Module 3.4

LSC Model

Curvefitting Using Linear Regression

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LSC Model Applications

- Chuck's problem: 60,000 interest rate swaps on investment banking book of business
- Stephen's problem: $800m UST portfolio
- Andy's problem: 30,000 energy risk factors (5 commodities, 120 monthly maturities, 50 geographic locations)
- Each has the challenge to parsimoniously value a complex book of business

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Linear Regression

- Solution to set of linear equations

$$x_{11}\beta_1 + x_{12}\beta_2 + x_{13}\beta_3 + \cdots + x_{1n}\beta_n = y_1$$
$$x_{21}\beta_1 + x_{22}\beta_2 + x_{23}\beta_3 + \cdots + x_{2n}\beta_n = y_2$$
$$\vdots$$
$$x_{m1}\beta_1 + x_{m2}\beta_2 + x_{m3}\beta_3 + \cdots + x_{mn}\beta_n = y_m$$

- Ordinary least squares (OLS) solution

$$\underset{n\times 1}{\hat{\mathbf{b}}} = \left(\underset{n\times m}{\mathbf{X}^T}\ \underset{m\times n}{\mathbf{X}}\right)^{-1} \underset{n\times m}{\mathbf{X}^T}\ \underset{m\times 1}{\mathbf{Y}}$$

$$\underset{n\times 1}{\hat{\mathbf{b}}} = \left(\underset{n\times n}{\mathbf{NX}}\right)^{-1} \underset{n\times 1}{\mathbf{NY}}$$

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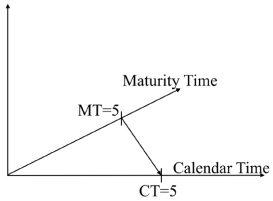
Two Time Concepts With Debt

- Calendar time (Time series)
- Maturity time (Cross-sectional)

Figure 3.4.1. Contrasting maturity time and calendar time



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Fitted Term Structure Models

- Calendar time
  - Crack and Nawalkha [2000]: "Up to 95 percent of the returns to U. S. Treasury security portfolios are explained by term-structure level shifts, slope shifts, and curvature shifts" (34)
- Maturity time
  - Non-stochastic shape of the term structure at a particular point in calendar time

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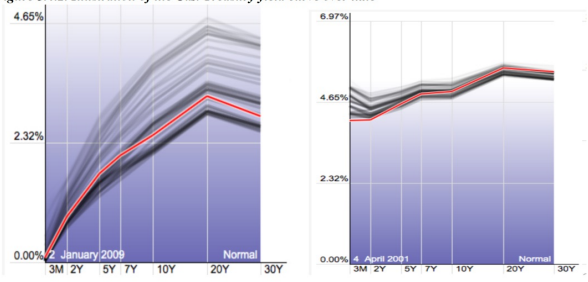
YC Shifts Often Non-Parallel

Figure 3.4.2. Illustration of the U.S. Treasury yield curve over time[4]



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LSC Model

- Willner (1996) posits that the desirable properties of a curve fitting routine must address the bond "portfolio manager's need for *intuitive*, *descriptive*, and *comprehensive* risk exposure information."
- Generalized and parsimonious model
- Non-linear fit using OLS

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LSC Model

- Linear factors: $y_i = \sum_{j=0}^{N} x_{i,j} f_j$
- Level: $x_{i,0} = 1$ Slope: $x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i / s_1}\right)$
- Curvatures: $x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i / s_j}\right) - e^{-\tau_i / s_j}; j > 1$
- LSC model has the lowest "average (across the sample) mean (across the curve) absolute yield error" (Steeley) when compared with splines, polynomials and Vasicek's model

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LSC Model Properties

- As maturity goes to infinity, then $y_i \to f_0$
- As maturity goes to zero, then $y_i \to f_0 + f_1$
- If the interest rate term structure is upward sloping then f_1 is negative.
- Spot rate factors greater than one measure the rate of curvature. Higher (lower) values lead to flatter (steeper) slopes

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LSC Model Illustrations

- Nine constant maturity treasuries (CMT)
- Linear interpolation
- Nine factor model (very good fit)
- Derive implied continuously compounded spot rates
- Estimate CMT and compare to fitted CMT

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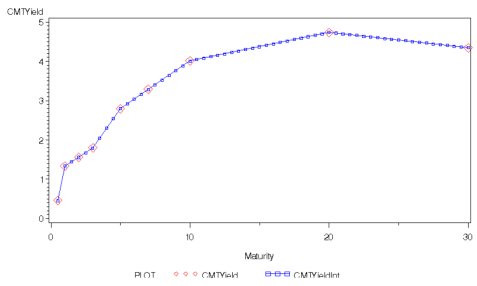
CMT and Linear Interpolation

Figure 3.4.3. Linear interpolation



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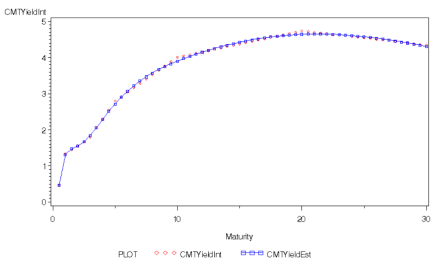
CMT and Fitted 9-Factor LSC

Figure 3.4.4. LSC model with nine factors



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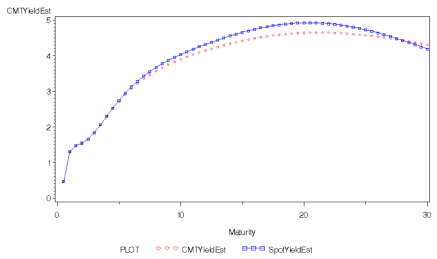
Implied Spot rates

Figure 3.4.5. Implied spot rates inferred from LSC model with nine factors



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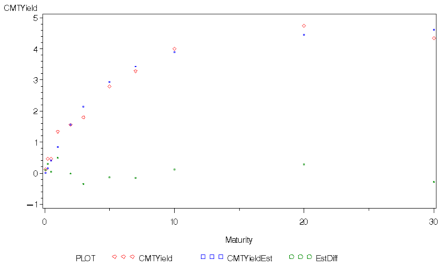
Original CMT and Fitted CMT

Figure 3.4.6. Original CMT compared with fitted CMT



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LSC Model Examples

- U.S. swap rates
- Good fit with low R^2
- Regression independent variables
- Different slope coefficients
- Different curvature coefficients

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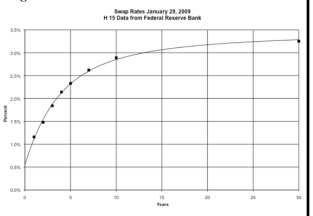
U.S. Swap Rates and Fitted LSC

Figure 3.4.7. U. S. swap rates with fitted LSC model along with numerical values

Level	3.50618%
Slope	-2.95534%
Curvature	-0.35538%
Scalar	2.00

Regression Variables			
Maturity	y	x1	x2
0.00001	na	1.00000	0.00000
1	1.16%	0.78694	0.18041
2	1.48%	0.63212	0.26424
3	1.84%	0.51791	0.29478
4	2.14%	0.43233	0.29700
5	2.33%	0.36717	0.28508
7	2.62%	0.27709	0.24689
10	2.89%	0.19865	0.19191
30	3.25%	0.06667	0.06667



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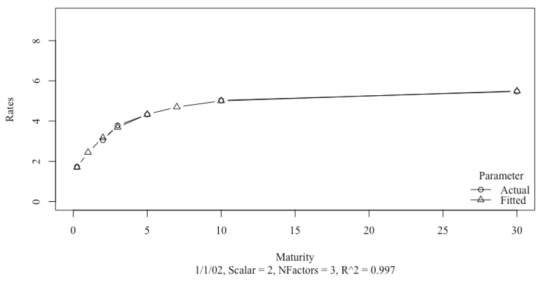
U.S. Swap Rates and Fitted LSC

Figure 3.4.8. U. S. swap rates with fitted LSC model in R



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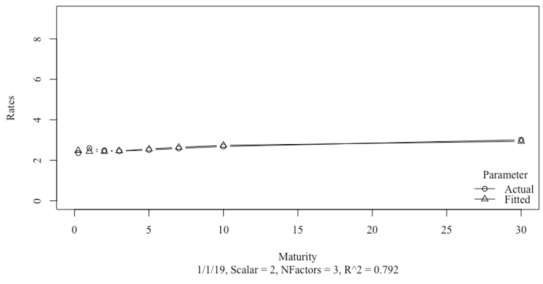
U.S. Swap Rates and Fitted LSC

Figure 3.4.9. U. S. swap rates with fitted LSC model when data lacks variability



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Good Fit With Low R^2

Figure 3.4.10. Fabricated rates with fitted LSC model with minimal variability

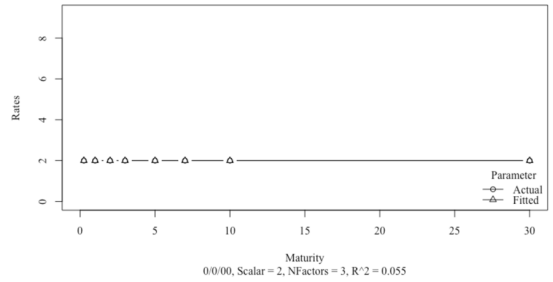

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Figure 3.4.11. LSC model regression independent variables, x_{ij}

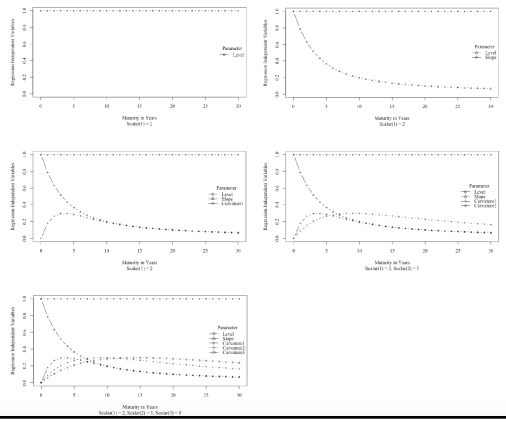

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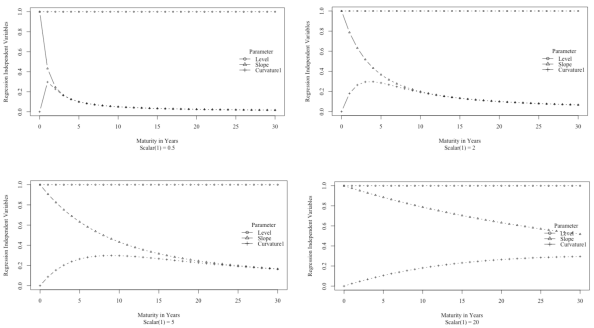
Figure 3.4.12. Three factor LSC model regression independent variables, x_{ij} with different scalars



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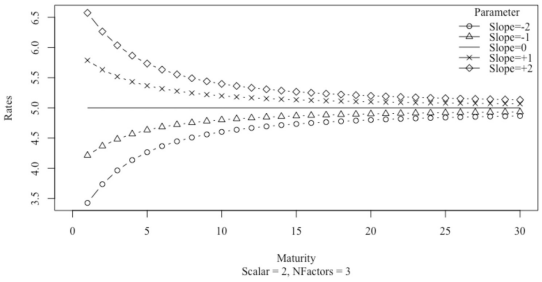
LSC Model Slope Coefficients

Figure 3.4.13. Three factor LSC model with different slope coefficients



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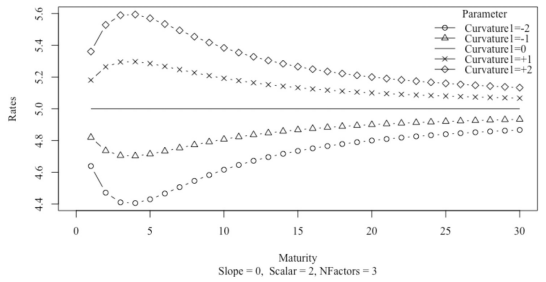
LSC Model Curvature (Slope=0)

Figure 3.4.14. Three factor LSC model with different coefficient1 coefficients (Slope = 0)



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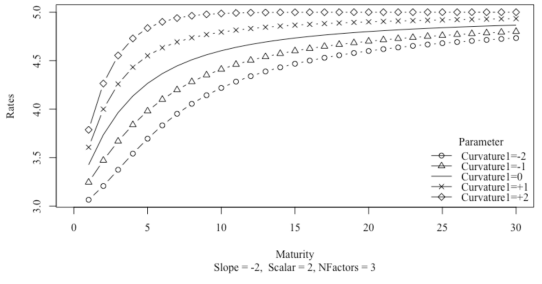
LSC Model Curvature (Slope=–2)

Figure 3.4.15. Three factor LSC model with different coefficient1 coefficients (Slope = –2)



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LSC Model Fits Well

- LSC applications
  - Various yield curves
  - Volatility curves
  - First differences (e.g., natural gas futures)
  - Percentage changes
- Extensively used throughout the remainder of this material

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Summary

- LSC model defined and illustrated extensively
- Extendable and captures numerous shapes while relying solely on OLS
- Fits empirical data very well
- Widely applicable in finance

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