in the error term. Therefore, the current bond value can be expressed as

$$\begin{aligned} V_B &= \sum_{i=1}^{N_t} CF_i DF_i = \sum_{i=1}^{N_t} CF_i e^{-y_i \tau_i} \\ &= \sum_{i=1}^{N_t} CF_i e^{-\left(r_i^{LSC} + sp_i^{LSC} + \varepsilon_i\right)\tau_i} = \sum_{i=1}^{N_t} CF_i e^{-\left(\sum_{j=0}^{N^r} x_{i,j} f_j^r + \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp} + \varepsilon_i\right)\tau_i} \end{aligned} . \tag{4.2.2}$$

As calendar time will be an important component, we add the subscript t to the LSC model notation.

From Module 3.5 and 4.1, we apply the LSC model to bond yields, we have

$$y_i = \sum_{j=0}^{N^r} x_{i,j} f_j^r + \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp} + \varepsilon_i . \tag{4.2.3}$$

We define the portion of the yield to maturity attributable to the base rate as

$$r_i^{LSC} \equiv \sum_{j=0}^{N^r} x_{i,j} f_j^r , \tag{4.2.4}$$

and the portion of the yield to maturity attributable to the spread as

$$sp_i^{LSC} \equiv \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp} . \tag{4.2.5}$$

Recall from Module 3.5, we have (note the calendar time t subscript here)

$$x_{i,0,t} = 1 , \; x_{i,1,t} = \frac{s_1}{\tau_{i,t}}\left(1 - e^{-\tau_{i,t}/s_1}\right), \text{ and } \; x_{i,j,t} = \frac{s_j}{\tau_{i,t}}\left(1 - e^{-\tau_{i,t}/s_j}\right) - e^{-\tau_{i,t}/s_j} ; j > 1. \tag{4.2.6}$$

Thus, the approximate value of the bond, based on both LSC models (base rate and spread), is expressed as

$$V_B \cong V_B^{LSC} = \sum_{i=1}^{N_t} CF_i DF_i^{LSC} = \sum_{i=1}^{N_t} CF_i e^{-\left(r_i^{LSC} + sp_i^{LSC}\right)\tau_i} . \tag{4.2.7}$$

With enough LSC factors, the residual error will be negligible. Thus, we define the discount factor based solely on the LSC model applied to the base rates and the LSC model applied to spreads as

$$DF_i^{LSC} \equiv e^{-\left(r_i^{LSC} + sp_i^{LSC}\right)\tau_i} . \tag{4.2.8}$$

Summary

In this chapter, we reviewed several unique aspects of corporate bonds, including the influence of term, credit rating, and holding period returns. With this foundation, we extended our work to illustrate valuing corporate bonds with R code from a somewhat independent source—the BB curve.

References

See references in Module 4.1.