Module 3.6

Embedded Parameters

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Overview

- Explore solving for embedded parameters
- Develop capacity to solve for any embedded parameter
- Illustrate with implied yield to maturity
- Review selected R code

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Numerical Methods

- Bracketed: Increment through range, solution when sign changes
- Bisection: Find the midpoint, consider $f(a)$ and $f(m)$ compared to $f(m)$ and $f(b)$, where $m = (a + b)/2$, select region with sign change, repeat
- Secant: Use slope at initial guess to estimate next evaluation point, repeat

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Numerical Methods

- Newton-Rhapson: Need first derivative function, can be unstable
- Inverse quadratic interpolation: Relies on three points to approximate, more efficient than simple linear interpolation
- Brent (used in R): Bracket the root(s) prior to starting the analysis, select numerical accuracy where the search will cease (combines bisection, secant, and inverse quadratic interpolation

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Simple Bond Yield to Maturity

- Bond value:

$$V = \sum_{i=1}^{N} \frac{C}{(1+y)^{t_i}} + \frac{Par}{(1+y)^{t_N}}$$

- Implied yield to maturity:

$$f(y) = \sum_{i=1}^{N} \frac{C}{(1+y)^{t_i}} + \frac{Par}{(1+y)^{t_N}} - P$$

- Goal: $f(y) = 0$

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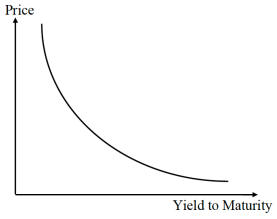
Bond Price-Yield Relationship

Figure 3.6.1. Relationship between bond prices and yield to maturity



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Bond Value Function in R

```
#
# Bond value function
#
FRMBondValue <- function(tempYieldToMaturity, tempCouponRate, tempParValue,
  tempYearsToMaturity){
  PV = 0.0 # Present value variable
  for (i in 1:tempYearsToMaturity){
    PV = PV + ((tempCouponRate/100.0)*tempParValue) /
      ((1.0 + (tempYieldToMaturity/100.0))^i)
  }
  return(PV + tempParValue /
    ((1.0 + (tempYieldToMaturity/100.0))^tempYearsToMaturity))
}
# Test the function
BondValue = FRMBondValue(YieldToMaturity, CouponRate, ParValue,
  YearsToMaturity)
BondValue
```

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Bond YtM Function (Version 1)

```
#
# Yield to maturity function with bond function (Version 1)
#
# Function that finds the difference between market and model bond prices
BondValue <- 90
FRMPriceDifferenceWFunction <- function(tempYieldToMaturity, tempCouponRate,
  tempParValue, tempYearsToMaturity, tempActualPrice){
  tempBV <- FRMBondValue(tempYieldToMaturity, tempCouponRate, tempParValue,
    tempYearsToMaturity)
  return(abs(tempActualPrice - tempBV))
}
TestDifference1 = FRMPriceDifferenceWFunction(YieldToMaturity, CouponRate,
  ParValue, YearsToMaturity, BondValue)
TestDifference1
```

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Bond YtM Function (Version 2)

```
#
# Yield to maturity function with embedded bond value calculation (Version 2)
#
# Function that finds the difference between market and model bond prices
FRMPriceDifference <- function(tempYieldToMaturity, tempCouponRate,
  tempParValue, tempYearsToMaturity, tempActualPrice){
  PV = 0.0 # Present value variable
  for (i in 1:tempYearsToMaturity){
    PV = PV + ((tempCouponRate/100.0)*tempParValue) /
      ((1.0 + (tempYieldToMaturity/100.0))^i)
  }
  return(abs(tempActualPrice - (PV + tempParValue
    / ((1.0 + (tempYieldToMaturity/100.0))^tempYearsToMaturity))))
}
# Test the function -- should be 0 if using BondValue from calculation above
BondValue = 90
TestDifference2 = FRMPriceDifference(YieldToMaturity, CouponRate, ParValue,
  YearsToMaturity, BondValue)
TestDifference2
TestDifference2 - TestDifference1
```

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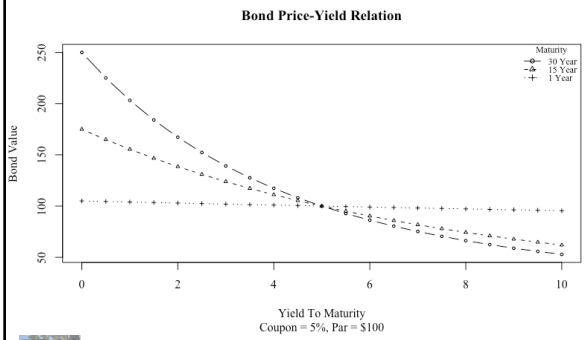
Sensitivity to Maturity



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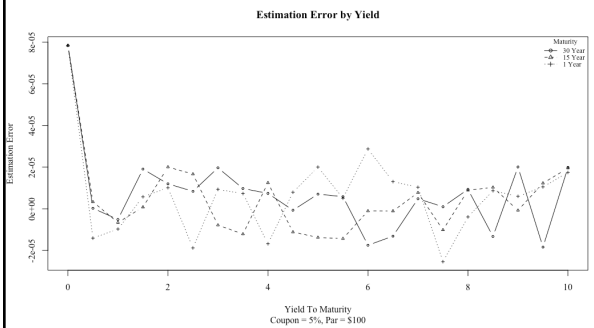
Yield to Maturity Estimation Error



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Summary

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- Developed capacity to solve for any embedded parameter
- Illustrated with implied yield to maturity
- Reviewed selected R code

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