Module 4.2

Valuation Corporate Bonds

1

Overview

- Corporate bonds have additional risks
  - Default risk (Measured by credit spread)
  - Liquidity risk
  - Embedded optionality (not addressed here)
    - Callable
    - Defaultable
    - Convertible
- Modeled as an additional LSC model curve

2

Notation Review

- Coupon – annual dollar coupon
- m – coupons per year
- Par – notional amount (principal)
- f – fraction of payment period elapsed since last coupon (NAD/NTD)
- N – number of remaining cash flows
- CF_i – ith cash flow, i = n: CF_n = (Coupon/m) + Par, otherwise CF_i = (Coupon/m)

3

Selected Notation

- y_d – yield to maturity with discrete compounding (dc)
- y_c – yield to maturity with continuous compounding (cc)
- $V_{B,dc}$ – bond value based on dc
- $V_{B,cc}$ – bond value based on cc

4

Bond Valuation (BV)

- Discrete compounding, bond equivalent yield $V_B = \sum_{i=1}^{N} \frac{\left(\frac{Coupon}{m}\right)Par}{\left(1+\frac{y_d}{m}\right)^{i-f}} + \frac{Par}{\left(1+\frac{y_d}{m}\right)^{N-f}} = \sum_{i=1}^{N} \frac{CF_i}{\left(1+\frac{y_d}{m}\right)^{i-f}}$
- Continuous compounding

$$V_B = \sum_{i=1}^{N}\left(\frac{Coupon}{m}\right)Par\left(e^{-y_c\tau_i}\right) + Par\left(e^{-y_c\tau_i}\right) = \sum_{i=1}^{N} CF_i\left(e^{-y_c\tau_i}\right) = \sum_{i=1}^{N} CF_i\left[e^{-y_c\left(\frac{i-f}{m}\right)}\right]$$

- Note that the bond value does not change, rather the yields are different

5

Central Finance Concepts

- Empirical observations
  - 3M Libor and 3M CMT
  - Spreads
    - Term
    - Credit
  - Bond yields

6

Digression: UST Issues

- Exempt from state and local taxes
- High demand at times from repo market
- Higher levels of liquidity
- “Flight to quality” during global stress
- HPRs negative correlation with equities
- US Treasury can print money

7

Digression: UST Issues

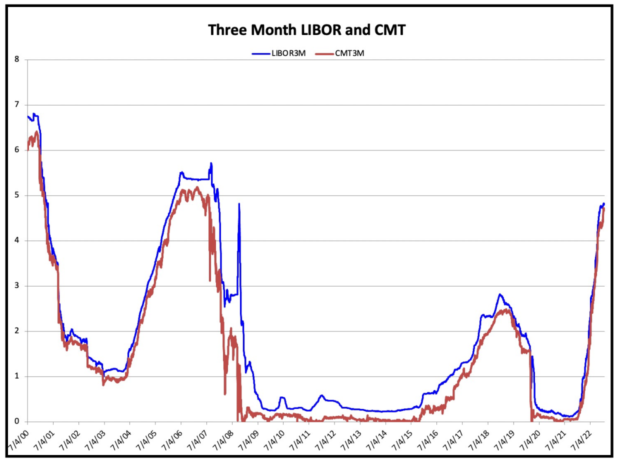
- Unique benefits to banks (zero capital set aside)
- Muncipal bond advanced refunding vehicle
- Libor-based rate is a good proxy for the *marginal dealer’s cost of funds*
  - Candidate replacements: OIS, SOFR, and AMERIBOR

8

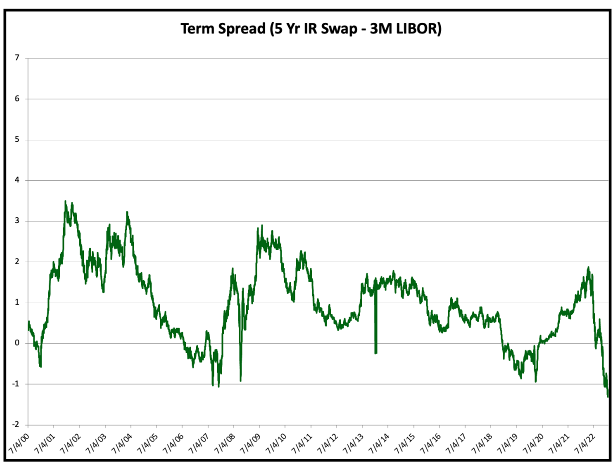
Selected Empirical Observations

- Base curve selection
  - UST-based
  - Libor-based
  - Other
- Interest swaps good proxy for term spreads
  - Collateralized, AA-rated banks
  - Longer time to maturity, higher spreads
- Credit spreads

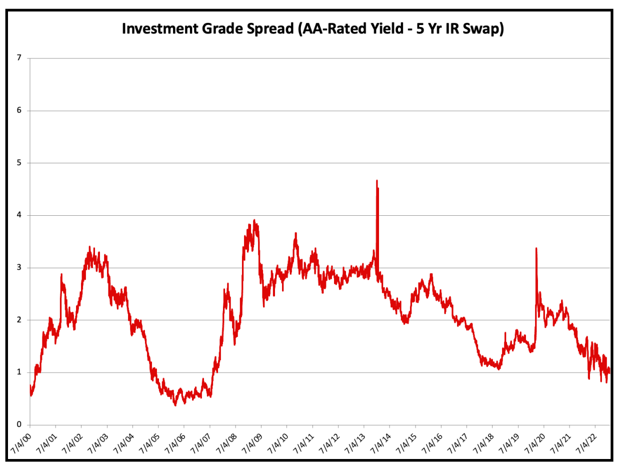
9



10



11

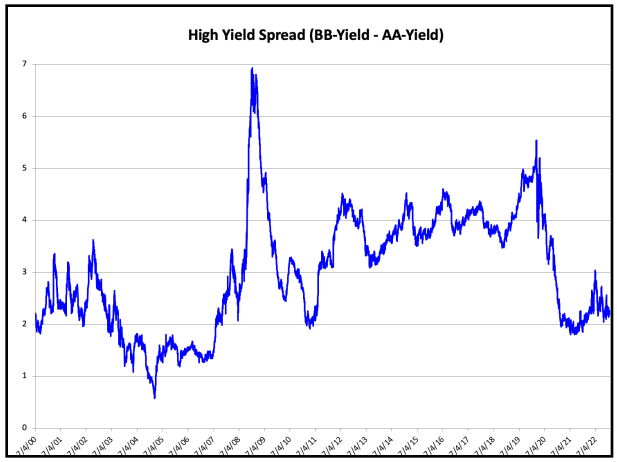


12

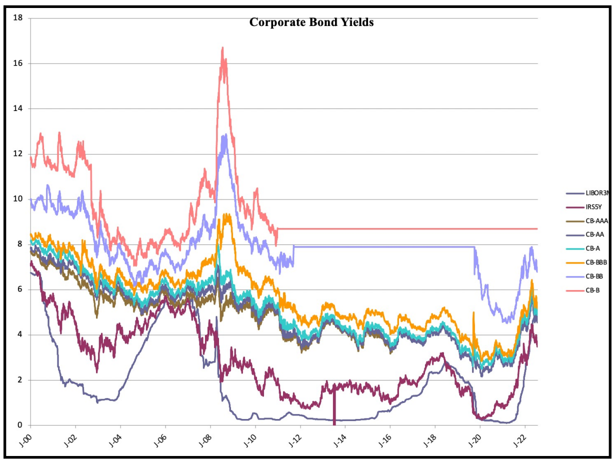
More on Credit Spreads

- Lower rating, higher spread (BB and B no longer reported)
- AA-rated bond fund is risky, but positive average return
- BB-rated fund is riskier, but may have higher average returns (period specific)

13



14

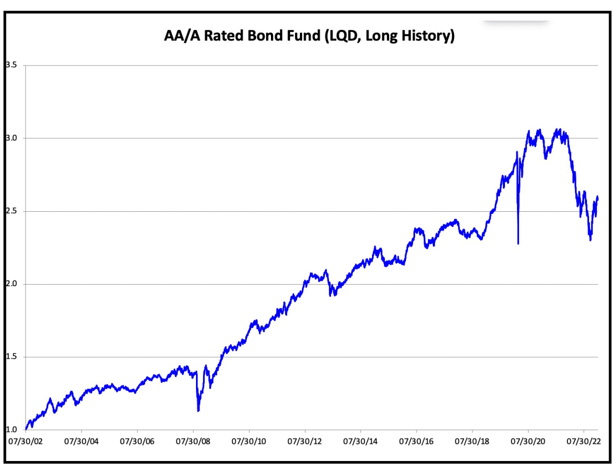


15

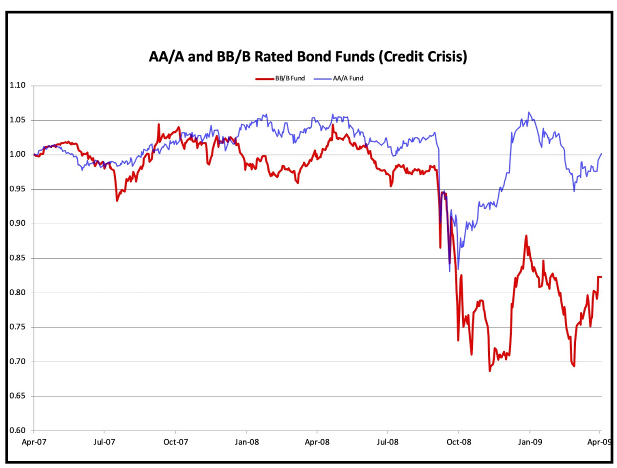
Analysis of Credit Spreads

- Credit spread reflects
  - Expected loss given default
  - Likelihood of default
  - Additional risk premium for bearing default risk
- Holding period return does not equal yield to maturity, even if held to maturity

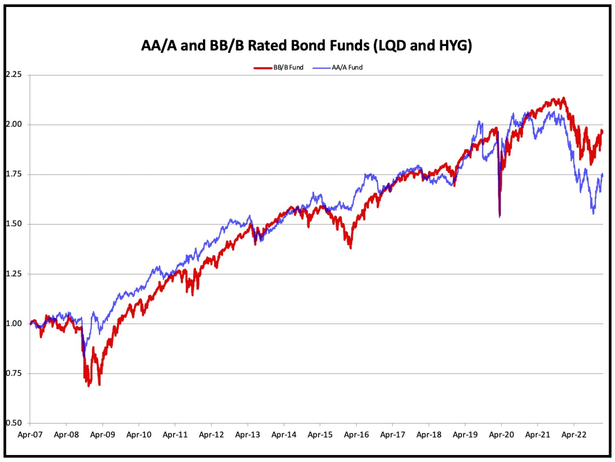
16



17



18



19

Quantitative Finance Materials

- LSC model applied to
  - Base curve
  - Credit spreads
  - Bond valuation
- Illustrate with BB yields

20

Application of LSC Model

- Corporate bond valuation model

$$V_B = \sum_{i=1}^{N_t} CF_i DF_i = \sum_{i=1}^{N_t} CF_i e^{-y_i \tau_i}$$

$$= \sum_{i=1}^{N_t} CF_i e^{-\left(r_i^{LSC} + sp_i^{LSC} + \varepsilon_i\right)\tau_i} = \sum_{i=1}^{N_t} CF_i e^{-\left(\sum_{j=0}^{N^r} x_{i,j} f_j^r + \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp} + \varepsilon_i\right)\tau_i}$$

- LSC model fit to both based rate (r) and spread(s) (sp)

$$y_i = \sum_{j=0}^{N^r} x_{i,j} f_j^r + \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp} + \varepsilon_i$$

21

LSC Curves

- Base rate curve

$$r_i^{LSC} \equiv \sum_{j=0}^{N^r} x_{i,j} f_j^r$$

- Spread curves

$$sp_i^{LSC} \equiv \sum_{j=0}^{N^{sp}} x_{i,j} f_j^{sp}$$

- Bond valuation

$$V_B \cong V_B^{LSC} = \sum_{i=1}^{N_t} CF_i DF_i^{LSC} = \sum_{i=1}^{N_t} CF_i e^{-\left(r_i^{LSC} + sp_i^{LSC}\right)\tau_i}$$

22

Data Inputs

- Bond inputs
  - CMT yield
  - BB spreads
- LSC inputs
  - Scalars
  - Number of factors

23

Required Functions

- Bond functions
  - Number of coupons, fraction period elapsed
  - Accrued interest, bond value (based on y), y
- LSC functions
  - Bond value based on LSC parameters
  - LSC rate for given maturity (for discount factors)

24

CMT Discount Rate Solution

- Discount rates based on CMT rates

$$V_B = Par = \sum_{i=1}^{N}\left(\frac{Coupon_N}{m}\right)Par\left(e^{-r_i^{LSC}\tau_i}\right)+Par\left(e^{-r_N^{LSC}\tau_N}\right)$$

$$1.0=\left(\frac{Coupon_N}{m}\right)\sum_{i=1}^{N}\left(e^{-r_i^{LSC}\tau_i}\right)+\left(e^{-r_N^{LSC}\tau_N}\right)$$

$$Coupon_N = m\frac{1.0-e^{-r_N^{LSC}\tau_N}}{\sum_{i=1}^{N} e^{-r_i^{LSC}\tau_i}} = m\frac{1.0-e^{-\sum_{j=0}^{N^r} x_{N,j}^r f_j^r \tau_N}}{\sum_{i=1}^{N} e^{-\sum_{j=0}^{N^r} x_{N,j}^r f_j^r \tau_i}} = m\frac{1.0-DF_N}{\sum_{i=1}^{N} DF_i}$$

25

R Code Optimization

- Minimize deviations between Coupon vector and inputted CMT rates

```
OptOutput <- optimx(par=x, fn=DiffSwRates, NFac = NFactors, S = Sc,
  NCMTs = NBaseCurve, MSR = MarketCMTRates,
  method=c('nlminb'), control=list(save.failures=FALSE, maxit=2500))
```

- Note if nlminb does not work, several other methods are attempted (BFGS, Nelder-Mead, and L-BFGS-B). See optimx documentation for details of methods

26

Audit Checks for CMT

- CMT spot rate function

```
# Based on LSC parameters, y, and other inputs, NFactors, Sc, NBaseCurve,
#  provide estimates of fitted input rates
SREstimates <- CMTRates(y, NFactors, Sc, NBaseCurve)
SREstimates
```

- CMT discount rate function

```
# Based on LSC parameters, y, and other inputs, NFactors, Sc, NBaseCurve,
#  provide estimates of fitted discount rates
DREstimates <- DiscountRates(y, NFactors, Sc, NBaseCurve)
DREstimates
```


27

BB Discount Rate Solution

- Discount rates based on BB rates

$$V_B = Par = \sum_{i=1}^{N}\left(\frac{Coupon_N}{m}\right)Par\left(e^{-r_i^{LSC,BB}\tau_i}\right)+Par\left(e^{-r_N^{LSC,BB}\tau_N}\right)$$

$$1.0=\left(\frac{Coupon_N}{m}\right)\sum_{i=1}^{N}\left(e^{-r_i^{LSC,BB}\tau_i}\right)+\left(e^{-r_N^{LSC,BB}\tau_N}\right)$$

$$Coupon_N = m\frac{1.0-e^{-r_N^{LSC,BB}\tau_N}}{\sum_{i=1}^{N} e^{-r_i^{LSC,BB}\tau_i}} = m\frac{1.0-e^{-\sum_{j=0}^{N^{BB}} x_{N,j}^{BB} f_j^{BB} \tau_N}}{\sum_{i=1}^{N} e^{-\sum_{j=0}^{N^{BB}} x_{i,j}^{BB} f_j^{BB} \tau_i}} = m\frac{1.0-DF_N^{BB}}{\sum_{i=1}^{N} DF_i^{BB}}$$

28

Computing Spread Factors

- Spread Factor Relations

$$\sum_{j=0}^{N^{BB}} x_{N,j}^{BB} f_j^{BB} = \sum_{j=0}^{N^r} x_{N,j}^r f_j^r + \sum_{j=0}^{N^{sp}} x_{N,j}^{sp} f_j^{sp}$$

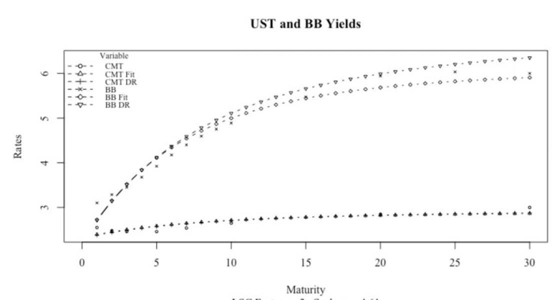
$$\sum_{j=0}^{N^{sp}} x_{N,j}^{sp} f_j^{sp} = \sum_{j=0}^{N^{BB}} x_{N,j}^{BB} f_j^{BB} - \sum_{j=0}^{N^r} x_{N,j}^r f_j^r$$

$$= \sum_{j=0}^{N^{BB}} x_{N,j}^{BB}\left(f_j^{BB}-f_j^r\right) = \sum_{n=0}^{N^{BB}} x_{N,j}^{BB} f_j^{sp}$$

$$\left(\text{Assuming } N_F^{BB}=N_F^r=N_F^{sp} \text{ and } s_n=s_n^{BB}=s_n^r=s_n^{sp}.\right)$$

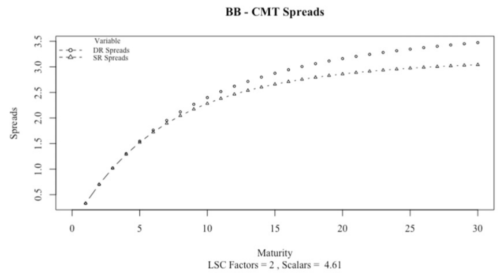
29

Figure 4.2.9 CMT and BB Yields With Fitted CMT and BB Yields and Fitted Discount Factors



30

Figure 4.2.10 CMT and BB Yields Less CMT Spread with Fitted Spot Rates and Fitted Discount Factors



Xerox BB Bond (2 factors)

- Expect quoted bond price with yield to maturity to fit well but not LSC BB-curve fit (interpret with caution)

```
> QuotedBondPrice; QuotedBondPriceEstYTM; QuotedBondPriceEstLSC
[1] 99.289
[1] 99.28767
[1] 99.13166
> PriceEstDiffYTM; PriceEstDiffLSC; PriceEstDiffLSCPct
[1] -0.001328109
[1] -0.1573414
[1] -0.1584681
```


Figure 4.2.11 CMT and BB Yields With Fitted CMT and BB Yields and Fitted Discount Factors

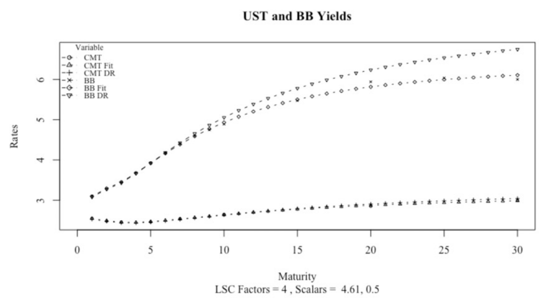
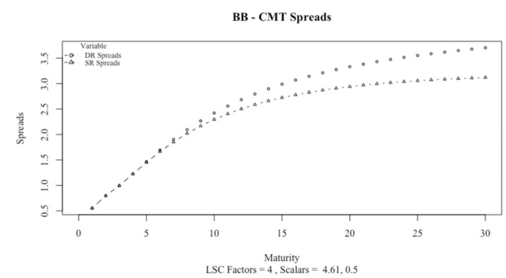


Figure 4.2.12 CMT and BB Yields Less CMT Spread with Fitted Spot Rates and Fitted Discount Factors



Xerox BB Bond (4 factors)

- More LSC factors does not mean closer fit

```
> QuotedBondPrice; QuotedBondPriceEstYTM; QuotedBondPriceEstLSC
[1] 99.289
[1] 99.28767
[1] 99.90041
> PriceEstDiffYTM; PriceEstDiffLSC; PriceEstDiffLSCPct
[1] -0.001328109
[1] 0.6114147
[1] 0.615793
```


Summary

- Corporate bond features
  - Maturity
  - Credit rating
  - Liquidity
  - Embedded optionality
- LSC decomposition
  - Base rate
  - Spread(s)