Module 3.2

CDF and Inverse CDF

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Introduction

- Explore method to compute the standard normal cumulative distribution function and its inverse
- Identify and illustrate machine error

2

Standard Normal CDF

- Open integral

$$N(d) = \int_{-\infty}^{d} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

- Solves for the CDF value, $N()$, given a known value of the percentage point (d)
- No known analytic solution
- Many available approximations but require a computer

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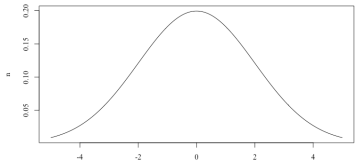
Properties of N(d)

- Percentage point range: $-\infty < d < \infty$
- CDF range: $0 \le N(d) \le 1$
- Probability density function n(d):



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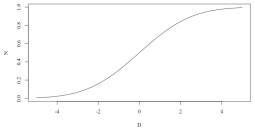
Properties of N(d)

- Cumulative distribution function N(d):



- Approximation (d>0):

$$N(d) = 0.5 + \frac{1}{\pi}\sum_{r=0}^{12} \frac{e^{-\frac{\left(r+\frac{1}{2}\right)^2}{9}} \sin\left\{\frac{d\left(r+\frac{1}{2}\right)\sqrt{2}}{3}\right\}}{\left(r+\frac{1}{2}\right)}$$

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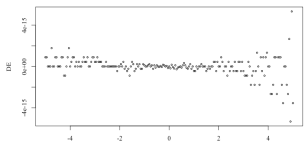
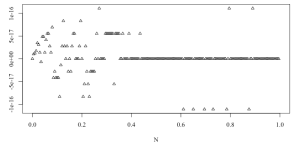
Estimation Error in d

- Error in d:



- Error in N:



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Summary

- Explored methods to compute the standard normal cumulative distribution function and its inverse
- Identified and illustrated machine error