Chapter 2

Approaches to Valuation

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Valuation Challenges

- How to value a residential house?
  - Evaluate similar houses
- How to value a common stock?
  - Present value of future dividend payments
- How to value an option contract?
  - Risk-neutral valuation methods
- Why are these three approaches different?

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Valuation Presuppositions

- Well-defined property rights
  - "Rights to control, use, or transfer things (broadly conceived), including rights in intangibles such as intellectual property." Case, 2003, p. 1.
  - Common stock and naked short selling

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Valuation Presuppositions

- Clear rule of law
  - "(T)he degree to which the society is bound by law, is committed to processes that allow property rights to be secure under legal rules that will be applied predictably and not subject to the whims of particular individuals, matters." Case, 2003, p. 2
  - Financial instruments and insider trading

Valuation Presuppositions

- Culture of trust
  - Features of trust
    - Rely on someone with something valuable
      - Make oneself vulnerable in confidence
      - Trusted will not exploit and will care
  - Means of making business simpler and safer
  - Thomas Hobbes, *Leviathan* (1651), considering life without a state, there would be "war…of every man against every man," life would be "solitary, poor, nasty, brutish, and short." (Bailey, p. 3.)

Valuation Presuppositions

- Mapping Uncertainties
  - Uncertainties related to future activities and events are mappable
  - Mapping is almost always subjective within the context of financial valuation

Standard Assumptions

- Finite time horizon
- Complete probability space $(\Omega, \Im, P)$
  - Ω – set of all possible realizations
  - $\Im$ – sigma field of distinguishable events
  - P – probability measure on $\Im$
- Uncertainty is modeled with Brownian motion

Review of Expectations

- $E(x)$ – expected value of x
  - Flip of coin: Heads = +\$3, Tails = –\$1
  - $E_p(x) = (1/2)3 + (1/2)(-1) = \$1$
- Risk adjusted expected value of x
  - Probability of Tails = 60% (dislike loss, subjective)
  - $E_q(x) = (0.4)3 + (0.6)(-1) = 0.6$
  - Alternative measure of probability (worth only \$0.6)

Review of Present Value

- Present value at risk free rate, r_f:
  - $PV_{rf}(x) = x/(1 + r_f)^T$
- Present value at risk adjusted rate, k:
  - $PV_k(x) = x/(1 + k)^T = x/(1 + r_f + RP)^T$
  - RP – subjective risk premium

Financial Valuation

- "Underlying all practical problems in connection with the financial aspects of the corporation, there is the problem of value." Arthur Stone Dewing [1941]
- "Value in Exchange" vs "Value in Use"
- Value => useful, scarce, require sacrifice
- Reallocation of consumption through financial assets satisfies "value"

Three Approaches

$$V_i = \sum_{\substack{k=1 \\ k \neq i}}^{s} \alpha_k P_k \text{ (Market comparable approach).}$$

$$V_i = \sum_{t=1}^{T} \sum_{j=1}^{m} \frac{1}{\left(1 + r_t + RP_{i,t,j}\right)^t} p_{t,j} CF_{i,t,j} \text{ (Discount factor adjusted approach).}$$

$$V_i = \sum_{t=1}^{T} PV(\$1,t) \sum_{j=1}^{m} q_{t,j} CF_{i,t,j} = \sum_{t=1}^{T} PV(\$1,t) E_q\left(CF_{i,t}\right) \text{ (Cash flow adjusted approach).}$$

Approaches to Valuation
How would you value a house?

- Present value of rental income
  - Estimate expected rental income
  - Estimate appropriate discount rate
- Sales price of similar homes
  - Identify "similar"
  - Adjustments
- Other methods

Market Comparables Approach

- "… when several services or commodities satisfy a human want equally well, the value of each one of them is determined not by the sacrifice necessary to obtain each, but rather by the sacrifice necessary to obtain the one most easily available, which may be substituted for any one of the others." Dewing [1941]

Market Comparables Approach

- Two investments that result in the same future cash flows, for sure, ought to have the same current value
- Share of common stock trading on two exchanges
- Capitalizing on mis-pricing of this nature is called arbitrage
- Does not require dynamic trading

Market Comparables Approach

- $V = sum[w_i V_i]$
- $V_i = \sum_{\substack{k=1 \\ k \neq i}}^{s} \alpha_k P_k$
- No subjective items
  - No discount rate required, risk adjusted or not
  - No future distribution of outcomes assumed based on individual's subjective probability beliefs (p) or any other probability beliefs

MCA Key Assumptions

- There exists a set of securities that produce future cash flows in each state identical to the security being valued (even states that are currently unimaginable).
- Trading costs and other market frictions are minimal.
- Short selling is allowed.

MCA Justification

- Well-defined state space (time t, space j)

$$CF_{i,t,j} = \sum_{\substack{k=1 \\ k \neq i}}^{s} \alpha_k CF_{k,t,j}$$

- Absence of static arbitrage implies

$$P_i = \sum_{\substack{k=1 \\ k \neq i}}^{s} \alpha_k P_k$$

MCA Example: Put-Call Parity

- Futures option put-call parity

$$c_t = PV(\$1, T-t)(F_{t,T} - X) + p_t = \frac{F_{t,T} - X}{(1+r)^{T-t}} + p_t$$

- Four markets
  - Call option
  - Put option
  - Futures contract
  - Financing

Arbitrage Cash Flow Table

Table 2.1. Forward put-call parity cash flow table

Strategy	Today (t)	At Expiration (T) $F_{T,T} < X$	At Expiration (T) $F_{T,T} > X$
Sell call	$+c_t$	\$0	$-(F_{T,T} - X)$
Lend or Borrow	$-PV(\$1,T-t)(F_{t,T} - X)$	$+(F_{t,T} - X)$	$+(F_{t,T} - X)$
Buy put	$-p_t$	$+(X - F_{T,T})$	\$0
Net		$+(F_{t,T} - F_{T,T})$	$+(F_{t,T} - F_{T,T})$
Long Forward	\$0	$+(F_{T,T} - F_{t,T})$	$+(F_{T,T} - F_{t,T})$
NET	???	\$0	\$0

Alternative CF Table

Table 2.2. Alternative forward put-call parity cash flow table

Strategy	Today (t)	At Expiration (T) $F_{T,T} < X$	At Expiration (T) $F_{T,T} > X$
Buy call	$-c_t$	\$0	$+(F_{T,T} - X)$
Borrow or Lend	$+PV(\$1,T-t)(F_{t,T} - X)$	$-(F_{t,T} - X)$	$-(F_{t,T} - X)$
Sell put	$+p_t$	$-(X - F_{T,T})$	\$0
Net		$-(F_{t,T} - F_{T,T})$	$-(F_{t,T} - F_{T,T})$
Short Forward	\$0	$-(F_{T,T} - F_{t,T})$	$-(F_{T,T} - F_{t,T})$
NET	+ by assumption	\$0	\$0

Numerical Example

Table 2.3. Arbitrage example with forward put-call parity cash flow table

Strategy	Today (t)	At Expiration (T) $F_{T,T} < X$	At Expiration (T) $F_{T,T} > X$
Sell call	$+c_t = \$0.53$	\$0	$-(F_{T,T} - X)$ $= -(F_{T,T} - \$3.5)$
Lend or Borrow	$-PV(\$1,T-t)(F_{t,T} - X)$ $-0.95238(\$3.5 - \$3.5)$ $= \$0$	$(F_{t,T} - X)$ $(\$3.5 - \$3.5) = \$0$	$+(F_{t,T} - X)$ $= +(\$3.5 - \$3.5) = \$0$
Buy put	$-p_t = \$0.52$	$+(X - F_{T,T})$ $= +(\$3.5 - F_{T,T})$	\$0
Net		$+(F_{t,T} - F_{T,T})$ $= +(\$3.5 - F_{T,T})$	$+(F_{t,T} - F_{T,T})$ $= +(\$3.5 - F_{T,T})$
Long Forward	\$0	$+(F_{T,T} - F_{t,T})$ $= (F_{T,T} - \$3.5)$	$+(F_{T,T} - F_{t,T})$ $= +(F_{T,T} - \$3.5)$
NET	+\$0.01	\$0	\$0

Insights from MCA

- Transaction costs assumptions critical
- No assumptions regarding future behavior of underlying
- No assumption on risk aversion of investor
- No intermediate trading required
- Strong beliefs on direction of underlying may influence spot prices, but will have no influence on the derivative due to arbitrage

MCA Summary

- Static replication of future cash flows
- No risk information provided
- Based on the law of one price
- Widely applied
  - Mutual fund net asset value
  - Value of portfolio
  - Derivatives static arbitrage

Cash Flow Adjusted Approach

- Requires active trading based on “self-financing, dynamic replication” (SFDR) (no intermediate cash flows in trading strategy)
- High level of marketabililty required (Williams was wrong, marketability does influence value)
- If SFDR possible, value “as if” traders are risk-neutral

Cash Flow Adjusted Approach

- $V = PV_{rf}\{sum[E_q(CF)]\}$
- $V_i = \sum_{t=1}^{T} PV(\$1,t)\sum_{j=1}^{m} q_{t,j} CF_{i,t,j} = \sum_{t=1}^{T} PV(\$1,t) E_q\left[CF_{i,t}\right]$
- Two *objective* items
  - r_f – risk *free* discount rate
  - Future distribution of outcomes assumed based on “objective” probability beliefs (q)

CFAA Key Assumptions

- There exists a stochastic process (or processes) that accurately depicts the future potential outcomes; that is, the state space is well defined.
- There exists a trading strategy that produces future cash flows in each state identical to the security being valued.
- Trading costs and other market frictions are minimal (short selling allowed).

CFAA Justification

- Well-defined state space (time t, space j)

$$P_i = \sum_{t=1}^{T}\sum_{j=1}^{m} SC_{t,j} CF_{i,t,j}$$

- State claims

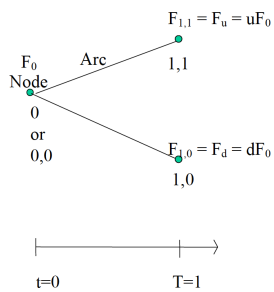
$$SC_{t,j} = PV(\$1,t) q_{t,j}$$

- Absence of dynamic arbitrage implies

$$P_i = \sum_{t=1}^{T} PV(\$1,t)\sum_{j=1}^{m} q_{t,j} CF_{i,t,j} = \sum_{t=1}^{T} PV(\$1,t) E_q\left[CF_{i,t}\right]$$

CFAA Example: Futures Call

- Single period binomial framework
- Three markets
  - Call option
  - Futures contract
  - Financing



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CFAA Calculations

$$u = \frac{F_u}{F_t} = \exp\left(\sigma\sqrt{T-t}\right) = \exp\left(0.40\sqrt{1}\right) = 1.491825 \text{ (forward price relative - up event),}$$

$$d = \frac{F_d}{F_t} = \frac{1}{u} = \frac{1}{1.491825} = 0.670320 \text{ (forward price relative - down event),}$$

$$q_u = \frac{1-d}{u-d} = \frac{1-0.670320}{1.491825-0.670320} = 40.13123\% \text{(EMP – up event), and}$$

$$q_d = \frac{u-1}{u-d} = 1-q_u = 1-0.4013123 = 59.86877\% \text{ (EMP – down event).}$$

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CFAA Selected Calculations

- Delta: $\Delta_c = \dfrac{c_u - c_d}{F_u - F_d} = \dfrac{\$1.72139 - \$0}{\$5.22139 - \$2.34612} = 0.598688$
- Financing: $B^* = \dfrac{F_t - dF_t}{1+r} = \dfrac{\$3.5 - (0.670320)\$3.5}{1+0.05} = \$1.09893$
- No arb. value

$$\Pi_t = \frac{1}{\Delta_c}c_t - B^* = \frac{1}{\Delta_c}c_t - \frac{F_t - dF_t}{1+r} = \frac{1}{\Delta_c}c_t - \$1.09893$$

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CFAA Valuation

- EMM: $c_t = \dfrac{1}{1+r}E_q(c_T) = \dfrac{1}{1+r}(q_u c_u + q_d c_d)$

$$= \frac{1}{1+0.05}\left[0.4013123(\$1.72139) + 0.5986877(\$0)\right] = \$0.657918$$

- State Claims:

$$SC_{T,u} = \frac{1}{1+r}q_u = \frac{1}{1+0.05}0.4013123 = \$0.382202 \text{ (up state) and}$$

$$SC_{T,d} = \frac{1}{1+r}q_d = \frac{1}{1+0.05}0.5986877 = \$0.570179 \text{ (down state).}$$

Therefore, the value of this call option is

$$c_t = SC_{T,u}c_{T,u} + SC_{T,d}c_{T,d} = 0.382202(\$1.72139) + 0.570179(\$0) = \$0.657918.$$

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Insights from CFAA

- Transaction costs assumptions critical
- Requires assumptions regarding future behavior of underlying instrument
- No assumption on risk aversion of investor
- Intermediate trading required (SFDR)
- Strong beliefs on direction of underlying may influence spot prices, but will have no influence on derivative due to arbitrage

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CFAA Summary

- Dynamic replication of future cash flows
- Some risk information provided (EMM)
- Based on self-financing, dynamic replication
- Widely applied
  - BSMOVM
  - Binomial OVM
  - Real option analysis

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How would you value a stock?

- Forecast future expected cash flows and discount at risk-adjusted interest rate
- "…the value of a stock (is) the present worth of all dividends to be paid upon it …" John Burr Williams [1938]
- "…neither marketability nor stability should be permitted to enter into the meaning of the term investment value." Williams [1938]

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Discount Factor Adjusted Approach

- "stability" does influence market value
- present value by risk-adjusted discount rate

$$Value = \sum_{t=1}^{\infty} \frac{E_0(CF_t)}{(1+k)^t}$$

$$k = rf + RP$$

- What discount rate (k) and whose expectation [$E(\ )$]?

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Discount Factor Adjusted Approach

- $V = PV_k\{sum[E_p(CF)]\}$
- $V_i = \sum_{t=1}^{T}\sum_{j=1}^{m} \frac{1}{(1+r_t+RP_{i,t,j})^t} p_{t,j} CF_{i,t,j}$
- Two subjective items
  - k – risk adjusted discount rate
  - Future distribution of outcomes assumed based on individual's subjective probability beliefs (p)

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DFAA Key Assumptions

- There ***may not*** exist a stochastic process (or processes) that accurately depicts the future potential outcomes; that is, the state space is well defined.
- There ***may not*** exists a trading strategy that produces future cash flows in each state identical to the security being valued.
- Trading costs and other market frictions may be significant.

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DFAA Justification

- No assumption regarding well-defined state space
- No appeal to arbitrage
- Intuitive appeal to:
  - Risk adjusted discount rate
  - Subjective expectation

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DFAA Example: Futures Option

- Futures option call value:

$$c_t = \frac{1}{1+r+RP} E_p(c_T) = \frac{1}{1+r+RP}(p_u c_u + p_d c_d)$$

$$= \frac{1}{1+0.05+0.075061}\left[0.43(\$1.72139)+0.57(\$0)\right] = \$0.657918$$

- Implied risk premium:

$$RP = \left[\frac{E_p(c_T)}{c_t}\right]^{1/T-t} - (1+r) = \left[\frac{0.740198}{0.657918}\right]^{1/1} - (1+0.05) = 0.075061$$

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Insights from DFAA

- Transaction costs assumptions *not* critical
- Requires assumptions regarding future behavior of underlying instrument by investor
- Assumption on risk aversion of investor
- Intermediate trading *not* required
- Strong beliefs on direction of underlying may influence spot prices as well as derivative prices, assuming arbitrage not possible

DFAA Summary

- No static or dynamic replication required
- Subjective risk information possible
- Investor risk aversion often implied
- No intermediate trading required
- Strong beliefs directly influence valuations and risk measures

Valuation Assumptions

Table 2.4. Major assumptions of the three approaches to valuation

Assumptions	**MCA**	**CFAA**	**DFAA**
Short Selling Allowed With Full Use of Proceeds	Strong	Strong	NR*
Trading Cost Minimal	Weak	Strong	NR
Set of Securities Exist to Replicate Payoffs	Strong	NR	NR
Stochastic Process to Model Risk Variable	NR	Strong	Weak
Trading Strategy Exist to Replicate Payoffs	NR	Strong	NR
Explicit Risk Adjustment	NR	NR	Strong

* Not Relevant

Ranking of Approaches

- #1) Market Comparables Approach
  No future distribution assumptions
- #2) Cash Flow Adjusted Approach
  No assumptions about risk premium
- #3) Discount Factor Adjusted Approach
  Requires future distribution assumptions
  Requires risk premium
  But used often because #1 and #2 not possible

Summary

- Market comparables approach
  - $V = sum[w_i V_i]$
  - Stock portfolio, mutual fund, put-call parity
- Cash flow adjusted approach
  - $V = PV_r\{sum[E_q(CF)]\}$
  - Option valuation
- Discount factor adjusted approach
  - $V = PV_k\{sum[E_p(CF)]\}$
  - Individual stock valuation

