Module 4.3: Valuation Stocks

Learning objectives

- Review the traditional dividend discount model
- Extending the traditional dividend discount model to handle practical considerations
- Derive a robust *N*-stage dividend discount model that can handle multiple growth stages, multiple discount rates, and quarterly payments
- Explore ways to simplify stock valuation with LSC model

Executive summary

In this chapter, we review the historical performance of the U.S. stock market. We also introduced an N-Stage dividend discount model as well as applying the LSC model to lower the number of factors driving stock valuation, illustrated with R sample code. Further, we also apply the LSC valuation model and illustrated it with two factor LSC models applied to the cash flow growth rates and forward discount rates.

Central finance concepts

In this module we explore traditional dividend discount models with an effort to accommodate well-known attributes, such as paid quarterly but adjusted annually, apply multiple discount rates depending on a particular stage of dividend payments, and applying the LSC model to the present value of dividends. Further, we also introduce the LSC valuation model and illustrate it with two factor LSC models applied to the cash flow growth rates and forward discount rates. But first we make a quick review of the U.S. stock market history.

Historical review of U.S. stock market

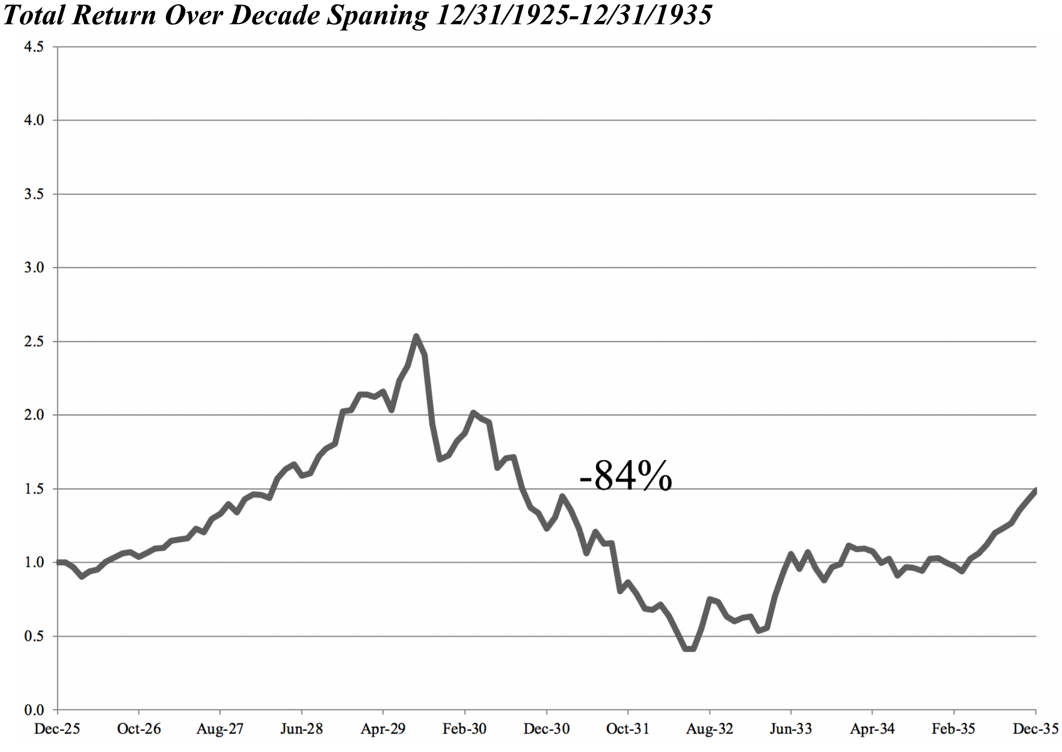
We briefly review the historical performance of the U.S. stock market starting on December 31, 1925. The main insights from this section are as follows. First, on average, stock prices rise significantly over time. Second, the percentage returns offered on broad stock market returns is consistent over time as demonstrated by the log scale. Figure 4.3.1 illustrates these two insights.

Figure 4.3.1 A Quick Tour of the U.S. Stock Market
Panel A Total Return per $1 Invested in12/31/25

Panel B Log Scale of Total Return per $1 Invested in12/31/25

Third, investing in the U.S. stock market is risky. Figure 4.3.2 documents that every decade there is at least one significant downturn. The stock market crash of 1929 was the greatest with a net loss from the peak to trough of –80%. Notice, however, that had you invested $1 on December 31, 1925 you would have ended the next decade up about 50%. Interestingly, every single decade ends up higher at the end than at the beginning. Also, every decade suffered a significant decline during the decade. The number indicated in each graph is the rate of return loss from peak to trough. Thus, if you never want to endure double digit losses, then you should avoid investing in the stock market.

Figure 4.3.2 Value-Weighted Returns in the U.S. Stock Market by Decade
Panel A Total Return Over Decade Spaning 12/31/1925-12/31/1935



Panel B Total Return Over Decade Spaning 12/31/1935-12/31/1945

Panel C Total Return Over Decade Spaning 12/31/1945-12/31/1955

Panel D Total Return Over Decade Spaning 12/31/1955-12/31/1965

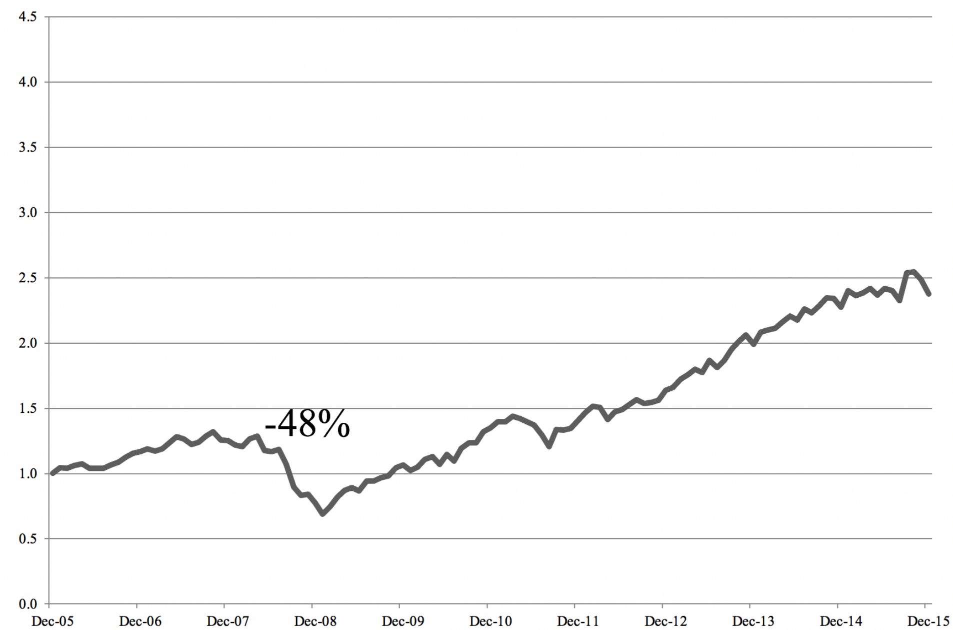
Panel E Total Return Over Decade Spaning 12/31/1965-12/31/1975

Panel F Total Return Over Decade Spaning 12/31/1975-12/31/1985

Panel G Total Return Over Decade Spaning 12/31/1985-12/31/1995

Panel H Total Return Over Decade Spaning 12/31/1995-12/31/2005

Panel I Total Return Over Decade Spaning 12/31/2005-12/31/2015



Panel J Total Return Over Decade Spaning 12/31/2015-12/31/2025

The key takeaway from this brief tour of the U.S. stock market is that those who are very long-term investors can easily see past the a significant downturn and not cash out. We now turn to introduce selected dividend discount models for valuing common stock.

The Gordon single factor dividend discount model

As discussed in Chapter 2, the traditional approach to common stock valuation is to forecast some future expected cash flows, typically dividends, and then to take the present value of this future expected cash flow stream. In 1959, Gordon argued that the appropriate discount rate increases with the degree of uncertainty related to the future dividend stream. Interestingly, there has been little effort to assign different discount rates across all dividends. Many market professionals seem to have concluded that the additional complexity is not worth the additional benefits. In most applications, the discount rate applied is the same for all dividends.

The dividend discount model (DDM) explored quantitatively falls within the discount factor adjusted approach to valuation. Recall from Chapter 2 that the DFAA method does not alter the investor's subjective cash flow probability distribution, rather the risk adjustment is taken in the discount rate applied. With DFAA, there must be sufficient structure imposed upon the state-space to compute at least the expected future cash flows and the appropriate risk premium.

In most applications of DDMs, the single stage growth rate is insufficient. We now explore one potential general solution to enriching the model.

The *N*-Stage dividend discount model

Based on Brooks and Helms (1990)[1] *N*-Stage Dividend Discount Model (NDDM), we extend their analysis by including stage varying discount rates as well as stage varying growth rates. The results covered in the quantitative section below provide a "closed-form" solution in the sense that an infinite sum is not required.

The NDDM comprises three parts: The initial stub period that address the portion of the current stage remaining until the beginning of the next stage. The series periods that comprise the finite series stages addressing different growth and discount rates. The final period that comprises the final infinite stage.

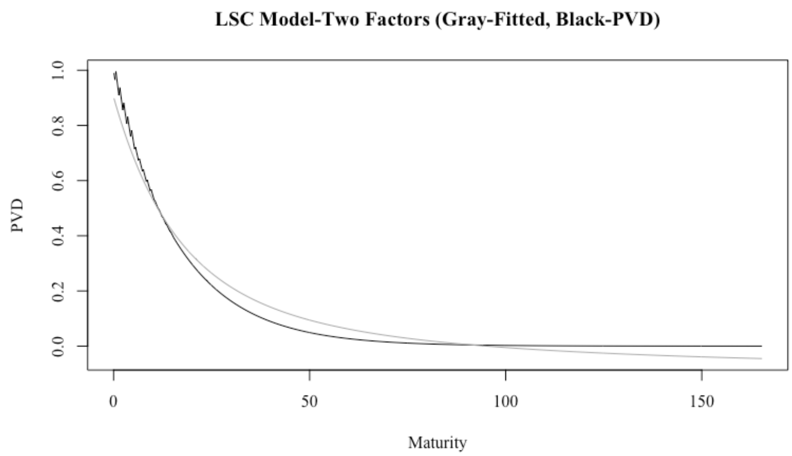
---

[1]See "An N Stage, Fractional Period, Quarterly Dividend Discount Model," *Financial Review*, 25 (November 1990), 651-657.

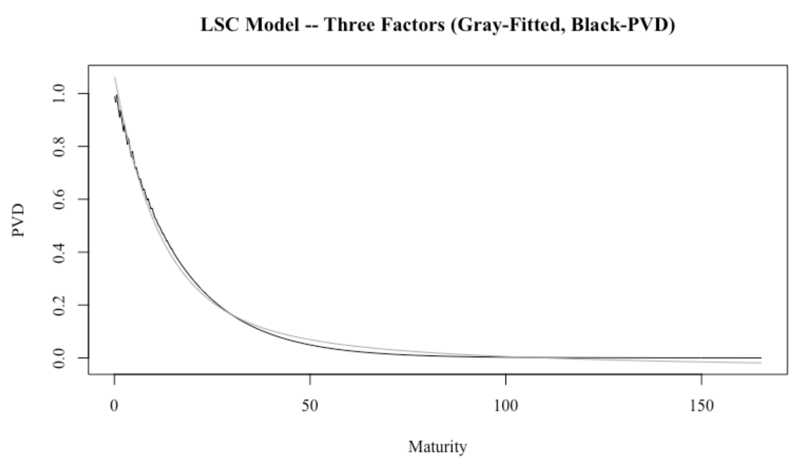
The NDDM is illustrated with several numerical examples and an initial effort is explored at applying the LSC model to the present value of dividends (PVD) function. The quantitative and coding is simply an illustration of the potential extensions feasible with a combination of quantitative capacity and R coding.

Figure 4.3.3 illustrates the four state DDM with LSC fit with 2, 3, and 4 factors. Thus, the 4 factor LSC fit appears adequate to model the present value of dividends.

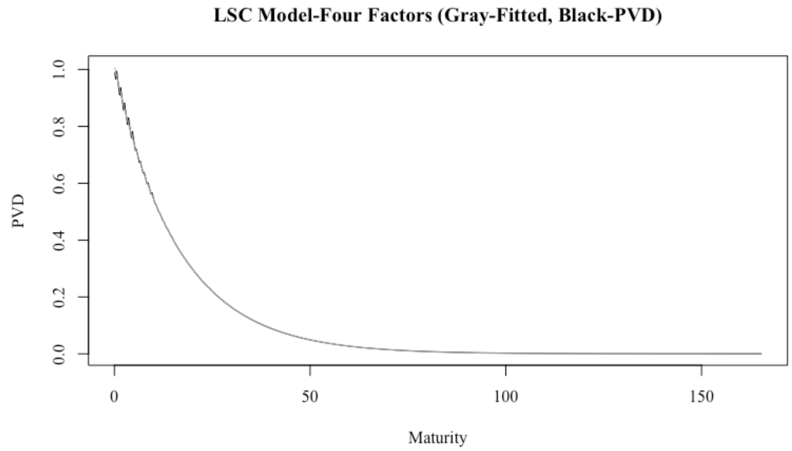
Figure 4.3.3 Four Stage Dividend Discount Model With LSC Fit
Panel A: Two Factor LSC Model Fit



Panel B: Three Factor LSC Model Fit



Panel C: Four Factor LSC Model Fit



LSC valuation model based on growth rates and forward discount rates

The LSC valuation model presented here is parsimonious and has numerous applications. For example, this approach can be applied to valuing common stocks as well as the entire firm. Our focus here will be on some generic instrument valuation, but it is straightforward to reframe it for other valuation tasks. Specifically, the presentation assumes some generic valuation task involving an expected future series of cash flows with known positive initial cash flow. If the initial known cash flow is zero or negative, then some simple modifications will be necessary but are not covered here.

We illustrate this approach assuming annual cash flows avoiding the nuances of quarterly payments. We assume cash flow rates are modeled within the LSC model framework. Specifically, we estimate the perpetual (level) growth rate, the short-term (slope) growth rate, and as many growth rate curvature factors as desired. In a similar fashion, we assume forward discount rates can also be modeled within the LSC model framework. Specifically, we estimate the perpetual (level) forward discount rate, the short-term (slope) forward discount rate, and as many forward discount rate curvature factors as desired.

This deployment of the LSC model is illustrated with the broad market ETF as well as several sector ETFs. We demonstrate interesting differences between these ETFs based on this unique perspective.

Quantitative finance materials

After a quick quantitative review of the Gordon single index DDM, we explore a more robust dividend discount model that addresses the salient factors related to actual dividend-paying stocks. We then turn to a unique application of the LSC model to stock valuation.

Gordon's single factor dividend discount model

From Chapter 2, we had the value of some financial instrument or portfolio as

$$P_i = \sum_{t=1}^{T} \sum_{j=1}^{m} \frac{1}{\left(1 + r_t + RP_{i,t,j}\right)^t} p_{t,j} CF_{i,t,j}, \tag{4.3.1}$$

where $p_{t,j}$ denotes the subjective probability based on a particular individual's perspective on future cash flows occurring at time t when state j occurs. For the application to common stocks (V_S) developed here, we assume the investor has a forecast of the expected future dividends. Thus, we have

$$V_S \equiv \sum_{i=1}^{\infty} PV\left(\tau_i, t\right)\left\{E_t\left[D\left(\tau_i, t\right)\right]\right\} = \sum_{i=1}^{\infty} DPV_i, \tag{4.3.2}$$

where $PV(\tau_i, t)$ denotes the present value of the ith dividend occurring τ_i years in the future and $D(\tau_i, t)$ denotes the uncertain ith dividend amount. Thus, the value of a stock today is simply the sum of the present value of all expected future dividend payments (DPV_i).

One simple application of this approach is known as the Gordon Growth Model. In this case, expected dividends are assumed to be paid annually, are assumed to grow as some constant growth rate, g, and these expected dividend payments are discounted at a constant rate, k. With these assumptions, we have

$$V_S = \frac{D_0(1+g)}{k-g}. \tag{4.3.3}$$

The discount rate is the cost of equity capital or the investor's required rate of return. The typical way the investor's required return is estimated is by using the risk-free rate plus a risk premium. One example is the Capital Asset Pricing Model (CAPM) which can be expressed here as

$$k = r + \beta\left[E(r_M) - r\right], \tag{4.3.4}$$

where r denotes the risk-free rate, r_M denotes the return on the market portfolio, and □ denotes the sensitivity of this stock to changes in the excess expected return of the market portfolio over the risk free rate.

The *N*-Stage dividend discount model

Recall from Equation (4.3.2) that V_S denotes the underlying instrument value at calendar time, t, as a function of the present value of expected future cash payments. We first review critical assumptions. Next, we provide an exhaustive notation list for convenience. Finally, we develop the NDDM.

Assumptions

- Dividends are paid quarterly at the time points in time each year, and further, the maturity time between payments is constant, $\Delta\tau = \Delta\tau_j = \tau_j - \tau_{j-1}$
- N stages of constant, continuously compounded, dividend growth (g_i) with corresponding constant, continuously compounded, discount rate (k_i),
- Initial maturity time is zero, $\tau_0 = 0$,
- By construction, initial growth rate is zero (as model next dividend payment), $g_0 = 0$,
- Each stage is for m_l years except the last stage, m_N has infinite payments,
- Dividends are constant for four quarters (1 year), and
- H dividends remain for the current year ($H \le 4$).

Detailed Notation

t – current time, expressed as a fraction of a year,
N – number of stages,
i – counter of quarterly dividends,
y – dividend years, $y = 0$ denotes current dividend year with up to 4 quarterly dividends remaining,
DPV_i – present value of quarterly dividend i,
P_y – present value of dividends paid in year y (not just one dividend, usually 4),
PV_y – present value of \$1 paid at last dividend in year y,
D_y – quarterly dividend paid in year y,
D_{-1} – last paid quarterly dividend,
$\Delta\tau_1 = \tau_1$ – time until next dividend payment at time t,
$\Delta\tau_q = \tau_q - \tau_{q-1} = \Delta\tau$ – time between two dividend payments ($q > 1$) at time t,
$4\Delta\tau = 1$ – four quarterly periods is one year,
f_y – annualized, continuously compounded, forward discount rate appropriate for year y,

g_y – annualized, continuously compounded, dividend growth rate appropriate for year y applied to the quarterly dividend payment, and

m_j – number of years for stage m_j.

Infinite and finite series with continuous compounding

With continuously compounded forward rates as well as continuously compounded growth rates, we note the following well-known results:

$$B_\infty = \sum_{i=1}^{\infty} e^{-x(i)} = \frac{1}{e^x - 1} \text{ and (Infinite series)} \tag{4.3.5}$$

$$B_N = \sum_{i=1}^{N} e^{-x(i)} = \frac{1 - e^{-x(N)}}{e^x - 1}. \text{ (Finite series)} \tag{4.3.6}$$

Proof: Expanding the infinite series,

$$B_\infty = \sum_{i=1}^{\infty} e^{-x(i)} = e^{-x} + e^{-2x} + e^{-3x} + \ldots.$$

Dividing by e^{-x},

$$\hat{B}_\infty = \frac{B_\infty}{e^{-x}} = 1 + e^{-x} + e^{-2x} + \ldots.$$

Taking the difference,

$$\hat{B}_\infty - B_\infty = 1.$$

Substituting,

$$\hat{B}_\infty - B_\infty = \frac{B_\infty}{e^{-x}} - B_\infty = 1.$$

Rearranging,

$$\hat{B}_\infty - B_\infty = B_\infty\left(\frac{1}{e^{-x}} - 1\right) = B_\infty \frac{1 - e^{-x}}{e^{-x}} = 1.$$

Thus, solving for B_∞,

$$B_\infty = \frac{e^{-x}}{1 - e^{-x}} = \frac{1}{e^x - 1}.$$

The finite series result is just the difference between two infinite series results, factoring $e^{-x(N)}$.

N-stage dividend discount model development

The present value of dividends paid in the current dividend year ($y = 0$, stub year) can be expressed as

$$P_0 = D_{-1} \sum_{q=1}^{H} e^{-f_0 \sum_{k=1}^{q} \Delta\tau_k}. \tag{4.3.7}$$

Every quarter is assigned to have 0.25 years between dividend payments except for the initial period. Thus, $\Delta\tau_k = 0.25$ except for $k = 1$. The initial time period, $k = 1$, we note $\Delta\tau_1 = \tau_1 \leq 0.25$. The time summation accounts for the first stub period. The present value of \$1 at the end of year 0 is

$$PV_0 = e^{-f_0 \sum_{k=1}^{H} \Delta\tau_k}. \tag{4.3.8}$$

For example, suppose there are two quarters left in this dividend year ($H = 2$). Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, and $f_0 = 10\%$, we have the present value of all dividends paid in the stub year is

$$P_0 = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} = \$1\left[e^{-0.1(0.1)} + e^{-0.1(0.1+0.25)}\right] = \$0.990050 + \$0.965605 = \$1.955655. \quad (4.3.9)$$

and the present value of the last dividend paid in the stub year is

$$PV_0 = e^{-f_0\sum_{k=1}^{H}\Delta\tau_k} = \$1e^{-0.1(0.1+0.25)} = \$0.965605. \quad (4.3.10)$$

For all subsequent years ($y > 0$), there are four constant dividend payments each made $\Delta\tau_1 = 0.25$ from the prior dividend payment. The present value at the end of the last year (beginning of year y) of the four dividend payments in year y is

$$D_y\left[e^{-f_y\Delta\tau(1)} + e^{-f_y\Delta\tau(2)} + e^{-f_y\Delta\tau(3)} + e^{-f_y\Delta\tau(4)}\right]. \quad (4.3.11)$$

Dividend growth implies $D_y = D_{y-1}e^{g_y}$ and PV_{y-1} is the present value of cash flows received at time τ_{y-1}, the time of the last dividend paid in year $y - 1$. Thus, the present value of the four dividends paid in year y can be expressed as

$$P_y = PV_{y-1}\left(D_{y-1}e^{g_y}\right)\left[e^{-f_y\Delta\tau(1)} + e^{-f_y\Delta\tau(2)} + e^{-f_y\Delta\tau(3)} + e^{-f_y\Delta\tau(4)}\right]. \quad (4.3.12)$$

Recall $4\Delta\tau = 1$, thus rearranging,

$$P_y = PV_{y-1}D_{y-1}\left[e^{f_y\Delta\tau(3)} + e^{f_y\Delta\tau(2)} + e^{f_y\Delta\tau(1)} + 1\right]e^{-(f_y - g_y)}. \quad (4.3.13)$$

Note $PV_{y-1}D_{y-1}$ is the present value of the last dividend paid in the prior year. Let the present value of the last dividend in the prior year *grossed up* by the future value of a \$1 annuity over the next year be expressed as

$$P_{y-} \equiv PV_{y-1}D_{y-1}\left[e^{f_y\Delta\tau(3)} + e^{f_y\Delta\tau(2)} + e^{f_y\Delta\tau(1)} + 1\right]. \quad (4.3.14)$$

Thus, the present value of the dividends in one particular year can be bootstrapped from the prior year simply as

$$P_y = P_{y-}e^{-(f_y - g_y)}. \quad (4.3.15)$$

Note that P_{y-} is not P_{y-1}. P_{y-} is the present value of the last dividend in the prior year grossed up by the future value of a \$1 annuity over the next year, a convenient notation for analysis below. Recall P_{y-1} is the present value of the dividends paid in year $y - 1$.

For example, given the data above where there are two quarters left in this dividend year ($H = 2$), $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, and $f_0 = 10\%$. We further assume a dividend growth rate of 5%. Thus, for the first full year of Stage 1, we have the present value of the four dividends paid is

$$
\begin{aligned}
P_1 &= PV_0 D_0\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]e^{-(f_1-g_1)} \\
&= 0.965605(\$1)\left[e^{0.1(0.25)(3)}+e^{0.1(0.25)(2)}+e^{0.1(0.25)(1)}+1\right]e^{-(0.1-0.05)} \\
&= \$0.965605\left[1.077884+1.051271+1.025315+1\right]0.951229 \\
&= \$0.965605(4.154470)0.951229 = \$3.815928
\end{aligned}
\tag{4.3.16}
$$

Thus, the present value of dividends in year 1 at the beginning of the first full year (y = 1) is $\$3.951852 = \$1(4.154470)0.951229$ and thus the present value at time t is \$3.815928.

Stage N

The beginning of the first dividend year in the N^{th} stage is n_N years from the evaluation date or

$$
n_N = \sum_{l=1}^{N-1} m_l . \tag{4.3.17}
$$

For example, assume $N = 3$ stages, $H = 2$ quarters, $m_1 = 5$ years, and $m_2 = 10$ years. Thus, $n_N = 15$ and the final stage would begin at the end of quarter 62 [= 15(4) + 2]. Stage 1 would begin at the end of quarter 2 (start of quarter 3). Stage 2 would begin at the end of quarter 22 (start of quarter 23).

Stage N present value is (based on the infinite series results above)

$$
\hat{S}_N = P_{n_N-}\sum_{i=1}^{\infty} e^{-(f_N-g_N)i} = P_{n_N-}\frac{1}{e^{(f_N-g_N)}-1}. \tag{4.3.18}
$$

Stage j (j<N)

For stage j ($j < N$), we have (based on the finite series results above)

$$
\hat{S}_j = P_{n_j-}\sum_{i=1}^{m_j} e^{-(f_j-g_j)i} = P_{n_j-}\frac{1}{e^{(f_j-g_j)}-1} - P_{(n_j+m_j)-}\frac{1}{e^{(f_j-g_j)}-1} = P_{n_j-}\frac{1-e^{(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1}, \tag{4.3.19}
$$

where PV_{m_j} denotes the present value at f_j (appropriate discount rate for stage j) applied for m_j years.

N-Stage dividend discount model

Combining the results, we have

$$
V_S = \sum_{i=1}^{\infty} P_i = P_0 + \sum_{j=1}^{N}\hat{S}_j = P_0 + \sum_{j=1}^{N-1} P_{n_j-}\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1} + P_{n_N-}\frac{1}{e^{(f_N-g_N)}-1}. \tag{4.3.20}
$$

Thus, the value of the stock at time t is simply the present value of the infinite sum of future dividends. With the N-stage dividend discount model (NDDM), we reduce the infinite sum of present values of each future dividend payment (P_i) to the present value of the stub period (P_0) plus the present value of N sets of dividend payments ($\hat{S}_j$).

Substituting back to the original inputs, we have

$$
\begin{aligned}
V_S = P_0 &+ \sum_{j=1}^{N-1} PV_{n_j-1}D_{n_j-1}\left[e^{f_j\Delta\tau(3)}+e^{f_j\Delta\tau(2)}+e^{f_j\Delta\tau(1)}+1\right]\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1} \\
&+ PV_{n_N-1}D_{n_N-1}\left[e^{f_N\Delta\tau(3)}+e^{f_N\Delta\tau(2)}+e^{f_N\Delta\tau(1)}+1\right]\frac{1}{e^{(f_N-g_N)}-1}
\end{aligned}
\tag{4.3.21}
$$

Further substitutions yields the closed-form NDDM:

$$
\begin{aligned}
V_S &= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} \\
&+\sum_{j=1}^{N-1}\left(e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\prod_{i=1}^{j-1}e^{-f_i m_i}\right)\left(D_{-1}\prod_{i=1}^{j-1}e^{g_i m_i}\right)\left[e^{f_j\Delta\tau(3)}+e^{f_j\Delta\tau(2)}+e^{f_j\Delta\tau(1)}+1\right]\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1}. \\
&+\left(e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\prod_{i=1}^{N-1}e^{-f_i m_i}\right)\left(D_{-1}\prod_{i=1}^{N-1}e^{g_i m_i}\right)\left[e^{f_N\Delta\tau(3)}+e^{f_N\Delta\tau(2)}+e^{f_N\Delta\tau(1)}+1\right]\frac{1}{e^{(f_N-g_N)}-1}
\end{aligned}
\tag{4.3.22}
$$

Rearranging and simplifying, the NDDM contains three components:

$$Stub = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k}, \text{ (Stub Component)} \tag{4.3.23}$$

$$Series = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\sum_{j=1}^{N-1}\left[\prod_{i=1}^{j-1}e^{-(f_i-g_i)m_i}\right]\left[e^{f_j\Delta\tau(3)}+e^{f_j\Delta\tau(2)}+e^{f_j\Delta\tau(1)}+1\right]\frac{1-e^{-(f_j-g_j)m_j}}{e^{(f_j-g_j)}-1}, \text{(Series Component)} \tag{4.3.24}$$

$$Final = D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[\prod_{i=1}^{N-1}e^{-(f_i-g_i)m_i}\right]\left[e^{f_N\Delta\tau(3)}+e^{f_N\Delta\tau(2)}+e^{f_N\Delta\tau(1)}+1\right]\frac{1}{e^{(f_N-g_N)}-1}. \text{ (Final Component)} \tag{4.3.25}$$

The N-stage DDM can be expressed simply as

$$V_S = Stub + Series + Final. \tag{4.3.26}$$

We now take this model out for a test drive.

N-stage dividend discount model examples
To illustrate the NDDM, we briefly compute stock valuations for three cases, one stage no growth model, one stage growth model, and two stage growth model. With this foundation, the NDDM can easily be deployed with as many stages as required.

One stage no growth model (N = 1, g = 0): Suppose the dividend growth rate is zero ($N = 0$, $g = 0$) and there are two quarters left in this dividend year ($H = 2$). Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, and $f_0 = 10\%$, we have

$$
\begin{aligned}
V_S &= \sum_{i=1}^{\infty}P_i = P_0 + \sum_{j=1}^{1}\hat{S}_j = \hat{S}_0 = D_{-1}\sum_{q=1}^{H}e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + P_{1-}\sum_{q=1}^{\infty}e^{-f_0\Delta\tau q} \\
&= D_{-1}\sum_{q=1}^{H}e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + PV_0 D_0\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]\frac{1}{e^{f_1}-1} \qquad . \\
&= D_{-1}\sum_{q=1}^{H}e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_0\Delta\tau(3)}+e^{f_0\Delta\tau(2)}+e^{f_0\Delta\tau(1)}+1\right]\frac{1}{e^{f_0}-1}
\end{aligned}
\tag{4.3.27}
$$

Or in this particular case,

$$
\begin{aligned}
V_S &= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_0\Delta\tau(3)}+e^{f_0\Delta\tau(2)}+e^{f_0\Delta\tau(1)}+1\right]\frac{1}{e^{f_0}-1}\\
&= \$1\left[e^{-0.1(0.1)}+e^{-0.1(0.1+0.25)}\right]+\$1e^{-0.1(0.1+0.25)}\left[e^{0.1(0.25)(3)}+e^{0.1(0.25)(2)}+e^{0.1(0.25)(1)}+1\right]\frac{1}{e^{0.1}-1}\\
&= 0.990050+0.965605+0.965605\left(1.077884+1.051271+1.025315+1\right)\frac{1}{1.105171-1}\\
&= 1.955655+0.965605\left(4.154470\right)\left(9.508325\right)=40.099033
\end{aligned} \quad . \quad (4.3.28)
$$

One stage growth model (N = 1, g > 0): Suppose this one stage dividend growth rate is 5% ($N = 1$, g = 5%) and there are two quarters left in this dividend year ($H = 2$). Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, f_0 = 10%, and f_1 = 12%, we have

$$
\begin{aligned}
V_S &= \sum_{i=1}^{\infty} P_i = P_0 + \sum_{j=1}^{1}\hat{S}_j = P_0 + \hat{S}_1 = D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + P_{1-}\sum_{q=1}^{\infty} e^{-(f_1-g_1)\Delta\tau q}\\
&= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + PV_0 D_0\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]\frac{1}{e^{(f_1-g_1)}-1}\\
&= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]\frac{1}{e^{(f_1-g_1)}-1}
\end{aligned} \quad . \quad (4.3.29)
$$

Or in this particular case,

$$
\begin{aligned}
V_S &= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} + D_{-1}e^{-f_0\sum_{k=1}^{H}\Delta\tau_k}\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]\frac{1}{e^{(f_1-g_1)}-1}\\
&= \$1\left[e^{-0.1(0.1)}+e^{-0.1(0.1+0.25)}\right]+\$1e^{-0.1(0.1+0.25)}\left[e^{0.12(0.25)(3)}+e^{0.12(0.25)(2)}+e^{0.12(0.25)(1)}+1\right]\frac{1}{e^{(0.12-0.05)}-1}\\
&= 1.955655+0.965605\left(1.094174+1.061837+1.030455+1\right)\frac{1}{1.072508-1}\\
&= 1.955655+0.965605\left(4.186466\right)13.791582=\$57.707746
\end{aligned} \quad . \quad (4.3.30)
$$

Two stage model (N = 2): Suppose this two stage model has dividend growth rates of 5% (Stage 1) and 2% (Stage 2) ($N = 2$, $g_1 = 5\%$, $g_2 = 2\%$). There are two quarters left in this dividend year ($H = 2$) and 5 years in Stage 1. Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, $f_0 = 10\%$, $f_1 = 12\%$, and $f_2 = 9\%$ we have

$$
\begin{aligned}
V_S &= \sum_{i=1}^{\infty} P_i = P_0 + \sum_{j=1}^{2}\hat{S}_j = P_0 + \hat{S}_1 + \hat{S}_2\\
&= D_{-1}\sum_{q=1}^{H} e^{-f_0\sum_{k=1}^{q}\Delta\tau_k}\\
&+ e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} D_{-1}\left[e^{f_1\Delta\tau(3)}+e^{f_1\Delta\tau(2)}+e^{f_1\Delta\tau(1)}+1\right]\frac{1-e^{-(f_1-g_1)m_1}}{e^{(f_1-g_1)}-1}\\
&+ \left(e^{-f_0\sum_{k=1}^{q}\Delta\tau_k} e^{-f_1 m_1}\right)\left(D_{-1}e^{g_1 m_1}\right)\left[e^{f_2\Delta\tau(3)}+e^{f_2\Delta\tau(2)}+e^{f_2\Delta\tau(1)}+1\right]\frac{1}{e^{(f_2-g_2)}-1}
\end{aligned} \quad . \quad (4.3.31)
$$

Or in this particular case,

$$
\begin{aligned}
V_S &= \$1\left[e^{-0.1(0.1)} + e^{-0.1(0.1+0.25)}\right] \\
&+\$1e^{-0.1(0.1+0.25)}\left[e^{0.12(0.25)(3)} + e^{0.12(0.25)(2)} + e^{0.12(0.25)(1)} + 1\right]\frac{1-e^{-(0.12-0.05)(5)}}{e^{(0.12-0.05)}-1} \\
&+\$1e^{-0.1(0.1+0.25)}e^{-0.12(5)}e^{0.05(5)}\left[e^{0.09(0.25)(3)} + e^{0.09(0.25)(2)} + e^{0.09(0.25)(1)} + 1\right]\frac{1}{e^{(0.09-0.02)}-1} \\
&= 1.955655 + 0.965605(1.094174 + 1.061837 + 1.030455 + 1)\frac{1-0.704688}{1.072508-1} \\
&+0.965605(0.548811)(1.069830 + 1.046028 + 1.022755 + 1)13.791582 \\
&= 1.955655 + 0.965605(4.186466)4.072818 \\
&+0.965605(0.548811)(1.284025)4.138613(13.791582) \\
&= 1.955655 + 16.464256 + 38.838714 = \$57.258625
\end{aligned}
\tag{4.3.32}
$$

Three stage model (N = 3): Now suppose a three stage model has dividend growth rates of 6% (Stage 1), 3% (Stage 2), and 0% (Stage 3) ($N = 3$, $g_1 = 5\%$, $g_2 = 2\%$, and $g_3 = 0\%$). There are two quarters left in this dividend year ($H = 2$), 5 years in both Stage 1 and Stage 2. Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, $f_0 = 10\%$, $f_1 = 12\%$, $f_2 = 9\%$, and $f_3 = 6\%$, Table 4.3.1 is generated by the R program:

Table 4.3.1. Three stage dividend discount model

Stage	ForwardRate	GrowthRate	YearsInStage	SeriesValue	InitialDividend
0	10.0%		2	$1.955655	$1.000000
1	12.0%	6.0%	5	$16.943629	$1.000000
2	9.0%	3.0%	5	$12.408675	$1.349859
3	6.0%	0.0%		$35.064727	$1.568312
			Stock Value	$66.372687	
NOTE: YearsInStage 0 denotes remaining dividends.					

Four stage model (N = 4): Now suppose a four stage model has dividend growth rates of 6% (Stage 1), 3% (Stage 2), 1% (Stage 3), and 0% (Stage 4) ($N = 4$, $g_1 = 5\%$, $g_2 = 2\%$, $g_3 = 1\%$, and $g_4 = 0\%$). There are two quarters left in this dividend year ($H = 2$), 5 years in Stage 1, Stage 2, and Stage 3. Assuming $\Delta\tau_1 = \tau_1 = 0.1$, $D_{-1} = \$1$, $f_0 = 10\%$, $f_1 = 12\%$, $f_2 = 9\%$, $f_3 = 7\%$, and $f_4 = 6\%$, Table 4.3.2 is generated by the R program:

Table 4.3.2. Four stage dividend discount model

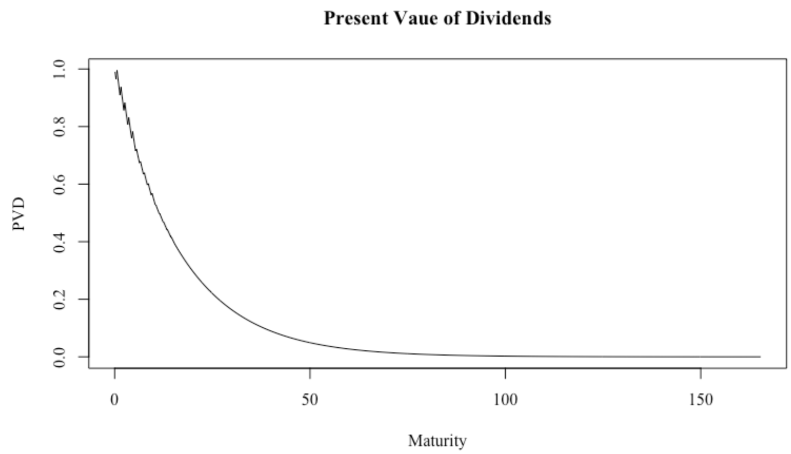
Stage	ForwardRate	GrowthRate	YearsInStage	SeriesValue	InitialDividend
0	10.0%		2	$1.955655	$1.000000
1	12.0%	6.0%	5	$16.943629	$1.000000
2	9.0%	3.0%	5	$12.408675	$1.349859
3	7.0%	1.0%	5	$9.122746	$1.568312
4	6.0%	0.0%		$25.976589	$1.648721
				$66.407295	

We now turn to an innovative application of the LSC model to the present value of dividend payments.

Applying the LSC model to the present value of expected dividend payments

From Module 4.1 and 4.2, we learned the mechanics of the LSC model applied to bond yields. We now apply the LSC model to the present value of dividends for a single stock. The four stage model above is illutrated in Figure 4.3.4. The jaggedness near zero highlights the assumption that dividends are changes once per year at a stated growth rate and the four quarterly dividends are based upon it. Thus, higher dividends will result in higher present values but as we move out into the future the discounting effect dominates. Thus, except for the first few years, the present value function is relatively smooth. Thus, the LSC model may help in factor reduction.

Figure 4.3.4 Present Value of Dividends Based on Four Stage Model



Thus, applying the LSC model, we have

$$y_{PVD,i} = \sum_{j=0}^{N} x_{i,j} f_j \,, \tag{4.3.33}$$

where $y_{PVD,i}$ denotes some input maturity time variables such as an interest rate for some maturity corresponding to i, $x_{i,j}$ denotes input LSC coefficients based on some maturity and some factor, and f_j denotes the output factors. Like the LSC model in general form, this LSC model application assumes

$$x_{i,0} = 1,\ x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i/s_1}\right), \text{ and } x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i/s_j}\right) - e^{-\tau_i/s_j}; j > 1, \tag{4.3.34}$$

where the variables are as previously defined in Module 3.5. We illustrate this LSC model fit with several different number of factors in the coding section below.

LSC valuation model based on growth rates and forward discount rates

We assume a generic valuation task involving an expected future series of cash flows (CF_i) with known positive initial cash flow (CF_0). If the initial known cash flow is zero or negative, then some simple modifications will be necessary but are not covered here.

We illustrate this approach assuming annual cash flows avoiding the nuances of quarterly payments. We assume cash flow rates (g_i) are modeled within the LSC model framework. Specifically, we estimate the perpetual (level) growth rate, the short-term (slope) growth rate, and as many growth rate curvature factors as desired. In a similar fashion, we assume forward discount rates (f_i) can also be modeled within the LSC model framework. Specifically, we estimate the perpetual (level) forward discount rate, the short-term (slope) forward discount rate, and as many forward discount rate curvature factors as desired.

Based on the notation introduced above, the asset value (V) can be expressed as

$$V = CF_0 \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i} g_j \tau_j}}{e^{\sum_{j=1}^{i} f_j \tau_j}} = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} \left(g_j - f_j\right)\tau_j} = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left(f_j - g_j\right)\tau_j} \,. \tag{4.3.35}$$

We illustrate this approach with a two factor LSC model for both the growth rate and forward discount rate. Each rate is estimated based on the LSC model as (here we let $f_0 = L_g$ or L_f and $f_1 = S_g$ or S_f)

$$g_j = L_g + sc_{g,j} S_g \text{ and} \tag{4.3.36}$$

$$f_j = L_f + sc_{f,j} S_f, \tag{4.3.37}$$

where (we consider the possibility where the scalar for the expected cash flow growth rates is different from the scalar for the forward discount rates)

$$sc_{g,j} = \frac{s_g}{\tau_i}\left(1 - e^{-\tau_i/s_g}\right) \text{ and} \tag{4.3.38}$$

$$sc_{f,j} = \frac{s_f}{\tau_i}\left(1 - e^{-\tau_i/s_f}\right). \tag{4.3.39}$$

Substituting Equations (4.3.36) and (4.3.37) into Equation (4.3.35), we have

$$V = CF_0 \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[L_f + sc_{f,j}S_f - \left(L_g + sc_{g,j}S_g\right)\right]\tau_j}. \tag{4.3.40}$$

Equation (4.3.40) is an elegant yet concise way to express instrument valuation.

The level parameter for the growth rate could be estimated based on some long-term base interest rate, such as the fixed rate on the 30-year Libor-based interest rate swap. A company would not be expected to sustain a growth rate in excess of the base perpetual interest rate. A company would also not be expected to survive if they could not sustain a growth rate at least equal to the base perpetual interest rate. [2]

We now explore some ideas related to the level forward discount rate. Note by rearranging Equation (4.3.40), we have

$$V = CF_0 \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)}. \tag{4.3.41}$$

We can express the value per unit of cash flow (*VCF*) as

$$VCF = \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}S_f - sc_{g,j}S_g\right)}. \tag{4.3.42}$$

Note, we often have the cash flow yield as an input, denoted *cy*. Thus,

$$VCF = \frac{V}{CF_0} = \frac{V}{cy(V)} = \frac{1}{cy}. \tag{4.3.43}$$

As maturity tends to positive infinity, the slope regression independent variables tend to zero or

$$\lim_{\tau_i \to \infty}\left(sc_{g,j}\right) = \lim_{\tau_i \to \infty}\left[\frac{s_g}{\tau_i}\left(1 - e^{-\tau_i/s_g}\right)\right] = 0 \text{ and} \tag{4.3.44}$$

$$\lim_{\tau_i \to \infty}\left(sc_{f,j}\right) = \lim_{\tau_i \to \infty}\left[\frac{s_f}{\tau_i}\left(1 - e^{-\tau_i/s_f}\right)\right] = 0. \tag{4.3.45}$$

[2] See Aswath Damodaran's CFA Institute's 2020 Annual Conference presentation. He uses the 10-year U.S. Treasury rate with a two stage model. See https://annual.cfainstitute.org/speakers/aswath-damodaran/.

For some very distant maturity (n), the VCF ratio will tend to (where the indicator function, i, increments from the future point in time, n, or

$$VCF_n = \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right)}. \tag{4.3.46}$$

Multiply both sides by $e^{\left(L_f - L_g\right)}$, we get

$$VCFe^{\left(L_f - L_g\right)} = \sum_{i=0}^{\infty} e^{-i\left(L_f - L_g\right)} = 1 + \sum_{i=1}^{\infty} e^{-i\left(L_f - L_g\right)}. \tag{4.3.47}$$

Subtracting Equation (4.3.46) from Equation (4.3.47), we have simply

$$VCF\left[e^{\left(L_f - L_g\right)} - 1\right] = 1 \text{ or}$$

$$VCF = \frac{1}{e^{\left(L_f - L_g\right)} - 1}. \tag{4.3.48}$$

If we assume the long run growth rate is proxied by some long run base interest rate ($\hat{L}_g$), then we can infer the long run forward discount rate ($\hat{L}_f$), based on an estimate of a financial instrument's long run projected VCF, denoted $\widehat{VCF}_L$, or

$$\hat{L}_f = \ln\left(1 + \frac{1}{\widehat{VCF}_L}\right) + \hat{L}_g. \tag{4.3.49}$$

Thus, we assume that VCF ratio will mean revert to the average VCF ($\overline{VCF}$), but they do not necessarily completely revert. Rather we can assume a dampener, D, where

$$\widehat{VCF}_L = VCF + D\left(\overline{VCF} - VCF\right). \tag{4.3.50}$$

Finally, as maturity tends to zero, the slope regression independent variables tend to one or

$$\lim_{\tau_i \to 0}\left(sc_{g,j}\right) = \lim_{\tau_i \to 0}\left(\frac{1 - e^{-\tau_i/s_g}}{\frac{\tau_i}{s_g}}\right) = \lim_{\tau_i \to 0}\left[\frac{\frac{\partial}{\partial \tau_i}\left(1 - e^{-\tau_i/s_g}\right)}{\frac{\partial}{\partial \tau_i}\left(\frac{\tau_i}{s_g}\right)}\right].$$
$$= \lim_{\tau_i \to 0}\left[\frac{\frac{1}{s_g}e^{-\tau_i/s_g}}{\frac{1}{s_g}}\right] = 1 \tag{4.3.51}$$

Similarly,

$$\lim_{\tau_i \to 0}\left(sc_{f,j}\right) = \lim_{\tau_i \to 0}\left[\frac{s_f}{\tau_i}\left(1 - e^{-\tau_i/s_f}\right)\right] = 1. \tag{4.3.52}$$

Thus, the remaining tasks for the analyst is to estimate reasonable values for the slope coefficients, S_g and S_f, where

$$VCF = \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[\hat{L}_f + sc_{f,j}\hat{S}_f - \left(\hat{L}_g + sc_{g,j}\hat{S}_g\right)\right]}. \tag{4.3.53}$$

The entire valuation exercise is reduced to estimating four critical values based on a two factor LSC valuation model. Clearly, one can easily assume more factors that would require more parameter estimates.

LSC valuation model application to exchange-traded funds tied to the S&P 500 index

The LSC valuation model is illustrated with the broad market ETF with ticker symbol SPY that seeks to replicate the performance of the S&P 500 index including dividends. Further, we also examine nine sector or industry ETFs that seek to replicate sectors of the S&P 500 index including dividends. Table 4.3.3 presents the inputs required for the two factor LSC valuation model. DY denotes the dividend yield and DR denotes the estimated annually compounded discount rate.

Table 4.3.3. Two factor LSC valuation model inputs

Industry	Ticker	Price	DY	DR
Broad Market	SPY	$311.17	1.90%	8.00%
Technology	XLK	$105.09	1.18%	9.00%
Financial	XLF	$22.99	2.71%	10.00%
Industrials	XLI	$68.70	2.30%	7.80%
Consumer Discretionary	XLY	$129.00	1.24%	8.20%
Materials	XLB	$56.10	2.21%	12.00%
Healthcare	XLV	$101.04	2.33%	9.80%
Utilities	XLU	$57.61	3.52%	7.50%
Consumer Stables	XLP	$58.92	2.83%	6.50%
Energy	XLE	$37.11	4.00%	14.00%

Selected other inputs are provided in Table 4.3.4, where *Nf* denotes number of factors, *sG*0 denotes the scalar applied to the LSC model for growth rates, *sF*0 denotes the scalar applied to the LSC model for forward discount rates, L_g denotes the assumed level growth rate for all instruments, and *D* denotes the damper required in Equation (4.3.50).

Table 4.3.4. Two factor LSC valuation model inputs

Input	Value
Nf	2
sG0	3.0
sF0	10.0
Lg	4.0%
D	50.0%

The goal here is to calibrate the two factor LSC valuation model based on these inputs. We proceed with two iterations.

Iteration 1: Solve for slope of growth rate

Assuming a constant discount rate (k), we solve for implied slope of the expected future cash flow, S_g, where

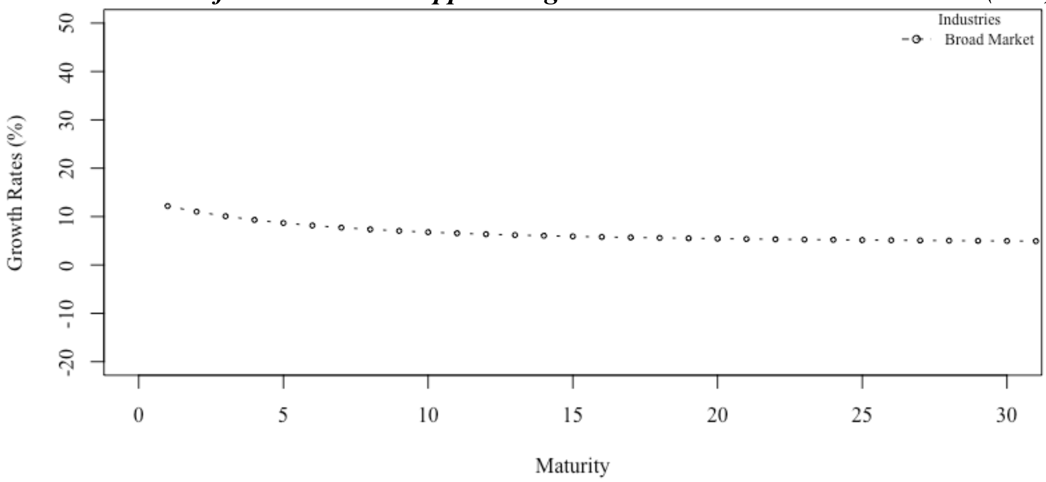
$$VCF = \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i}\left(\hat{L}_g + sc_{g,j}S_g\right)}}{\left(1+k\right)^i}. \tag{4.3.54}$$

For example, for the Broad Market (SPY), we substitute the known values and have

$$\frac{1}{0.019} = \sum_{i=1}^{\infty} \frac{e^{\sum_{j=1}^{i}\left[0.04 + \frac{3}{j}\left(1-e^{-j/3}\right)S_g\right]}}{\left(1+0.08\right)^i}. \tag{4.3.55}$$

Solving for S_g, we find $\hat{S}_g = 0.096179$ or 9.6179%. Remember, a positive slope number implies a downward sloping growth rate curve. Figure 4.3.5 illustrates the growth rates for the first 30 year based on this parameterization of the LSC model.

Figure 4.3.5 Illustration of the LSC model applied to growth rates with the Broad Market (SPY)



With the estimate of S_g, we are ready to solve for the final parameter, S_f.

Iteration 2: Solve for slope of forward discount rate

Given results from Iteration 1, we now address estimating forward discount rate. First, we focus on estimating the forward discount rate level, $\hat{L}_f$. Based on the data provided on the sector ETFs, we have

$$\begin{aligned}\widehat{VCF}_L &= VCF + D\left(\overline{VCF} - VCF\right)\\ &= 52.6316 + 0.5\left(47.5313 - 52.6316\right).\\ &= 50.08145\end{aligned}$$

Solving for the forward discount rate level parameter, we have

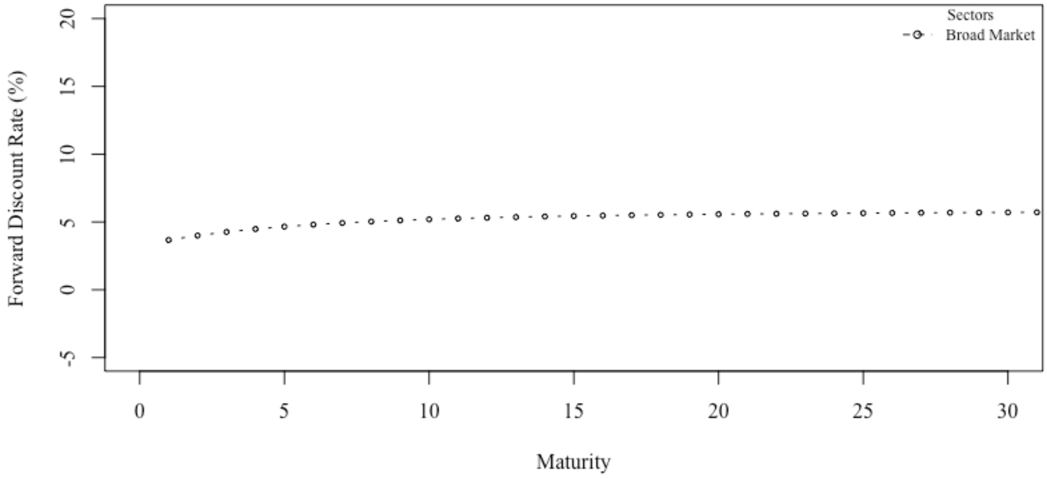
$$\begin{aligned}\hat{L}_f &= \ln\left(1 + \frac{1}{\widehat{VCF}_L}\right) + \hat{L}_g\\ &= \ln\left(1 + \frac{1}{50.08145}\right) + 0.04.\\ &= 0.059771\end{aligned}$$

Thus for the Broad Market (SPY), we substitute the known values and have

$$52.6316 = \sum_{i=1}^{\infty} e^{\sum_{j=1}^{i} -\left[0.059771 + \frac{10}{j}\left(1-e^{-j/10}\right)\hat{S}_f - \left(0.04 + \frac{3}{j}\left(1-e^{-j/3}\right)0.096179\right)\right]}. \tag{4.3.56}$$

Solving for S_f, we find $\hat{S}_f = -0.023815$ or –2.3815%. A negative slope number implies an upward sloping forward discount rate curve. Figure 4.3.6 illustrates the forward discount rates for the first 30 year based on this parameterization of the LSC model.

Figure 4.3.6. Illustration of LSC model applied to forward discount rate with the Broad Market (SPY)



Fully calibrated model

The fully calibrated model can be expressed as

$$VCF = \sum_{i=1}^{\infty} e^{-i\left(\hat{L}_f - \hat{L}_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}\hat{S}_f - sc_{g,j}\hat{S}_g\right)}. \tag{4.3.57}$$

$$V = CF_0 \sum_{i=1}^{\infty} e^{-i\left(\hat{L}_f - \hat{L}_g\right) - \sum_{j=1}^{i}\left(sc_{f,j}\hat{S}_f - sc_{g,j}\hat{S}_g\right)}. \tag{4.3.58}$$

Figure 4.3.7 presents the first 30 years of growth rates.

Figure 4.3.7. Growth rates based on LSC model

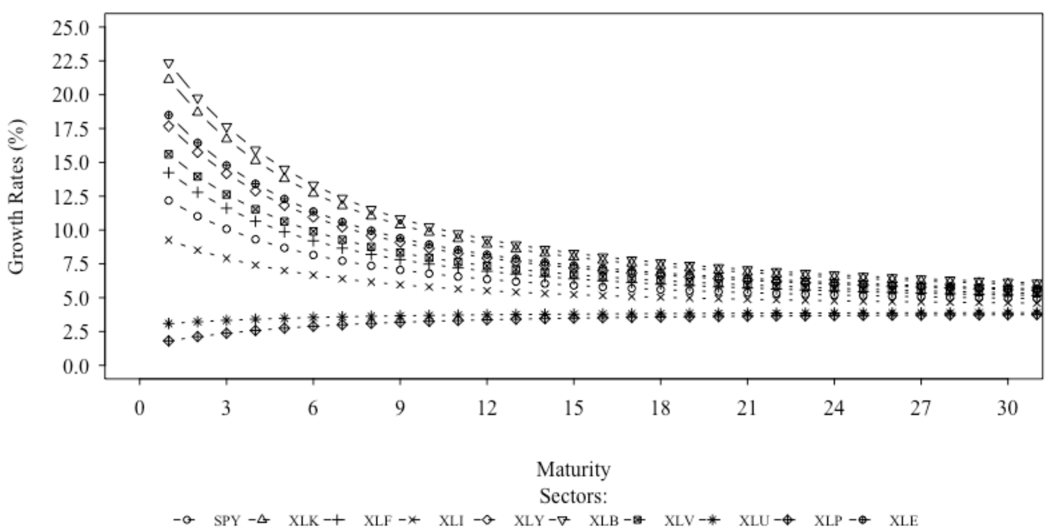


Figure 4.3.8 presents the first 30 years of forward discount rates.

Figure 4.3.8. Forward discount rates based on LSC model

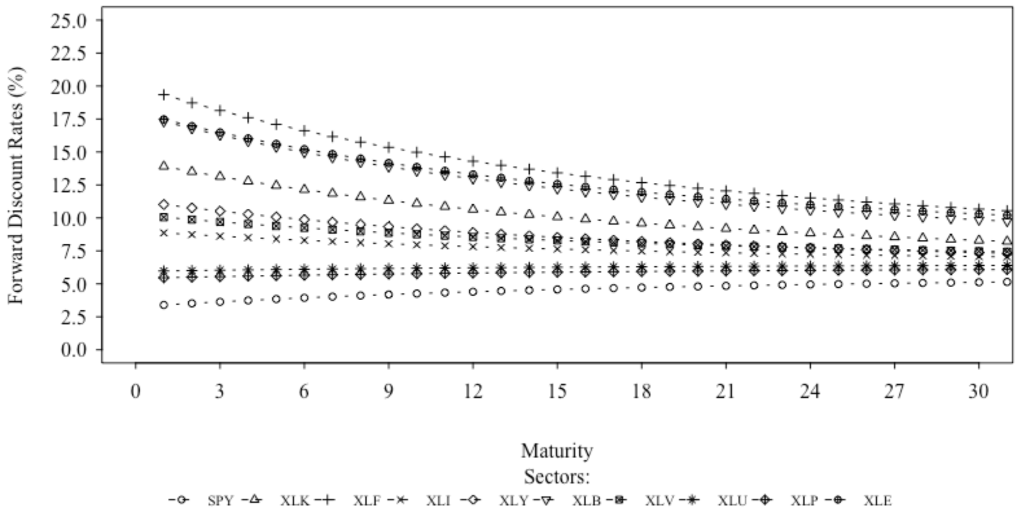


Table 4.3.5 reports the results for the fully calibrated LSC model. GSlope denotes the fitted LSC model slope parameter for growth and similarly WSlope denotes the fitted LSC model slope parameter for forward discount rates. WLevel is the fitted LSC model level parameter for forward discount rates.

Table 4.3.5. Calibrating the LSC Model

Industry	Ticker	Price	DY	DR	CF	PCF	GSlope	PCFL	WLevel	WSlope
Broad Market	SPY	$311.17	1.90%	8.00%	$5.9122	52.632	9.6179	50.08145	5.8822	-2.3816
Technology	XLK	$105.09	1.18%	9.00%	$1.2401	84.746	20.1333	66.13854	5.1731	9.86
Financial	XLF	$22.99	2.71%	10.00%	$0.6230	36.900	12.0371	42.21585	6.6739	13.0265
Industrials	XLI	$68.70	2.30%	7.80%	$1.5801	43.478	6.1766	45.50479	6.2739	2.5642
Consumer Discretionary	XLY	$129.00	1.24%	8.20%	$1.5996	80.645	16.0797	64.08824	5.2324	6.7896
Materials	XLB	$56.10	2.21%	12.00%	$1.2398	45.249	21.5877	46.3901	6.1859	11.6216
Healthcare	XLV	$101.04	2.33%	9.80%	$2.3542	42.919	13.6402	45.22489	6.3033	3.7498
Utilities	XLU	$57.61	3.52%	7.50%	$2.0279	28.409	-1.0676	37.97021	7.4595	-2.7163
Consumer Stables	XLP	$58.92	2.83%	6.50%	$1.6674	35.336	-2.5747	41.43351	6.7907	-1.9987
Energy	XLE	$37.11	4.00%	14.00%	$1.4844	25.000	17.0437	36.26566	7.9221	8.9215

Recall from the Model Inputs tab, we assumed the growth scalar was 3 years, the forward discount rate scalar was 10 years, the long-run growth rate was 2.5% (GLevel), and the damper was 50%. With this robust model, we are now ready for scenario analysis, sensitivity analysis, and/or Monte Carlo simulations.

Summary

In this chapter, we reviewed the historical performance of the U.S. stock market. Further, we introduced an N-Stage dividend discount model as well as applying the LSC model to lower the number of factors driving stock valuation. Ee also apply the LSC valuation model and illustrated it with two factor LSC models applied to the cash flow growth rates and forward discount rates.

References

Brooks, R. and B. Helms, "An N Stage, Fractional Period, Quarterly Dividend Discount Model," *Financial Review*, 25 (November 1990), 651-657.

Jordan, J. V. and S. A. Mansi, "How Well Do Constant-Maturity Treasuries Approximate the On-the-Run Term Structure?" *The Journal of Fixed Income* (September 2000), 1-11.