Chapter 2. Approaches to Valuation

"*Underlying all practical problems in connection with the financial aspects of the corporation, there is the problem of value.*" Arthur Stone Dewing (1941)

Learning objectives

- Introduce important presuppositions for financial markets to function, clear rule of law, clean property rights, and a culture of trust
- Review the typical assumptions related to quantitative finance
- Discuss three categories of approaches to financial valuation: the market comparable approach, the cash flow adjusted approach, and the discount factor adjusted approach

Introduction

Prior to launching into specific quantitative finance modules, we review here three categories of approaches to valuation. In our experience, there is a significant amount of confusion regarding how different valuation models interrelate and when various models should be reasonable to use.

We cannot omit to mention important presuppositions required for financial markets to function at all. In Appendix 2A, we explore the philosophical foundations of quantitative finance.

Finally, this is the only chapter not to have corresponding R code in the QF Repository or a chapter R Commentary.

Presuppositions for functioning financial markets

We suggest that there are at least four presuppositions for financial markets to reasonably function. A presupposition is a requirement that is antecedent in logic or fact, that is, it is what is assumed beforehand.

First, there needs to be clear rule of law. Ambiguity in law leads to tyranny in enforcement. For example, if the speed limit is set to be "reasonable," then any law enforcement officer can arrest anyone for speeding. The law enforcement officer can arbitrarily determine that your speed was unreasonable, especially if you are a member of the wrong political party.

Second, to execute a transaction, there needs to be clean property rights. If property ownership is uncertain, then buying or selling that property will result in disputes.

Case (2003) notes, "(T)he degree to which the society is bound by law, is committed to processes that allow property rights to be secure under legal rules that will be applied predictably and not subject to the whims of particular individuals, matters." (p. 2)

Third, financial markets are more efficient if built on a foundation of trust. Trust implies you rely on someone with something of value. If you trust, then you make yourself vulnerable in confidence. You are assuming the trusted will not exploit and will be concerned. For example, medical surgery would be impossible if doctors were not at least somewhat trustworthy. Clearly, trust makes cooperative activities, such as financial markets, possible.

Finally, we assume that the uncertainties related to future activities can reasonably be mapped to a subjective probability distribution of some form.

We turn now to enumerate the typical assumptions underlying quantitative finance models.

Typical assumptions for quantitative models

We review the standard set up for financial models. For more exhaustive details, see Harrison and Kreps (1979) and Harrison and Pliska (1981).

1. $\left[0,\hat{T}\right]$, for fixed $0 \le t \le \hat{T}$, finite time horizon. Thus, calendar time can be expressed as a finite segment of the real number line.
2. $\left(\Omega,\Im,\mathrm{P}\right)$, uncertainty is characterized by a complete probability space, where the state space Ω is the set of all possible realizations of the stochastic economy between time 0 and time $\hat{T}$ and has a typical element ω representing a sample path, $\Im$ is the sigma field of distinguishable events at time $\hat{T}$, and P is a probability measure defined on the elements of $\Im$. (See more detailed explanation below.)
3. $F=\left[\Im\left(t\right):t\in\left(0,\hat{T}\right)\right]$ the augmented, right continuous, complete filtration generated by the appropriate stochastic processes in the economy and assume that $\Im\left(\hat{T}\right)=\Im$. The augmented filtration, $\Im\left(t\right)$, is generated by Z. $\Im\left(0\right)$ contains only Ω and the null sets of P. In a finance context, the filtration is keeping track of what is known at a point in time and what is not known.
4. F is generated by a K-dimensional Brownian motion, $Z\left(t\right)=\left[Z_1\left(t\right),\cdots,Z_K\left(t\right)\right],t\in\left(0,\hat{T}\right)$ is defined on $\left(\Omega,\Im,P\right)$, where $\left[\Im\left(t\right)\right],t\in\left(0,\hat{T}\right)$ is the augmentation of the filtration $\left[\Im^Z\left(t\right)\right],t\in\left(0,\hat{T}\right)$ generated by $Z\left(t\right)$, and satisfies the usual conditions.
5. $E_{\mathrm{P}}\left(\cdot\right)$ denotes the expectation with respect to the probability measure P.
6. All stated equalities or inequalities involving random variables hold P-almost surely.
7. P is common for all agents implying uniqueness of the nature of the stochastic processes.
8. Conventional perfect market conditions are typically assumed, such as no transaction costs, no taxes, unrestricted short selling, and no regulatory or institutional constraints.
9. Future financial instrument values can be represented by some distribution.

We provide more details on a few of these assumptions.

- $\left(\Omega,\Im,\mathrm{P}\right)$ characterizes uncertainty using a complete probability space, where the state space Ω denotes the set of all possible realizations between time 0 and time $\hat{T}$, ω represents one sample path, $\Im$ denotes the sigma field of events known at time $\hat{T}$, and P is a probability measure defined on the sigma field, $\Im$. $\left(\Omega,\Im,\mathrm{P}\right)$ is a mathematical representation of our perceptions of unpredictable movements in underlying instrument prices
- Uncertainty means unpredictable change (both likelihood and outcome are unknown)
- Complete probability space – uncertainty is reduced to risk (both likelihood and outcome are known)
- Ω – state space, all possible sample paths representing a model of uncertainty
- 0 – time is measurable, our analysis is limited to a finite time length
- $\hat{T}$ – terminal point in time
- ω – a unique event (e.g., sample path), known only at time $\hat{T}$
- $\Im$ – sigma field, a collection of sets illustrated below
- P – a probability measure defined on $\Im$

Consider a three period binomial illustration where each period is 1 year and the likelihood of up is 3/5ths:

- Ω – {ϕ, {d}, {u}, {d,d}, {d,u}, {u,d}, {u,u}, {d,d,d}, {d,d,u}, {d,u,d}, {d,u,u}, {u,d,d}, {u,d,u}, {u,u,d}, {u,u,u}}
- 0 – initial period in binomial illustration, $\hat{T} - 3$
- ω – {u,d,u}, $\Im$ – keeps track of information (complete past sample path)
- t=0 {ϕ, {d}, {u}, {d,d}, {d,u}, {u,d}, {u,u}, {d,d,d}, {d,d,u}, {d,u,d}, {d,u,u}, {u,d,d}, {u,d,u}, {u,u,d}, {u,u,u}} (100%)
- t=1: {{d}, {d,d}, {d,u}, {d,d,d}, {d,d,u}, {d,u,d}, {d,u,u}} (40%)
{{u}, {u,d}, {u,u}, {u,d,d}, {u,d,u}, {u,u,d}, {u,u,u}} (60%)
- t=2: {{d,d}, {d,d,d}, {d,d,u}} (16%)
{{d,u}, {d,u,d}, {d,u,u}} (24%)
{{u,d}, {u,d,d}, {u,d,u}} (24%)
{{u,u}, {u,u,d}, {u,u,u}} (36%)
- t=3: {{d,d,d}} (6.4%)
{{d,d,u}} (9.6%)
{{d,u,d}} (9.6%)
{{d,u,u}} (14.4%)
{{u,d,d}} (9.6%)
{{u,d,u}} (14.4%)
{{u,u,d}} (14.4%)
{{u,u,u}} (21.6%)

Note the state space is the set of all possible realizations between time 0 and time $\hat{T}$. In this case, it is Ω or {ϕ, {d}, {u}, {d,d}, {d,u}, {u,d}, {u,u}, {d,d,d}, {d,d,u}, {d,u,d}, {d,u,u}, {u,d,d}, {u,d,u}, {u,u,d}, {u,u,u}}. The sigma field of distinguishable events keeps track of what is known at any point in time. Notice above that as each point in time passes, the set of possible distinguishable events is reduced.

We now turn to three categories of approaches to financial valuation.

Approaches to financial valuation

There are a wide variety of approaches to financial valuation. A general review of various approaches to financial valuation is provided here for the purpose of appropriately characterizing various valuation models.

There are a variety of objectives for financial valuation models, including relative valuation and risk management. Option valuation typically involves relative valuation. Relative valuation or partial equilibrium assumes that the underlying instrument is in equilibrium, then seeks to identify mispriced options within an overall set of instrument prices. For example, one may attempt to identify mispriced options within all the options trading on a single security or several securities. Option models can also be useful for risk management purposes. The option model can provide various statistics related to estimates of different types of risk exposures.

The general framework presented here is based on Brooks (1998, 2002) where the focus was on interest rate swaps and energy options. Financial instrument valuation models can be classified into one of three categories of valuation: the market comparable approach, cash flow adjusted approach, and discount factor adjusted approach.[1] Each of these categories is based on some form of rational approach. Each category is briefly reviewed here.

One approach to valuation is based on the notion of comparability or substitution. If two option-related investments result in the same future cash flows (same amount and timing) no matter what happens, then the appropriate values for these two investments should be the same. The two investments are assumed to produce the same future cash flows regardless of the assumed underlying return distribution (known or unknown). We assign the label market comparable approach (MCA) to these types of methods. This approach is based on the law of one price and does not require any intermediate trading activities.

[1]An example of the cash flow adjusted approach is risk neutral option valuation and an example of the discount factor adjusted approach is the traditional discounted cash flow used in project finance.

A second approach to valuation is also founded on the notion of comparability, but requires active trading based on the principles of self-financing and dynamic replication. The seminal works of Black and Scholes [1973] and Merton [1973] are based on the idea of synthetically creating the cash flows of a risk-free bond from dynamically trading a stock and a call option on that stock. Although there have been a multitude of research papers written using this type of procedure, there is one common thread. The future cash flows can be discounted at the risk-free rate once either the cash flows or the probability distribution has been adjusted. These valuation methods are often referred to as risk-neutral valuation because the discount rate is the risk-free rate. I refer to these adjusted probabilities as equivalent martingale measures because the probabilities have been adjusted so that the stochastic process follows a martingale, after adjusting for the time value of money. Also, the probability space of the original probabilities is equivalent to probability space after the adjustment. A martingale is a stochastic process (set of ordered random variables) where the conditional expectation of the subsequent outcome is equal to its current value. We assign the label cash flow adjusted approach (CFAA) to emphasize that these types of methods require some adjustment to the numerator of the valuation equation. It is interesting to point out that this category of approach is likely only legitimate if the related securities have a high level of marketability. The CFAA approach is the category most dependent on marketability to be viable.

The traditional approach to valuation is to forecast future expected cash flows and then to take the present value of this future expected cash flow stream. Many stock valuation models take this approach where the appropriate discount rate increases with the degree of uncertainty related to the future. I assign the label discount factor adjusted approach to these types of methods. The identifying criterion for a valuation method to fall in the DFAA category is that the adjustment for risk is made in the denominator of the valuation equation. The higher the risk (however defined), the higher the interest rate will be for discounting.

The beauty of CFAA to valuation is the discount rate does not have to be estimated. The appropriate discount rate is particularly difficult to estimate as it would be expected to be a function of both calendar time and the underlying asset price. This is due to the option expiration, and the degree of implied leverage in the firm (i.e., the higher the equity price, the lower the debt-to-asset ratio, hence the lower the financial risk of equity). Most option pricing models used today fall within the category of CFAA.

Mathematical details

The three general categories of approaches to valuation mathematically can be expressed as:

$$V_i = \sum_{\substack{k=1 \\ k \neq i}}^{s} \alpha_k P_k \text{ (Market comparable approach).} \tag{2.1}$$

$$V_i = \sum_{t=1}^{T} \sum_{j=1}^{m} \frac{1}{\left(1 + r_t + RP_{i,t,j}\right)^t} p_{t,j} CF_{i,t,j} \text{ (Discount factor adjusted approach).} \tag{2.2}$$

$$V_i = \sum_{t=1}^{T} PV\left(\$1,t\right) \sum_{j=1}^{m} q_{t,j} CF_{i,t,j} = \sum_{t=1}^{T} PV\left(\$1,t\right) E_q\left(CF_{i,t}\right) \text{ (Cash flow adjusted approach).} \tag{2.3}$$

where V_i denotes the value of some instrument *i*, α_k denotes number of units of instrument *k* held, r_t denotes the per-period risk-free interest rate, $RP_{i,t,j}$ denotes the risk premium on instrument *i*, at time *t*, when outcome *j* occurs, $CF_{i,t,j}$ denotes the cash flow on instrument *i*, at time *t*, when outcome *j* occurs, $PV(\$1,t)$ denotes the present value of \$1 from 0 to *t*, $q_{t,j}$ denotes the equivalent martingale measure probability of outcome *j* occuring at time *t*, and $E_q()$ denotes taking the expectation under the equivalent martingale measure. Note that the instrument's value may or may not correspond to the observed market price, say P_i.

Approaches to valuation in detail[2]

We now review and illustrate these three approaches to valuation in detail. The focus here is on financial "value in exchange" as opposed to "value in use." The objective is to offer three categories of approaches to

[2]See Peter C. Fusaro, "New Techniques in Energy Options," *Energy Convergence The Beginning of the Multi-Commodity Market* (New York, NY: John Wiley & Sons, Inc., 2002).

valuation with a particular focus on establishing criteria for selecting the appropriate category to use for any given quantitative finance problems. We use a forward option contract to illustrate the issues. Williams (1938) and Gordon (1959) were among the pioneers in applying valuation techniques to financial assets. The capital asset pricing model (CAPM), introduced by Sharpe (1964), Lintner (1965), and others, was an early attempt to quantify the equilibrium adjustment for risk. Within the CAPM, risk was measured by the asset's beta, and future expected cash flows were discounted at a risk-adjusted rate. We classify valuation models that discount at a risk-adjusted rate within the category the discount factor adjusted approach.

With the pioneering work of Arrow (1964), Black and Scholes (1973), Harrison and Kreps (1979), Cox and Ross (1976), and Hansen and Richard (1987), another approach to valuation has emerged that has been given a variety of names such as: state-claims valuation, equivalent-martingale valuation, stochastic discount factor valuation, or risk-neutral valuation. Within these methods of valuation, the adjustment for risk is taken in some way in the numerator of the valuation equation. For example, the typical way risk is adjusted in these methods is to adjust the probability measure. We will classify valuation models that adjust risk in this manner within the category called the cash flow adjusted approach. Cochrane (2000) demonstrates that these two general approaches to valuation can be reconciled with each other within the state-claims framework. Before reviewing cash flow adjusted approaches and discount factor approaches, we review a much simpler approach.

Market comparable approach (MCA)

One approach to valuation is based on the notion of comparability or substitution. Dewing (1941) expressed this approach as follows: “When several services or commodities satisfy a human want equally well, the value of each one of them is determined not by the sacrifice necessary to obtain each, but rather by the sacrifice necessary to obtain the one most easily available, which may be substituted for any one of the others.” If two investments will result in the same future cash flows (same amount and timing) no matter what happens, then the appropriate values for these two investments should be the same. The two investments are assumed to produce the same future cash flows regardless of the assumed underlying return distribution (presently known or unknown). We assign the label market comparable approach (MCA) to these types of methods. This approach is based on the law of one price and does not require any intermediate trading activities.

The least imposing mathematical framework would involve a situation where the set of possible outcomes is not explicitly defined, that is, the circumstances for whatever reason involve future events that defy an easy mapping into a state-space. We cannot assign probabilities to future events nor even express what these future events might entail. With so little information, one may think that it is not possible to derive a reasonable estimate of the market value of a particular derivative security. This is not true.

Suppose the state-space is not well-defined, but there is a set of actively traded securities such that

$$CF_{i,t,j} = \sum_{\substack{k=1 \\ k\neq i}}^{s} \alpha_k CF_{k,t,j} \text{ for all t and j,} \tag{2.4}$$

where s is the number of actively traded securities involved in replicating the cash flows (CF) for the i^{th} security at time t for state j. Let α_k denote number of units of security k, where positive implies long and negative implies short. That is, it is possible to replicate the cash flows for the i^{th} security with a set of other actively traded securities. If there are no trading costs, no other market frictions, and short-selling is allowed, arbitrage activities will cause

$$P_i = \sum_{\substack{k=1 \\ k\neq i}}^{s} \alpha_k P_k, \tag{2.5}$$

where P_i is the market price of security i. Clearly, security i is comparable in cash flow to a portfolio of other securities. Thus, we call approaches to valuation based on employing other securities the market comparable approach.

We emphasize the key assumptions when using market comparable approaches are suitable:

- There exists a set of securities that produce future cash flows in each state identical to the security being valued (even states that are currently unimaginable).

- Trading costs and other market frictions are minimal.
- Short selling is allowed.

The degree of confidence with the market comparable method will be directly related to the degree that these three key assumptions are reasonable. There are numerous examples of applications of the market comparable method. The value of a portfolio is merely the sum of the value of each security. Most of finance and accounting theories hinge critically on this view.

One can value options on forward contracts using the well-known put-call parity for European-style forward options (no early exercise). Stoll (1994) established the relationship between puts and calls; however, this relationship was well understood as far back as Russell Sage in 1869. (See Sarnoff (1965).) Put-call parity with forward contracts states that the current price of a call option (c_t) is equal to the difference between the current price of the forward contract ($F_{t,T}$) (observed at t and matures at T) and the strike price (X) discounted at the risk free rate (r where annual compounding is assumed or r_c where continuous compounding is assumed) plus the current price of the put (p_t). Let $PV(\$1, T-t)$ denote the present value at t of a dollar at time T. Therefore, we assume

$$PV\left(\$1,T-t\right)=\frac{\$1}{\left(1+r\right)^{T-t}}=\$1e^{-r_c(T-t)}. \tag{2.6}$$

Both options are assumed to have the same expiration, T (where $T-t$ is expressed in terms of fraction of a year). The forward put-call parity is

$$c_t = PV\left(\$1,T-t\right)\left(F_{t,T}-X\right)+p_t=\frac{F_{t,T}-X}{\left(1+r\right)^{T-t}}+p_t. \text{ (forward put-call parity equation)} \tag{2.7}$$

For put-call parity to hold, the previous three conditions must be reasonably true. If the put market is not liquid or if short-selling is not permissible, then we should not expect the forward put-call parity equation above to be consistently accurate in estimating the call price.

One way to validate put-call parity is with a cash flow table. This is the way most arbitrageurs view this potential opportunity. Suppose you rearranged put-call parity such that no investment was required at all:

$$c_t-\frac{F_{t,T}-X}{\left(1+r\right)^{T-t}}-p_t=0. \tag{2.8}$$

From this equation we construct a set of trades that exactly replicate these values. Specifically, $+c_t$ implies sell calls (positive cash flow means contract is sold), borrow ($F_{t,T} < X$) or lend ($F_{t,T} > X$) the discounted difference between the forward price and the strike price, and buy puts. Due to the net cash flows from these three trades, we also enter a long position in a forward contract. Table 2.1 is the cash flow table illustrating the cash flows both today and at expiration.

Table 2.1. Forward put-call parity cash flow table

Strategy	**Today (t)**	**At Expiration (T) $F_{T,T} < X$**	**At Expiration (T) $F_{T,T} > X$**
Sell call	$+c_t$	\$0	$-(F_{T,T}-X)$
Lend or Borrow	$-PV(\$1,T-t)(F_{t,T}-X)$	$+(F_{t,T}-X)$	$+(F_{t,T}-X)$
Buy put	$-p_t$	$+(X-F_{T,T})$	\$0
Net		$+(F_{t,T}-F_{T,T})$	$+(F_{t,T}-F_{T,T})$
Long Forward	\$0	$+(F_{T,T}-F_{t,T})$	$+(F_{T,T}-F_{t,T})$
NET	???	\$0	\$0

How much should a portfolio that pays \$0 for sure be worth today? No matter what discount rate you use, the present value is zero. If ??? is positive, you have a money machine or arbitrage profits. If ??? is negative in the table above, then enter the opposite trades, and you have a money machine. Table 2.2 illustrates this case.

Table 2.2. Alternative forward put-call parity cash flow table

Strategy	Today (t)	At Expiration (T) $F_{T,T} < X$	At Expiration (T) $F_{T,T} > X$
Buy call	$-c_t$	\$0	$+(F_{T,T} - X)$
Borrow or Lend	$+PV(\$1,T-t)(F_{t,T} - X)$	$-(F_{t,T} - X)$	$-(F_{t,T} - X)$
Sell put	$+p_t$	$-(X - F_{T,T})$	\$0
Net		$-(F_{t,T} - F_{T,T})$	$-(F_{t,T} - F_{T,T})$
Short Forward	\$0	$-(F_{T,T} - F_{t,T})$	$-(F_{T,T} - F_{t,T})$
NET	+ by assumption	\$0	\$0

Consider the following numerical example: Suppose the forward price for a one year forward contract is \$3.5, the strike price is \$3.5, the call option premium is \$0.53, the put option premium is \$0.52, the time to expiration is one year, and the continuously compounded interest rate is 5%. Because the forward price equals the strike price, in equilibrium, the call price should equal the put price. Therefore, put-call parity does not hold. Because the put price is less than the call price, we will sell the call, buy the put, and enter a long forward position. Table 2.3 illustrates the cash flow table.

Table 2.3. Arbitrage example with forward put-call parity cash flow table

Strategy	Today (t)	At Expiration (T) $F_{T,T} < X$	At Expiration (T) $F_{T,T} > X$
Sell call	$+c_t = \$0.53$	\$0	$-(F_{T,T} - X)$ $= -(F_{T,T} - \$3.5)$
Lend or Borrow	$-PV(\$1,T-t)(F_{t,T} - X)$ $-0.95238(\$3.5 - \$3.5)$ $= \$0$	$(F_{t,T} - X)$ $(\$3.5 - \$3.5) = \$0$	$+(F_{t,T} - X)$ $= +(\$3.5 - \$3.5) = \$0$
Buy put	$-p_t = \$0.52$	$+(X - F_{T,T})$ $= +(\$3.5 - F_{T,T})$	\$0
Net		$+(F_{t,T} - F_{T,T})$ $= +(\$3.5 - F_{T,T})$	$+(F_{t,T} - F_{T,T})$ $= +(\$3.5 - F_{T,T})$
Long Forward	\$0	$+(F_{T,T} - F_{t,T})$ $= (F_{T,T} - \$3.5)$	$+(F_{T,T} - F_{t,T})$ $= +(F_{T,T} - \$3.5)$
NET	+\$0.01	\$0	\$0

Thus, we pocket \$0.01 per underlying unit with no risk in the future. Notice that the arbitrage produces exactly the monetary difference based on the forward put-call parity equation.

What makes the valuation category of market comparable approach so potent is the lack of any distributional assumptions regarding future uncertainty and how this uncertainty is priced. One security is created from trading others. We now review the cash flow adjusted approach.

Cash flow adjusted approach (CFAA)

A second approach to valuation is also founded on the notion of comparability, but requires active trading based on the principle of self-financing and dynamic replication. The seminal works of Black and Scholes (1973) and Merton (1973) are based on the idea of synthetically creating the cash flows of a risk-free bond from dynamically trading a stock and a call option on that stock.

Although there have been a multitude of research papers written using this type of procedure, there is one common thread. The future cash flows can be discounted at the risk-free rate once either the cash flows or the probability distribution has been adjusted. These valuation methods are often referred to as risk-neutral valuation because the discount rate is the risk-free rate. We refer to these adjusted probabilities as equivalent martingale measures because the probabilities have been adjusted so that the stochastic process follows a martingale, after adjusting for the time value of money. Also, the probability space of the original probabilities is equivalent to probability space after the adjustment. We assign the label cash flow adjusted approach (CFAA) to emphasize that these types of methods require some adjustment to the numerator of the valuation equation. It is interesting to point out that this category of approach is only viable if the related securities have a high level of marketability. The CFAA approach is the category most dependent on marketability to be viable.

When it is not possible to synthetically create the cash flows from existing securities without any assumptions about the state space, it may be possible to synthetically create a particular security's cash flows when there is sufficient structure assumed about the state space. As we will observe, either the cash flow for state *j* or the probability of observing state *j* will be adjusted to account for risk. This structure has taken many different forms depending on the valuation needs.

Underlying each of the valuation techniques classified under the CFAA is the ability to derive state-claims for all possible states in the sample space. A state-claim is the current price of receiving one unit ($1) at time t only if a particular outcome in the state-space occurs (state *j*) and zero units ($0) otherwise.

Assuming the state-space is well-defined, and enough structure exists to obtain state-claims, then the price of the i^{th} security can be expressed as

$$P_i = \sum_{t=1}^{T} \sum_{j=1}^{m} SC_{t,j} CF_{i,t,j}, \tag{2.9}$$

where SC denotes state claims and CF denotes cash flow. It can be demonstrated (see Cochrane (2000) for example) that the state-claim is equal to the discounted equivalent martingale measure or

$$SC_{t,j} = PV(\$1,t) q_{t,j} \text{ for all t and j,} \tag{2.10}$$

where r is assumed to be the appropriate continuously compounded risk-free rate and q denotes the equivalent martingale measure. Substituting for this definition of a state-claim and factoring out the discount function yields

$$P_i = \sum_{t=1}^{T} PV(\$1,t) \sum_{j=1}^{m} q_{t,j} CF_{i,t,j} = \sum_{t=1}^{T} PV(\$1,t) E_q(CF_{i,t}). \tag{2.11}$$

The current market price of security i is the discounted future expected cash flow based on equivalent martingale measures and the discounting is at the risk-free interest rate.

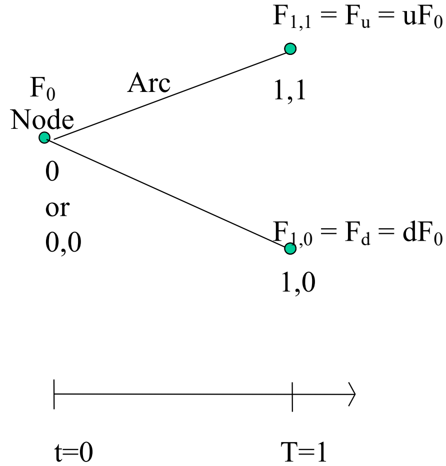
The key assumptions to reasonably use the CFAA are:

- There exists a stochastic process (or processes) that accurately depicts the future potential outcomes; that is, the state space is well defined.
- There exists a trading strategy that produces future cash flows in each state identical to the security being valued.
- Trading costs and other market frictions are minimal.
- Short selling is allowed.

Cash flow adjusted approach to valuation is built on the ability to construct reliable dynamic hedges. The famous Black-Scholes (1973) option pricing model assumes that a dynamic strategy can be designed using call options and the underlying stock to simulate a risk-free payoff in the future. Many derivative valuation models are built on the CFAA. The essence of this approach is to alter the probability distribution of future cash flows as to achieve a risk-free rate of return. As such, this approach is often referred to as an adjusted probability measure.

The CFAA is illustrated using options on natural gas forward contracts. A single period binomial framework is assumed with no market frictions of any kind and a riskless asset exists. The binomial model assumes either an up state (*u*) or a down state (*d*). Figure 2.1 illustrate a single period binomial framework for a futures contract.

Figure 2.1. Single period binomial illustration with a futures contract



In this single period model, there are three nodes (states) and two arcs (paths). The following market data is assumed:

S_0 = \$3 1/3 (spot price of asset observed at t)
$F_{t,T}$ = \$3.50 (forward price, observed at t, expiring at T)
X = \$3.50 (strike price)
r = 5% (annual compounded riskless rate)
$T - t$ = 1 year (time to expiration of forward contract)
σ = 40% (standard deviation of continuously compounded, annualized percentage price changes of forward contract)

Now several intermediate parameters are calculated. Remember the objective is to value the call option. The price relative of the forward contracts when the up and down states occur (consistent with the standard option valuation assumptions) as well as the equivalent martingale probabilities (EMP, q) are calculated:

$$u = \frac{F_u}{F_t} = \exp\left(\sigma\sqrt{T-t}\right) = \exp\left(0.40\sqrt{1}\right) = 1.491825 \text{ (forward price relative - up event)}, \tag{2.12}$$

$$d = \frac{F_d}{F_t} = \frac{1}{u} = \frac{1}{1.491825} = 0.670320 \text{ (forward price relative - down event)}, \tag{2.13}$$

$$q_u = \frac{1-d}{u-d} = \frac{1-0.670320}{1.491825-0.670320} = 40.13123\% \text{(EMP – up event), and} \tag{2.14}$$

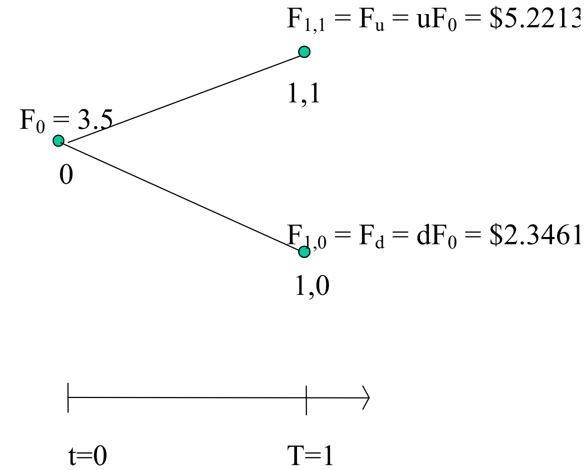
$$q_d = \frac{u-1}{u-d} = 1 - q_u = 1 - 0.4013123 = 59.86877\% \text{ (EMP – down event)}. \tag{2.15}$$

Note that the expected value of the forward price relatives is one or

$$E_q\left(\frac{F_T}{F_t}\right) = q_u u + q_d d = 0.4013123(1.491825) + 0.5986877(0.670320) = 1.0\,. \tag{2.16}$$

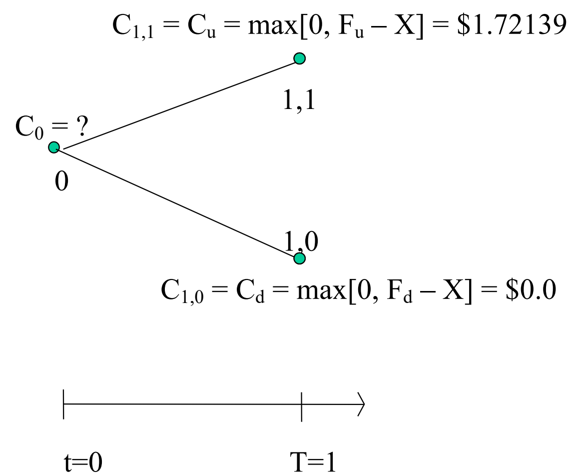
Thus, q is an equivalent martingale measure. Based on these parameters, Figure 2.2 illustrates the binomial tree results.

Figure 2.2. Numerical illustration of single period binomial model



Therefore, Figure 2.3 illustrates the binomial tree for the call option.

Figure 2.3. Numerical example of single period binomial model for call option



To find the value of the call option, an additional parameter is needed. The call option delta (Δ_C) measures the sensitivity of option prices to changes in the underlying forward price.

$$\Delta_c = \frac{c_u - c_d}{F_u - F_d} = \frac{\$1.72139 - \$0}{\$5.22139 - \$2.34612} = 0.598688. \tag{2.17}$$

Consider the unusual trading strategy of buying $1/\Delta_C$ call options, going short one forward contract, and borrowing the following amount (B*)

$$B^* = \frac{F_t - dF_t}{1+r} = \frac{\$3.5 - (0.670320)\$3.5}{1+0.05} = \$1.09893. \tag{2.18}$$

Hence the portfolio (Π_t) is valued at time t as (remember the cost of entering a forward contract is zero)

$$\Pi_t = \frac{1}{\Delta_c} c_t - B^* = \frac{1}{\Delta_c} c_t - \frac{F_t - dF_t}{1+r} = \frac{1}{\Delta_c} c_t - \$1.09893. \tag{2.19}$$

The values of this portfolio for the up and the down states are

$$\Pi_{T,u} = \frac{1}{\Delta_c} c_{T,u} + \left(F_t - uF_t\right) - B^*\left(1+r\right)$$

(up state) and (2.20)

$$= \frac{1}{0.598688}\$1.72139 + \left(\$3.50 - \$5.22139\right) - \$1.09893\left(1+0.05\right) = \$0$$

$$\Pi_{T,d} = \frac{1}{\Delta_c} c_{T,d} + \left(F_t - dF_t\right) - B^*\left(1+r\right)$$

(down state). (2.21)

$$= \frac{1}{0.598688}\$0.0 + \left(\$3.50 - \$2.34612\right) - \$1.09893\left(1+0.05\right) = \$0$$

Due to the zero future portfolio value, the value of the portfolio at t should also be zero. Therefore, the option price is

$$\Pi_t = \frac{1}{\Delta_c} c_t - B^* = \frac{1}{\Delta_x} c_t - \frac{F_t - dF_t}{1+r} = \$0 \qquad (2.22)$$
$$c_t = \Delta_c \frac{F_t - dF_t}{1+r} = \$0.657918$$

This equation is referred to as the no arbitrage method of valuing the option. There are two other perspectives that yield the same result. The equivalent martingale method takes the expected future call value and discounts it at the riskless rate.

$$c_t = \frac{1}{1+r} E_q\left(c_T\right) = \frac{1}{1+r}\left(q_u c_u + q_d c_d\right) \qquad (2.23)$$
$$= \frac{1}{1+0.05}\left[0.4013123\left(\$1.72139\right) + 0.5986877\left(\$0\right)\right] = \$0.657918$$

Alternatively, the state-claim method above can be deployed. Here the state-claim values for up and down states are

$$SC_{T,u} = \frac{1}{1+r} q_u = \frac{1}{1+0.05} 0.4013123 = \$0.382202 \text{ (up state) and} \qquad (2.24)$$

$$SC_{T,d} = \frac{1}{1+r} q_d = \frac{1}{1+0.05} 0.5986877 = \$0.570179 \text{ (down state).} \qquad (2.25)$$

Therefore, the value of this call option is

$$c_t = SC_{T,u} c_{T,u} + SC_{T,d} c_{T,d} = 0.382202\left(\$1.72139\right) + 0.570179\left(\$0\right) = \$0.657918. \qquad (2.26)$$

It is possible to demonstrate that these valuation procedures can be generalized to a multi-period setting. However, in the multi-period setting, intermediate trading is required to dynamically replicate the option payoffs (called a self-financing, dynamic replicating strategy). Clearly, for these valuation methods to yield reasonable results, the ability to actively trade the underlying asset (forward contract in this example) is required. The final category of valuation approaches, generically called the discount factor adjusted approach, is now covered.

Discount factor adjusted approach (DFAA)

The traditional approach to valuation is to forecast some future expected cash flows and then to take the present value of this future expected cash flow stream. John Burr Williams (1938) is usually credited with first articulating this procedure for common stocks. Williams states "The investment value of a stock [is] the present worth of all the dividends to be paid upon it adjusted for expected changes in the purchasing power of money." Interestingly, Williams goes on to argue "That neither marketability nor stability should be permitted to enter into the meaning of the term investment value." (See Ellis (1989), p. 153 and 156.)

In 1959, Gordon, when introducing the now famous dividend discount model bearing his name, argued that the appropriate discount rate increases with the degree of uncertainty related to the future dividend stream. Hence the "stability" of Williams does influence market value. From this foundational paper, a vast literature has developed extending and testing various aspects of this approach to valuation. We assign the label discount factor adjusted approach to these types of methods. The identifying criterion for a valuation method to fall in the DFAA category is that the adjustment for risk is made in the denominator of the valuation equation. The higher the risk (however defined), the higher the interest rate will be for discounting. When the nature of existing securities and/or the structure of the state space does not afford the ability to derive state-claims, then the valuation method typically adjusts for risk in the denominator by assuming a specific risk premium. This approach is the least favored due to the difficulty in accurately estimating required inputs and the resulting prices' sensitivity to these estimated inputs.

The discount factor adjusted method does not alter the cash flow probability distribution, rather the risk adjustment is taken in the interest rate at which the cash flows are discounted through time. There must be sufficient structure imposed upon the state-space to compute at least the expected future cash flows and the appropriate risk premium.

$$P_i = \sum_{t=1}^{T}\sum_{j=1}^{m}\frac{1}{\left(1+r_t+RP_{i,t,j}\right)^t}p_{t,j}CF_{i,t,j}, \tag{2.27}$$

where $p_{t,j}$ denotes the subjective probability based on a particular individual's perspective on future cash flows. Also, the size of the risk premium is a function of compounding method.

When sufficient structure exists to use the CFAA to valuation, using the DFAA requires a direct mapping between the risk premium and the assigned probabilities for future states, otherwise multiple values for the same security are obtained. In some sense, such one-to-one mapping does not always hold due to the vast amount of trading that occurs daily. Obviously, when a trader's probability beliefs and risk premium result in valuations sufficiently different from market prices, trading will occur.

A simple example of the DFAA is the standard Gordon growth model for valuing common stocks, $P_0 = D_1/(k-g)$, where k is the cost of equity capital or the investor's required rate of return. The typical way the investor's required return is estimated is by using the risk-free rate plus a risk premium (for example, CAPM $k = r + \beta_i(E(r_m) - r)$). Other examples of this approach are valuing mortgage backed securities with the option adjusted spread. These methods are extremely sensitive to parameter estimation error and are hard to externally verify. Because the DFAAs are used widely in practice, one would conclude that there is currently insufficient structure in some markets to apply either the MCA or CFAA.

The DFAA is placed within the CFAA using the binomial framework. Consider again the simple one period binomial framework in the previous section, the difference here is that each investor will impose their own subjective beliefs about the probability of the up and down state. For example, suppose an investor believed that the probability of an up event was 43% (as opposed to the equivalent martingale probability of 40.13123% identified earlier). Now we have two issues to address. First, what is the appropriate risk premium? Second, what is the appropriate value for the call option? Consider a constant risk premium of 7.5061% or

$$\begin{aligned} c_t &= \frac{1}{1+r+RP}E_p\left(c_T\right) = \frac{1}{1+r+RP}\left(p_u c_u + p_d c_d\right) \\ &= \frac{1}{1+0.05+0.075061}\left[0.43\left(\$1.72139\right)+0.57\left(\$0\right)\right] = \$0.657918 \end{aligned}, \tag{2.28}$$

which is the same result as CFAA methods. Clearly, they are the same by selecting the appropriate risk premium. Alternatively, we can solve for the implied risk premium.

$$RP = \left[\frac{E_p\left(c_T\right)}{c_t}\right]^{1/T-t} - \left(1+r\right) = \left[\frac{0.740198}{0.657918}\right]^{1/1} - \left(1+0.05\right) = 0.075061. \tag{2.29}$$

By combining the CFAA and DFAA approaches, interesting information can be gleaned from derivatives market values. The CFAA approach can be used to establish the appropriate volatility (or binomial tree) and the DFAA approach can be used with an investor's view to determine the implied risk premium. The implied risk premium is a useful measure for assessing hedging and speculative trading activities.

Selecting the best approach to valuation

Three categories of valuation methodologies encompass virtually all methods of valuation; market comparable approach (MCA), cash flow adjusted approach (CFAA), or discount factor adjusted approach

(DFAA). From a confidence perspective, market comparable is the best, followed by the cash flow adjusted method. Only as a last resort does one wish to go with a discount factor adjusted method. However, within energy markets considering the DFAA is reasonable due to lack of liquidity or other trading problems.

The next exhibit summarizes the major assumptions and their importance within the various approaches to valuation. For MCA, the existence of a set of securities that exactly replicate the future payoffs of a particular security and short selling are the critical assumptions. Is a public utility willing to short power in July? For CFAA, there are several assumptions that are critical, however, we no longer need the existence of a replicating set of securities. Finally, the critical assumption of DFAA is the ability to explicitly adjust for risk when discounting the future expected cash flows. Table 2.4 summarizes the major assumptions of the three approaches to valuation.

Table 2.4. Major assumptions of the three approaches to valuation

Assumptions	**MCA**	**CFAA**	**DFAA**
Short Selling Allowed With Full Use of Proceeds	Strong	Strong	NR*
Trading Cost Minimal	Weak	Strong	NR
Set of Securities Exist to Replicate Payoffs	Strong	NR	NR
Stochastic Process to Model Risk Variable	NR	Strong	Weak
Trading Strategy Exist to Replicate Payoffs	NR	Strong	NR
Explicit Risk Adjustment	NR	NR	Strong

* Not Relevant

Summary

In this chapter, three categories of approaches to valuation are reviewed to reduce the confusion regarding how different valuation models interrelate and when various models should be reasonable to use. Just for fun, Appendix 2A explores the philosophical foundations of quantitative finance.

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Appendix 2A. Philosophical Foundations of Finance

“*Ideas have both antecedents and consequences.*” Author unknown

Learning objectives

- Emphasize the importance of understanding finance as a social science
- Quantitative finance rests upon important philosophical foundations
- Important philosophical foundations can be categorized as logic, epistemology, metaphysics, and ethics
- Introduction to the notion of warrant and explain the differences between positivism and particularism

Introduction

In this appendix, we explore a topic generally ignored these days—philosophy. Prior to diving into the quantitative finance, we introduce some philosophical foundations of quantitative finance.

Warning: The material that follows in this review is rather dense and tough to follow for someone with minimal prior exposure. After decades of industry consulting, however, I view this content as vital to a financial analyst’s ultimate success. Numerous testimonials of bankrupt ideas witness to the insights presented next. Finance does not operate with the same epistemic certainty as physics.

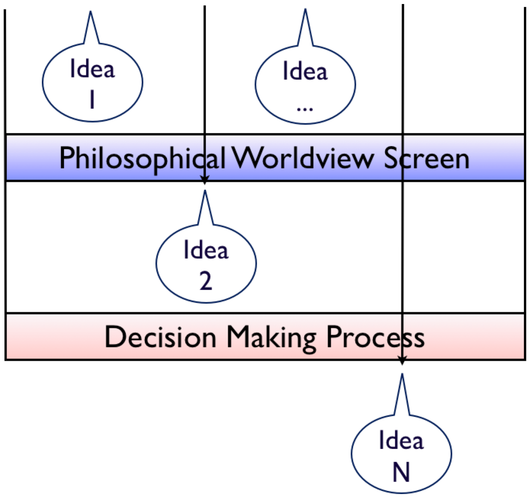
Philosophical foundations of quantitative finance

Thesis: Financial risk management will be improved when finance professionals better understand the basic philosophical foundations upon which this profession rests.

From expert testimony work to deciding which quantitative model to deploy on a trading floor, philosophical decisions are constantly there. For example, it is vital for an expert witness to provide independent analysis often leading to challenging moral issues. Also, how does one decide when a firm’s transaction fails to qualify as a “bona fide hedge” according to the Dodd-Frank Act? We will return to this question later in this introduction.

A philosophical worldview is a set of key propositions that one believes usually stemming from foundational beliefs. In this context, philosophical worldview means a perspective based on an ordered set of key propositions that govern all aspects of life. The word “worldview” is derived from a German word, Weltanschauung meaning wide world perspective; specifically, from Welt (world) and anschauung (perspective).

These key propositions tend to be presuppositions and core assumptions as opposed to the decision-making process taken for a particular task. Presuppositions are implicit assumptions about the world for it to make sense; in epistemology, presuppositions relate to the requirements necessary to put forth a coherent philosophical worldview.



The decision-making process means any method of assessing the "correctness" of a particular "idea" within a particular worldview. Ideas, or more precisely, propositions, are essentially truth claims. For example, one proposition is "Option prices reflect all publicly available information." Whether or not this proposition is deemed "valid" will depend on both the decision-making process deployed, as well as the philosophical worldview.

The philosophical worldview always precedes the decision-making process. The biggest influence on how decisions are made is often not the decision-making process, rather the philosophical worldview. This is particularly true in quantitative finance.

Little "Ideas"

One way to clarify the approach taken here is to consider how an individual assesses an "idea." Specific "ideas" arrive from many sources, data services (Bloomberg), visual media (TV), audio media (Radio), newspapers, books, conferences, podcasts, friends, and so forth. With limited time and limited resources, how does one decide which "ideas" warrant further consideration and which "ideas" get trashed right away. The ability to quickly toss out time consuming "ideas" before expending resources on them is a legitimate way to save precious resources.

The philosophical worldview *always* precedes the decision-making process. The biggest influence on how decisions are made is often not the decision-making process, but rather the philosophical worldview.

There are two key questions every "idea" must answer for the financial analyst to be warranted in incorporated the new "idea" into her intellectual arsenal.

For the philosophical worldview screen, one should ask, "Is the 'idea' *coherent* within the worldview?"

For the decision-making process screen, one should ask, "Does the 'idea' *correspond* to reality?"

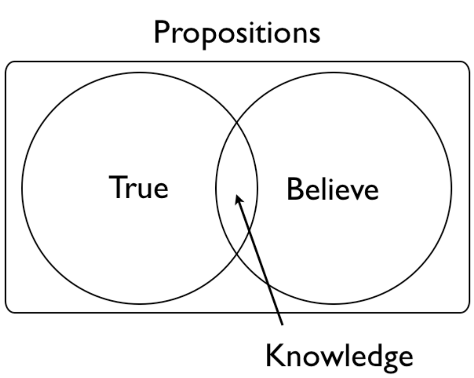
Philosophical foundations are based on logic, epistemology, metaphysics, and ethics.

Logic

Logic is the study of the rules of reasoning. The goal of logic is to reach a conclusion, specifically to improve one's ability to form good arguments as well as critically evaluate others' arguments. For example, the law of noncontradiction is widely held by financial analysts. Specifically, some "idea" P cannot be both true and false in the same sense at the same time.

Epistemology

Epistemology addresses how we know reality. It represents philosophical positions on the nature of knowledge – the branch of philosophy that studies knowledge. For many financial issues, it is just as important to know how we know what we know along with what we know.



Consider the figure nearby: Propositions contain the complete feasible set of "ideas." Assume True is a subset of propositional knowledge where the proposition is in fact true. Assume Believe is also a subset of propositional knowledge where the proposition is in fact believed by a particular analyst. Therefore, one definition of knowledge is the subset containing the intersection of Truth and Belief where there is warrant or justification. Unfortunately, as with most philosophical issues, it is not quite that simple.

Correspondence theory of truth

Consider two concepts, the truth-bearer and the truth-maker.

Truth-bearer is propositions that are true or false. For example, as of today the S&P 500 index is up this calendar year.

Truth-maker is the reality or the states of affairs, that is, the facts. For example, as of today the S&P 500 index is 50 points higher since the beginning of the year.

When the truth-maker corresponds with the truth-bearer, we say truth obtains. Correspondence refers to a notion that there is a relation between a proposition (truth-bearer) and the state of affairs (truth maker) that is its intentional object. Truth is grounded in intentionality. While evidence is truth-conducive, it is not the same thing as truth itself.

Metaphysics

Metaphysics is the study of what we know about reality. That is, the philosophical study of the nature of being and the ultimate categories or kinds of things that are real. Quantitative finance relies heavily on metaphysics as finance involves "ideas" and not physical objects. For example, consider the following quotes:

In his 1938 path-breaking work, Frederick R. Macaulay notes, "The concept of 'pure' or 'riskless' interest is metaphysical. The practical contrast is not between 'pure' and 'impure' but between 'promised' or 'expected' and 'actual' or 'realized'." (See *Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the United States since 1856*, NBER, p. 38.)

Jeffery M. Lipshaw concludes, "So, economics is a *science* in the logical positivist tradition. It ought not try to speculate why things are happening in a metaphysical sense, but simply to explain or predict regularities." (See "The Epistemology of the Financial Crisis: Complexity, Causation, Law, and Judgment," *Southern California Interdisciplinary Law Journal*, Vol. 19, 2009, p. 31.)

For many, the realm of metaphysics is denied. Before proceeding, we introduce one important philosophical question.

Do abstract entities exist?

One metaphysical debate is whether abstract entities even exist. Milton Friedman in the 1950s and 1960s successfully advocated for taking the logical positivist approach to economics. "Positive economics is in principle independent of any ethical position or normative judgments. As Keynes says, it deals with 'what is,' not with 'what ought to be.' Its task is to provide a system of generalizations that can be used to make correct predictions about the consequences of any change in circumstances." (See "The Methodology of Positive Economics," *Essays in Positive Economics*, 1966.) Thus, Friedman was one of the pioneers in migrating the economics profession away from the normative (what ought to be) to the positive (what is). Normative propositions fall within the category of abstract entities. Thus, as practicing financial analysts, one must decide whether abstract entities, such as integrity, exists.

Consider the following logical approach to this question.

EITHER {P: "Abstract entities exist"} OR ¬P: {"Abstract entities do not exist"}

Based on the law of noncontradiction, either P or ¬P is true, P and ¬P cannot both be true.

Consider the following definitions:

(U) **Universe** – "total spatiotemporal system of matter and (impersonal) energy, that is, as the sum total of material objects, in some way accessible to the senses and to scientific investigation."

(A) **Abstract objects** – "immaterial (i.e., nonphysical) entities that do not exist inside space and time; instead they are timeless and spaceless." (Moreland and Craig, 2003)

If $x \in U$ (read x is an element of U), then one can address where and when it is.

If $x \notin U \Rightarrow x \in A$ (read x is not an element of U implies x is an element of A), then one asking where and when it is incoherent.

x = {atoms, mountains, planes, stock certificate, mortgage document, plastic credit card}, $x \in U$

y = {properties (e.g., color, goodness), relations (e.g., greater than, father of), sets (e.g., {1, 2, 3, 4, 5}), numbers (e.g., 1, 2, 3), and propositions (e.g., grass is green)}, $y \in A$

(W) **World** – "sum total of everything whatever that exists including nonspatiotemporal abstract entities as well as the spatiotemporal universe of physical entities."

Based on logic, we can represent this issue with the following symbols (recall {P: "Abstract entities exist"}):

P v ¬P <=> W v U <=> (U & A) v U

where v denotes “or,” <=> denotes “if and only if,” and “&” denotes “and.”

Ethics

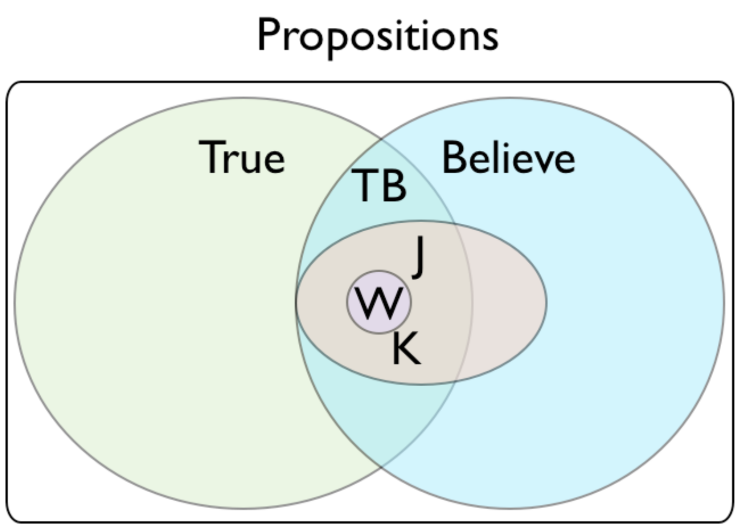
“(T)he study of what is right and what is wrong. Epistemology is concerned with the *true*, and ontology is with the *real*, but ethics with the *good*.”[3] “Ethics can be understood as the philosophical study of morality, which is concerned with our beliefs and judgments regarding right and wrong motives, attitudes, character and conduct.”[4] For example, in finance, not everything that has value is priced and not everything that has a price is valuable. Integrity has value but is not priced. Crime, such as insider trading, often has a price, but is not valuable.

We conclude this appendix with a brief sketch of the concept of warrant.

Warrant

Before leaving the philosophical foundations discussion, we introduce the notion of warrant using a rather analytical approach. Consider the following definition:

$$\exists W_{i,t} \equiv \left\{ \forall p_j \ni \left(p_j \in W_{i,t} \right) \Leftrightarrow Ind_{i,t} \ni w_{i,t} \succ Warranted \right\} \subset K_{i,t}$$



This definition can be read as follows: There exists a very small subset of knowledge $K_{i,t}$, denoted $W_{i,t}$, of propositions individual i has warrant at time t. The symbol “ $\ni$ ” denotes “such that” and “ $\succ$ ” denotes “an element of.” Knowledge is a subset of propositions that are both true and believed where there is also justification. Justification implies “one has sufficient evidence for the belief, one formed and maintained the belief in a reliable way ... or one’s intellectual and sensory faculties were functioning properly in a good intellectual environment when he formed the belief in question.” (Moreland and Craig, 74) In this setup, we allow for the possibility that a justified belief is in fact false, whereas knowledge represents a justified belief that is also true.

The warrant property could be time variant and individual variant. The warrant property is also binary, hence, either $p_j\left(k_{i,t,j} \succ True \wedge w_j \succ True\right) \equiv p_j\left(w_{i,t,j}\right)$ or

$$p_j\left(\left\{k_{i,t,j} \succ True \wedge w_j \succ \neg True\right\} \vee \left\{k_{i,t,j} \succ \neg True \wedge w_j \succ \neg True\right\} \vee \left\{k_{i,t,j} \succ \neg True \wedge w_j \succ True\right\}\right) \equiv p_j\left(\neg w_{i,t,j}\right)$$

where “ $\wedge$ ” denotes “and” and “ $\vee$ ” denotes “or.”

Warrant can be identified as “the belief was formed by cognitive faculties that are functioning properly and in accordance with a good design plan in a cognitive environment appropriate for the way those faculties were designed and when the design plan for our faculties is aimed at obtaining truth.” (Moreland and Craig, p. 103) As used here, warranted knowledge is often “properly basic,” such as the law of noncontradiction that does not have direct justification. Other examples of warranted knowledge include propositions such as, “other people have minds,” “abstract objects exist,” “integrity exists,” and so forth.

One way to categorize various philosophical worldviews is either being along the line of a particularist epistemology or along the line of a positivist epistemology. Without going into detail, these two categories of worldviews can analytically be expressed as follows:

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[3]Geisler and Feinberg, page 353.

[4]Moreland and Craig, page 393.

$$\exists W_{i,t} \equiv \left\{ \forall p_j \ni \left(p_j \in W_{i,t} \right) \Leftrightarrow Ind_{i,t} \ni w_{i,t} \succ Warranted \right\} = \left\{ \neg\varnothing \right\} \text{ Particularist epistemology}$$

$$\exists W_{i,t} \equiv \left\{ \forall p_j \ni \left(p_j \in W_{i,t} \right) \Leftrightarrow Ind_{i,t} \ni w_{i,t} \succ Warranted \right\} = \left\{ \varnothing \right\} \text{ Positivism}$$

In this framework, warrant is a very small subset of knowledge. We now provide one illustration of why understanding the philosophical foundations of quantitative finance is important.

Philosophy and finance illustration

When the risk management mission of a corporation is poorly defined, the corporation's stakeholders ultimately suffer. Without some minimal ethics viewpoint combined with financial performance renders the notion of hedging meaningless.

Like many regulations related to hedging, consider the 2010 Dodd-Frank Act. "Risk-mitigating hedging activities in connection with and related to individual or aggregated positions, contracts, or other holdings of a banking entity that are designed to reduce the specific risks to the banking entity in connection with and related to such positions, contracts, or other holdings" are permitted and hence referred to as a bona fide hedge in the Act. Bona fide (Latin – with good faith) in this Act appears to imply a lack of deceit.

Based on the prevailing philosophical worldview held by financial market participants today, almost any transaction will qualify as a bona fide hedge within existing regulations because of nebulous definitions, such as the one found in the Dodd-Frank Act. Thus, it should come as no surprise that several banks multi-billion-dollar speculative transactions that have gone awry would likely be classified as a bona fide hedge under the Act.

Although finance, like many other social sciences, is ultimately metaphysical (that is, transcending the physical), most academic finance professors have now taken the logical positivist approach together with many practitioners in our profession. One unfortunate consequence of the logical positivist worldview is that ethics, and for that matter metaphysics, is meaningless because it is non-physical in flavor.

Until we restore the metaphysical foundations at the core of our profession, concepts like "risk," "volatility," "interest rates," and even "hedging" will be ill-defined. And clearly what is ill-defined will remain poorly managed. In this context, the metaphysical foundations provide the rational basis for a particular ethical viewpoint.

As an illustration of the gravity of this lack of any metaphysical foundation within modern finance, just consider applications of hedging concepts. From the logical positivist perspective, most derivatives transactions cannot really be justified as either a "bona fide hedge" or not. Recall that the logical positivist's worldview renders ethical concepts like "deceit" and "good faith" meaningless.

Most firms have hundreds of positions with exposures to numerous market risks. These same firms have multiple stakeholders with different goals and objectives. There is no requirement in the Act, or for that matter in any other related regulations, for a financial performance benchmark to be clearly defined in advance. Therefore, almost any financial derivatives transactions can be deemed a "bona fide hedge" in the logical positivist tradition. All one must do is identify some existing exposure in the firm with the appropriate empirical correlation and voila, a derivatives transaction is a "bona fide hedge." But from almost any normative framework, such as the CFA Institute's Code of Ethics and Standards of Practice, many financial derivatives transactions today do not pass as a "bona fide hedge;" they would be deemed deceitful and in bad faith.

For students of the human condition, it would come as no surprise that corporate executives today assert that the activities of their traders are "bona fide hedges" in the Dodd-Frank Act sense. If these traders, however, were asked to justify their hedging transactions to their aging parents, they would struggle to do so without blushing (of course, assuming blushing was still a possibility for the traders).

Without the merest normative ethical framework justifying the corporation's existence, hedging will remain a vacuous concept. Without any preconceived and clearly stated risk management benchmarks, whether the corporation is on course or not is meaningless.

We now turn to important financial risk management tools developed in R. Note that these financial risk management tools are always first metaphysical in nature – just an idea. For example, the cumulative distribution function is merely an abstraction that has been implemented mathematically and we provide the R code to take the mathematical representations into machine language. Whether anything in finance adheres to this probability distribution is unlikely, but approximations of this nature assist in improving the decisions made by financial managers.

Appendix References

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