Module 4.3

Valuation Stocks

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Overview

- Review dividend discount models (DDM)
- Brief tour through stock market returns
- N-stage DDM
  - Unique growth for each stage
  - Unique *forward* rate for each stage
- Model illustrated
- LSC applied to PVD

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Central Finance Concepts

- Quick tour through the stock market
  - Stocks perform well in the long run for countries that have survived
  - Each decade witness significant losses
- Valuation is based on the present value of predicted future dividends

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Quick Tour-U.S. Stock Market

- Value-weighted returns
- Dividend included
- By decade
  - Generally positive
  - Each decade has severe decline
- General results are contingent on country surviving

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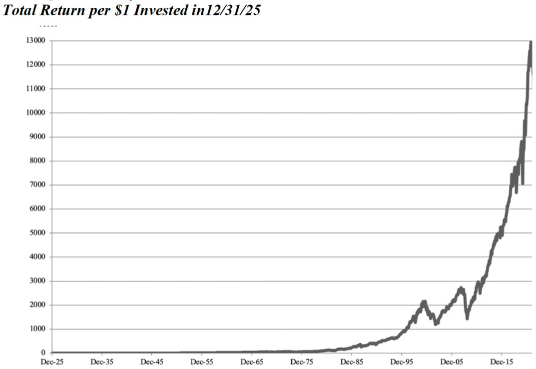
Figure 4.3.1 A Quick Tour of the U.S. Stock Market
Panel A Total Return per $1 Invested in12/31/25



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Panel B Log Scale of Total Return per $1 Invested in12/31/25

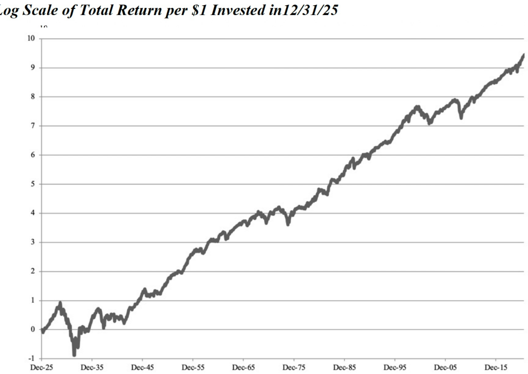

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Figure 4.3.2 Value-Weighted Returns in the U.S. Stock Market by Decade
Panel A Total Return Over Decade Spaning 12/31/1925-12/31/1935

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Panel B Total Return Over Decade Spaning 12/31/1935-12/31/1945

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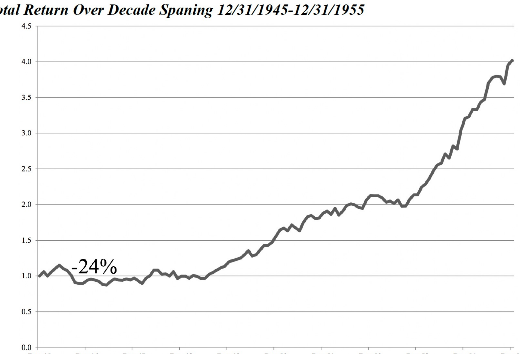
Panel C Total Return Over Decade Spaning 12/31/1945-12/31/1955



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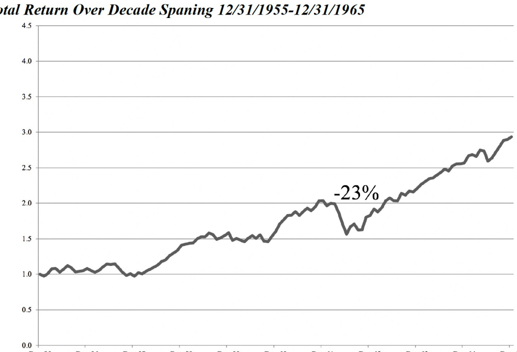
Panel D Total Return Over Decade Spaning 12/31/1955-12/31/1965



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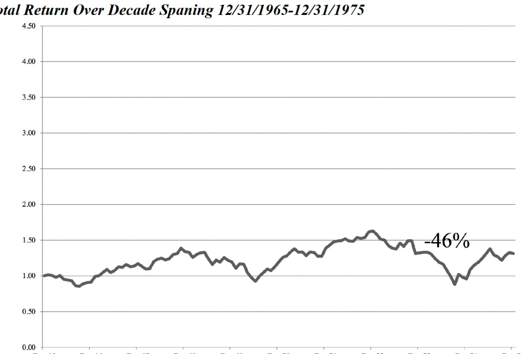
Panel E Total Return Over Decade Spaning 12/31/1965-12/31/1975



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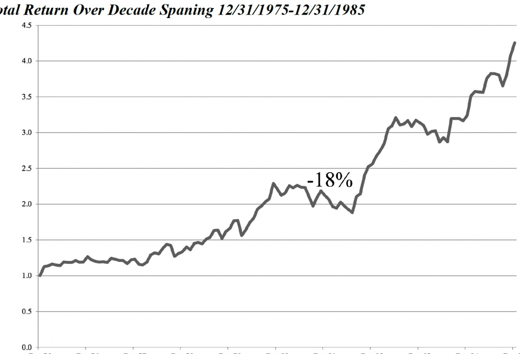
Panel F Total Return Over Decade Spaning 12/31/1975-12/31/1985



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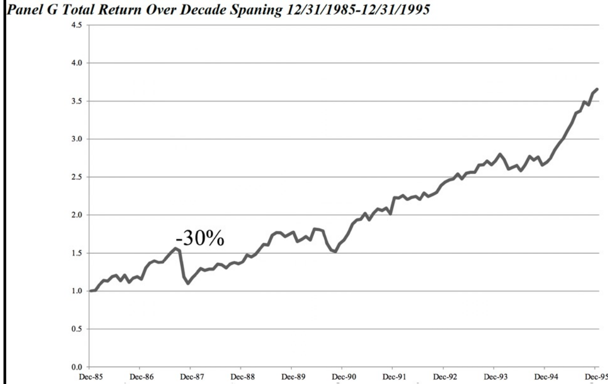
Panel G Total Return Over Decade Spaning 12/31/1985-12/31/1995



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Panel H Total Return Over Decade Spaning 12/31/1995-12/31/2005

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Panel I Total Return Over Decade Spaning 12/31/2005-12/31/2015

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Panel J Total Return Over Decade Spaning 12/31/2015-12/31/2025

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Dividend Discount Models

- Gordon (1959) single factor DDM
  - Constant growth rate
  - Constant discount rate
- N-stage DDM
  - Stage varying growth rates
  - Stage varying discount rates
- LSC model DDM
  - LSC fit to growth curve
  - LSC fit to forward discount rates

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Factor Reduction with LSC

- Complex PVD models still can be estimated with parsimonious LSC models
- Four factor DDM, quarterly pay
  - 2 factor LSC fit
  - 3 factor LSC fit
  - 4 factor LSC fit

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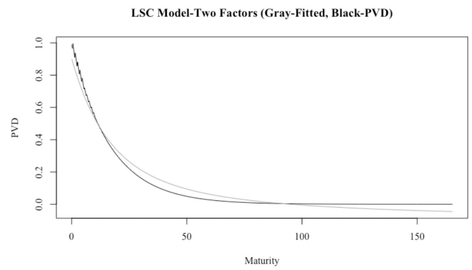
Figure 4.3.3 Four Stage Dividend Discount Model With LSC Fit
Panel A: Two Factor LSC Model Fit



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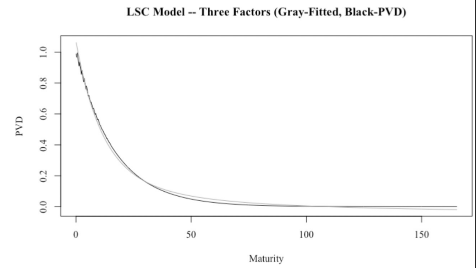
Panel B: Three Factor LSC Model Fit



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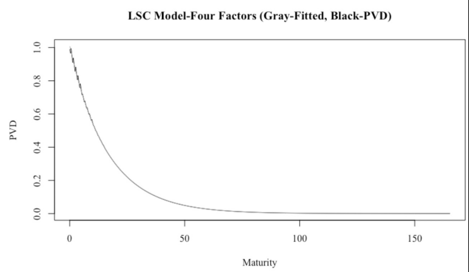
Panel C: Four Factor LSC Model Fit



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Quantitative Finance Materials

- Instrument valuation based on DFAA
- Generic DDM
- Gordon growth DDM
- N-stage justification and illustration
- LSC applied to PV dividends
- LSC applied to *f* and *g*

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Instrument Valuation

- Discount factor adjusted approach

$$P_i = \sum_{t=1}^{T}\sum_{j=1}^{m} \frac{1}{\left(1+r_t+RP_{i,t,j}\right)^t} p_{t,j} CF_{i,t,j}$$

  - $p_{t,j}$ – subjective probability, time *t*, state *j*
  - $CF_{i,t,j}$ – expected cash flow
  - $RP_{i,t,j}$ – risk premium
  - r_t – risk free rate

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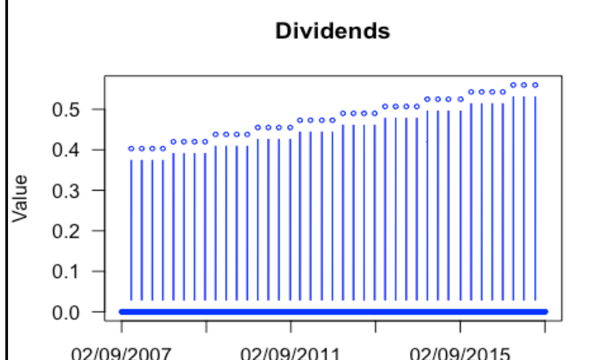
Dividend Policy Example: Southern Company, 10 Year Period



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Generic DDM

- Stock value

$$V_S \equiv \sum_{i=1}^{\infty} PV(\tau_i,t)\left\{E_t\left[D(\tau_i,t)\right]\right\} = \sum_{i=1}^{\infty} DPV_i$$

  - $PV(\tau_i,t)$ – present value of i^{th} dividend
  - $E_t\left[D(\tau_i,t)\right]$– expected dividend
  - DPV_i – present value of dividend
  - Competing issues
    - Discount factor declining
    - Expected dividends rising

Gordon Growth DDM

- Constant growth

$$V_S = \frac{D_0(1+g)}{k-g}$$

  - k – discount rate (constant)

$$k = r + \beta\left[E(r_M) - r\right]$$

  - g – dividend growth rate
  - Very simple, but crude
  - Is constant k and constant g internally coherent? Empirically corresponds?

N-Stage DDM Assumptions

- Quarterly dividends, change 1x per year
- N stages
  - Constant growth rate within each stage
  - Constant discount rate within each stage
- Stub (Initial) year has H remaining dividends

$$P_0 = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k}$$

Series with Continuous Compounding

- Infinite series

$$B_\infty = \sum_{i=1}^{\infty} e^{-x(i)} = \frac{1}{e^x - 1}$$

- Finite series

$$B_N = \sum_{i=1}^{N} e^{-x(i)} = \frac{1-e^{-x(N)}}{e^x - 1}$$

N-Stage DDM

- Stock Value: $V_S = Stub + Series + Final$
- Stub:

$$Stub = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k}$$

- Series:

$$Series = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\sum_{j=1}^{N-1}\left[\prod_{i=1}^{j-1} e^{-(f_i-g_i)m_i}\right]\left[e^{f_j\Delta\tau(3)} + e^{f_j\Delta\tau(2)} + e^{f_j\Delta\tau(1)} + 1\right]\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1}$$

- Final:

$$Final = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[\prod_{i=1}^{N-1} e^{-(f_i-g_i)m_i}\right]\left[e^{f_N\Delta\tau(3)} + e^{f_N\Delta\tau(2)} + e^{f_N\Delta\tau(1)} + 1\right]\frac{1}{e^{(f_N-g_N)}-1}$$

One Stage, No Growth

- N=0, g=0, H=2, $\Delta\tau_1$=0.1, D_{-1}=\$1, f_0=10%

$$V_S = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_0\Delta\tau(3)} + e^{f_0\Delta\tau(2)} + e^{f_0\Delta\tau(1)} + 1\right]\frac{1}{e^{f_0}-1}$$

$$= \$1\left[e^{-0.1(0.1)} + e^{-0.1(0.1+0.25)}\right] + \$1e^{-0.1(0.1+0.25)}\left[e^{0.1(0.25)(3)} + e^{0.1(0.25)(2)} + e^{0.1(0.25)(1)} + 1\right]\frac{1}{e^{0.1}-1}$$

$$= 0.990050 + 0.965605 + 0.965605(1.077884 + 1.051271 + 1.025315 + 1)\frac{1}{1.105171-1}$$

$$= 1.955655 + 0.965605(4.154470)(9.508325) = 40.099033$$

One Stage Growth

- N=1, g_1=5%, f_1=12%

$$V_S = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_1\Delta\tau(3)} + e^{f_1\Delta\tau(2)} + e^{f_1\Delta\tau(1)} + 1\right]\frac{1}{e^{(f_1-g_1)}-1}$$

$$= \$1\left[e^{-0.1(0.1)} + e^{-0.1(0.1+0.25)}\right] + \$1e^{-0.1(0.1+0.25)}\left[e^{0.12(0.25)(3)} + e^{0.12(0.25)(2)} + e^{0.12(0.25)(1)} + 1\right]\frac{1}{e^{(0.12-0.05)}-1}$$

$$= 1.955655 + 0.965605(1.094174 + 1.061837 + 1.030455 + 1)\frac{1}{1.072508-1}$$

$$= 1.955655 + 0.965605(4.186466)13.791582 = \$57.707746$$

Two Stage Model

- N=2, g_2=2%, f_2=9%, m_1=5 years

$$V_S = \$1\left[e^{-0.1(0.1)} + e^{-0.1(0.1+0.25)}\right]$$

$$+\$1e^{-0.1(0.1+0.25)}\left[e^{0.12(0.25)(3)} + e^{0.12(0.25)(2)} + e^{0.12(0.25)(1)} + 1\right]\frac{1-e^{-(0.12-0.05)(5)}}{e^{(0.12-0.05)}-1}$$

$$+\$1e^{-0.1(0.1+0.25)}e^{-0.12(5)}e^{0.05(5)}\left[e^{0.09(0.25)(3)} + e^{0.09(0.25)(2)} + e^{0.09(0.25)(1)} + 1\right]\frac{1}{e^{(0.09-0.02)}-1}$$

$$= 1.955655 + 0.965605(1.094174 + 1.061837 + 1.030455 + 1)\frac{1-0.704688}{1.072508-1}$$

$$+0.965605(0.548811)(1.069830 + 1.046028 + 1.022755 + 1)13.791582$$

$$= \$57.258625$$

Three Stage

Table 4.3.1. Three stage dividend discount model

Stage	ForwardRate	GrowthRate	YearsInStage	SeriesValue	InitialDividend
0	10.0%		2	$1.955655	$1.000000
1	12.0%	6.0%	5	$16.943629	$1.000000
2	9.0%	3.0%	5	$12.408675	$1.349859
3	6.0%	0.0%		$35.064727	$1.568312
			Stock Value	$66.372687	
NOTE: YearsInStage 0 denotes remaining dividends.					

Four Stage

Table 4.3.2. Four stage dividend discount model

Stage	ForwardRate	GrowthRate	YearsInStage	SeriesValue	InitialDividend
0	10.0%		2	$1.955655	$1.000000
1	12.0%	6.0%	5	$16.943629	$1.000000
2	9.0%	3.0%	5	$12.408675	$1.349859
3	7.0%	1.0%	5	$9.122746	$1.568312
4	6.0%	0.0%		$25.976589	$1.648721
				$66.407295	

Figure 4.3.9 Results contained in the Series.xlsx file

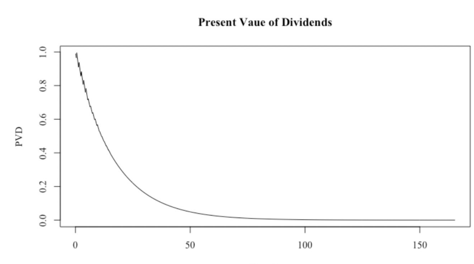
	A	B	C	D	E	F
1	Stage	ForwardRate	GrowthRate	YearsInStage	SeriesValue	InitialDividend
2	0	10%		2	$1.95566	$1.00000
3	1	12%	6%	5	$16.94363	$1.00000
4	2	9%	3%	5	$12.40868	$1.34986
5	3	7%	1%	5	$9.12275	$1.56831
6	4	6%	0%	150	$25.97338	$1.64872
7					$66.40409	

Figure 4.3.8 Excerpts from Dividend.xlsx file

	A	B	C	D
1	MaturityTime	DollarDividend	PV	PVD
2	0.10	$1.00	$0.99005	$0.99005
3	0.35	$1.00	$0.96561	$0.96561
4	0.60	$1.06	$0.93707	$0.99501
5	0.85	$1.06	$0.90937	$0.96561
6	1.10	$1.06	$0.88250	$0.93707
7	1.35	$1.06	$0.85642	$0.90937
8	1.60	$1.13	$0.83110	$0.93707
9	1.85	$1.13	$0.80654	$0.90937
10	2.10	$1.13	$0.78270	$0.88250
653	162.85	$1.65	$0.00003	$0.00006
654	163.10	$1.65	$0.00003	$0.00006
655	163.35	$1.65	$0.00003	$0.00005
656	163.60	$1.65	$0.00003	$0.00005
657	163.85	$1.65	$0.00003	$0.00005
658	164.10	$1.65	$0.00003	$0.00005
659	164.35	$1.65	$0.00003	$0.00005
660	164.60	$1.65	$0.00003	$0.00005
661	164.85	$1.65	$0.00003	$0.00005
662	165.10	$1.65	$0.00003	$0.00005
663	165.35	$1.65	$0.00003	$0.00005

PV(Expected Dividends)

Figure 4.3.4 Present Value of Dividends Based on Four Stage Model



LSC Model Details

- PVD

$$y_{PVD,i} = \sum_{j=0}^{N} x_{i,j} f_j$$

$$x_{i,0} = 1,\; x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i/s_1}\right), \text{ and } x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i/s_j}\right) - e^{-\tau_i/s_j};\, j > 1,$$

- Explore number of factors necessary to fit PVD curve
- Figures above, 4 factors fit well

LSC Model-Growth and Discount Rates

- Seek to apply to apply the LSC model framework to equity market data only
- Based solely on available equity-based information, seek alternative framework
- Two factor LSC applied to
  - Forward discount rates
  - Forward growth rates

Valuation Framework

- Forward discount rates and growth rates

$$V = CF_0 \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i} g_j \tau_j}}{e^{\sum_{j=1}^{i} f_j \tau_j}} = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} (g_j - f_j)\tau_j} = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -(f_j - g_j)\tau_j}$$

- Two factor LSC model applied to
  - Growth

$$g_j = L_g + sc_{g,j} S_g$$

  - Forward discount rates

$$f_j = L_f + sc_{f,j} S_f$$

LSC Valuation Details

- Scalars: $sc_{g,j} = \frac{s_g}{\tau_i}\left(1 - e^{-\tau_i/s_g}\right) \qquad sc_{f,j} = \frac{s_f}{\tau_i}\left(1 - e^{-\tau_i/s_f}\right)$
- LSC model valuation approximation

$$V = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[L_f + sc_{f,j} S_f - \left(L_g + sc_{g,j} S_g\right)\right]\tau_j}$$

- Value per unit of initial cash flow

$$VCF = \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j} S_f - sc_{g,j} S_g\right)}$$

In Perpetuity

- At some distant point in the future

$$VCF_n = \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right)}$$

- Closed form equivalent

$$VCF = \frac{1}{e^{\left(L_f - L_g\right)} - 1}$$

- Solving for the long-run forward rate

$$\hat{L}_f = \ln\left(1 + \frac{1}{\widehat{VCF}_L}\right) + \hat{L}_g$$

Model Assumptions

- Long-term growth rate proxied by long-term interest rate (e.g., Libor)
- Apply dampener to account for lack of complete adjustment

$$\widehat{VCF}_L = VCF + D\left(\overline{VCF} - VCF\right)$$

- Again, four critical values

$$VCF = \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[\hat{L}_f + sc_{f,j}\hat{S}_f - \left(\hat{L}_g + sc_{g,j}\hat{S}_g\right)\right]}$$

Iteration 1: $Slope_g$

- Long-run growth rate assumed to be some base curve Level parameter
- Given overall cost of equity and cash flow yield, solve for growth rate Slope

$$VCF = \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i}\left(\hat{L}_g + sc_{g,j} S_g\right)}}{\left(1+k\right)^i}$$

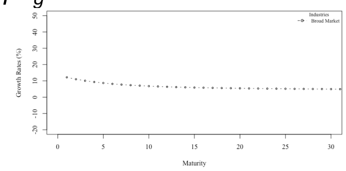
- Recall VCF = 1/Cash flow yield

Iteration 1 Illustration

- Broad market (SPY) calibration

$$\frac{1}{0.019} = \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i}\left[0.04+\frac{3}{j}\left(1-e^{-j/3}\right)S_g\right]}}{\left(1+0.08\right)^i}$$

- $Slope_g$ = 9.6179%.



Iteration 2: *$Level_f$*

- Based on damper and average VCF

$$\widehat{VCF}_L = VCF + D\left(\overline{VCF} - VCF\right)$$

- Applying to solve for $Level_f$

$$\hat{L}_f = \ln\left(1 + \frac{1}{\widehat{VCF}_L}\right) + \hat{L}_g$$

- Broad market (SPY), VCF = 52.6316, D = 50%, Average VCF = 47.5313: VCF^ = 50.08145 and $Level_f$^ = 5.9771%

Iteration 3: *$Slope_f$*

- Broad market (SPY)

$$52.6316 = \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[0.058822+\frac{10}{j}\left(1-e^{-j/10}\right)\hat{S}_f - \left(0.04+\frac{3}{j}\left(1-e^{-j/3}\right)0.096179\right)\right]}$$

- Thus, $Slope_f$ = –2.3815%
- Therefore, we have a fully specified stock valuation model based solely on equity market inputs
- Model ready for empirical assessment

LSC Valuation Illustration

Table 4.3.3. Two factor LSC valuation model inputs

Industry	Ticker	Price	DY	DR
Broad Market	SPY	$311.17	1.90%	8.00%
Technology	XLK	$105.09	1.18%	9.00%
Financial	XLF	$22.99	2.71%	10.00%
Industrials	XLI	$68.70	2.30%	7.80%
Consumer Discretionary	XLY	$129.00	1.24%	8.20%
Materials	XLB	$56.10	2.21%	12.00%
Healthcare	XLV	$101.04	2.33%	9.80%
Utilities	XLU	$57.61	3.52%	7.50%
Consumer Stables	XLP	$58.92	2.83%	6.50%
Energy	XLE	$37.11	4.00%	14.00%

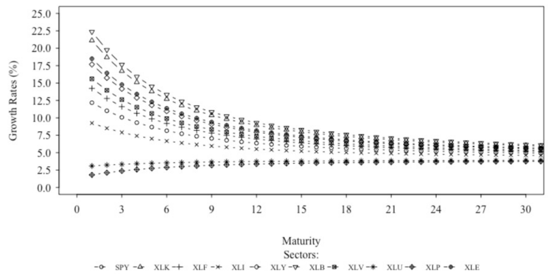
LSC Valuation Illustration

Table 4.3.4. Two factor LSC valuation model inputs

Input	Value
Nf	2
sG0	3.0
sF0	10.0
Lg	4.0%
D	50.0%

LSC Valuation Illustration

Figure 4.3.6. Growth rates based on LSC model



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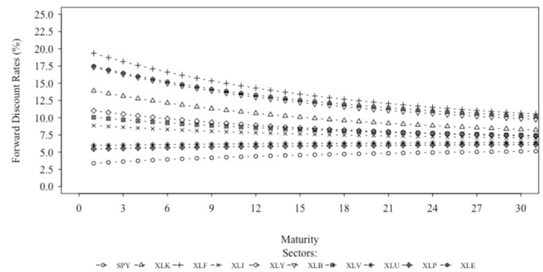
LSC Valuation Illustration

Figure 4.3.7. Forward discount rates based on LSC model



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LSC Valuation Illustration

Table 4.3.5. Calibrating the LSC Model

Industry	Ticker	Price	DY	DR	CF	PCF	GSlope	PCFL	WLevel	WSlope
Broad Market	SPY	$311.17	1.90%	8.00%	$5.9122	52.632	9.6179	50.08145	5.8822	-2.3816
Technology	XLK	$105.09	1.18%	9.00%	$1.2401	84.746	20.1333	66.13854	5.1731	9.86
Financial	XLF	$22.99	2.71%	10.00%	$0.6230	36.900	12.0371	42.21585	6.6739	13.0265
Industrials	XLI	$68.70	2.30%	7.80%	$1.5801	43.478	6.1766	45.50479	6.2739	2.5642
Consumer Discretionary	XLY	$129.00	1.24%	8.20%	$1.5996	80.645	16.0797	64.08824	5.2324	6.7896
Materials	XLB	$56.10	2.21%	12.00%	$1.2398	45.249	21.5877	46.3901	6.1859	11.6216
Healthcare	XLV	$101.04	2.33%	9.80%	$2.3542	42.919	13.6402	45.22489	6.3033	3.7498
Utilities	XLU	$57.61	3.52%	7.50%	$2.0279	28.409	-1.0676	37.97021	7.4595	-2.7163
Consumer Stables	XLP	$58.92	2.83%	6.50%	$1.6674	35.336	-2.5747	41.43351	6.7907	-1.9987
Energy	XLE	$37.11	4.00%	14.00%	$1.4844	25.000	17.0437	36.26566	7.9221	8.9215

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Alternative LSC Approach

- Three key curves
  - Base rate curve (UST or Libor)
  - Credit spread curve (BB – UST or Libor)
  - Growth rate curve (Equity market based)
- Model (not yet developed)

$$V_t \equiv D_{-1,t}\sum_{i=1}^{\infty} DF^g_{i,t} = D_{-1,t}\sum_{i=0}^{\infty} e^{-\left(r^{LSC}_{i,t}+sp^{LSC}_{i,t}+g^{LSC}_{i,t}\right)\tau_i}$$

- Explore implied growth parameters

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Summary

- Dividend discount models reviewed
- Brief tour through stock market returns
- N-stage DDM
  - Unique growth for each stage
  - Unique discount rate for each stage
- LSC applied to PVD
  - Four factors appear adequate
  - Equity market based LSC model

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