Module 14.1: Portfolio Risk Management

Learning objectives

- Explore various risk management issues related to stock portfolios
- Understand how diversification results in lower risk measures
- Introduce several explicit measures of the benefits from diversification

Executive summary

In this module, we introduce the notion of the measurable benefits from diversification. Specifically, we quantify in terms in a few ways the benefits of lower risk. We also provide extensive graphical analysis of selected individual stocks and ETFs.

Central finance concepts

Figure 14.1.1 illustrates why diversification is deemed beneficial with randomly selected individual stocks. This figure maps the value of a one dollar investment in SPY and nine randomly selected stocks. We see that by investing in SPY rather than individual stocks, one is assured of achieving the average performance rather than risk achieving the lowest return which in this case might be zero, while also assuring that one will not achieve the highest either.

Figure 14.1.1. Total Return Illustration with Nine Individual Stocks

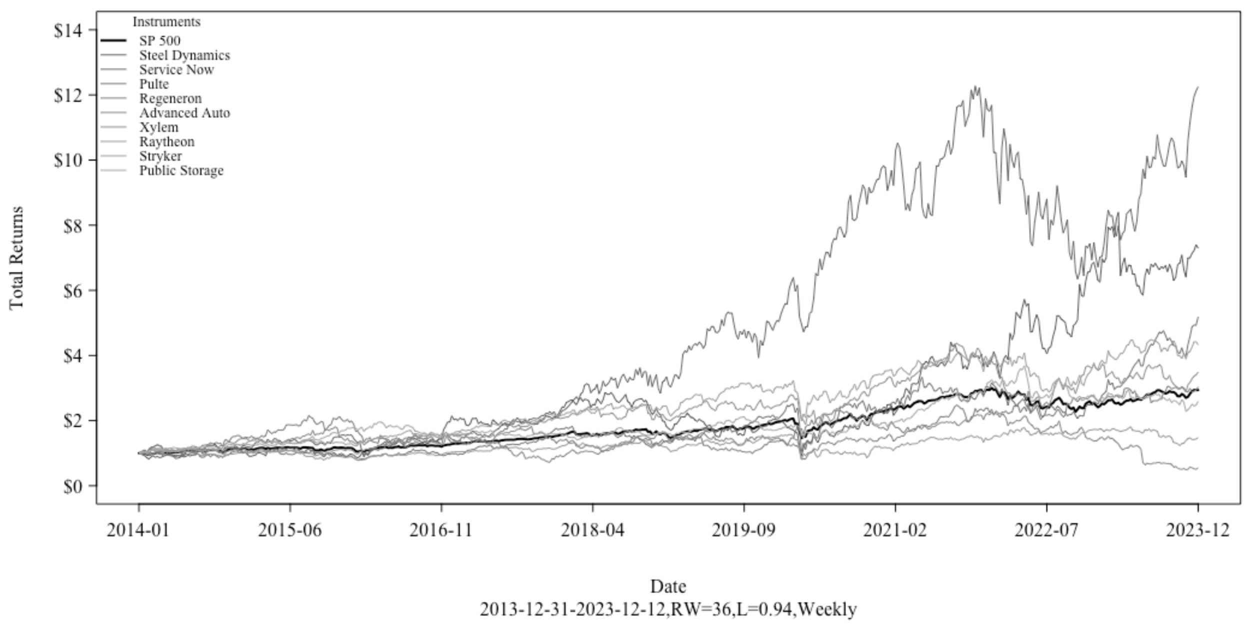


Figure 14.1.2 again illustrates why diversification is deemed beneficial, but this time illustrating the value of a one dollar investment in ten different ETFs. Specifically, the ETF that seeks to mimic the S&P 500 index (SPY, thickest line) as well as nine sector ETFs that seek to mimic nine of the sectors contained within the S&P 500 index (fainter lines). Note that the y-axis is one half of the prior figure ($0 to $7 rather than $0 to $14). One typically notes that by diversifying into SPY rather than a particular sector (say Energy, lowest line) one is assured of achieving the average performance rather than risk achieving the lowest while also assuring that one will not achieve the highest either. Thus, although the dispersion is significantly lower with sector ETFs, it remains considerable with exposures to different sectors.

Figure 14.1.2. Total Return Illustration with Nine Sector ETFs

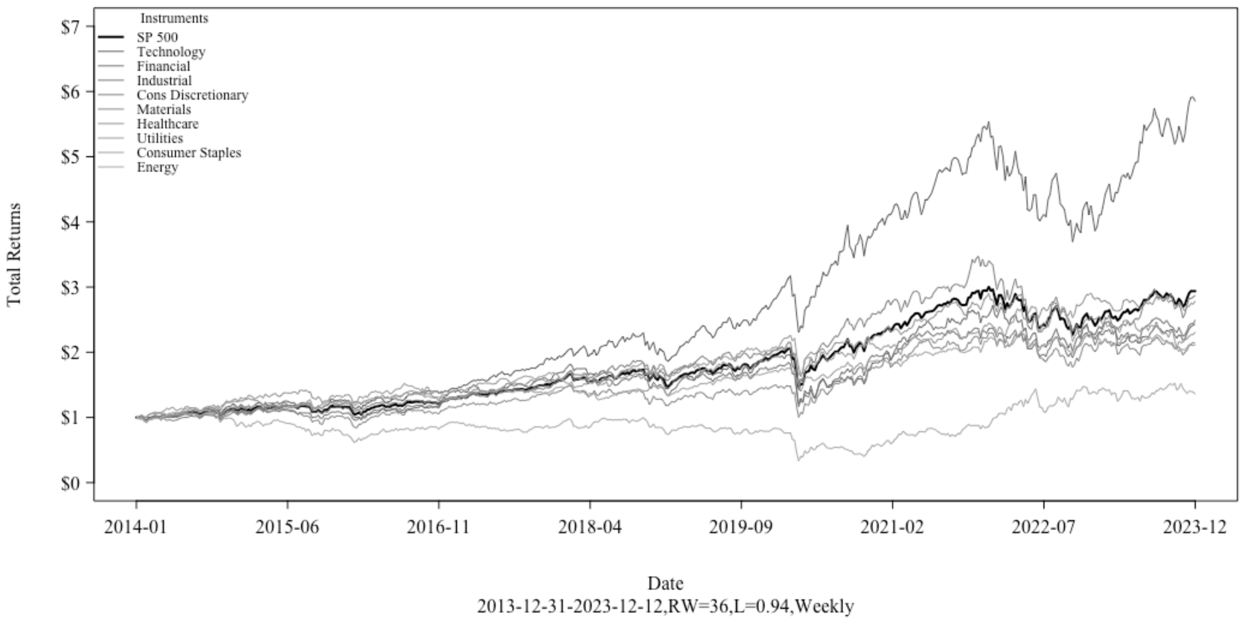


Figure 14.1.3 provides an alternative way to show that diversification is beneficial with randomly selected individual stocks. This figure maps the rolling standard deviation of returns on SPY and nine randomly selected stocks. The returns are based on weekly holding periods and a EWMA lambda of 0.94. We see that the standard deviation of SPY is consistently lower than almost all individual stocks. On rare occasions a particular rolling standard deviation falls below SPY. Notice that risk as measured by rolling standard deviation varies significantly over time.

Figure 14.1.3. Rolling Standard Deviation Illustration with Nine Individual Stocks

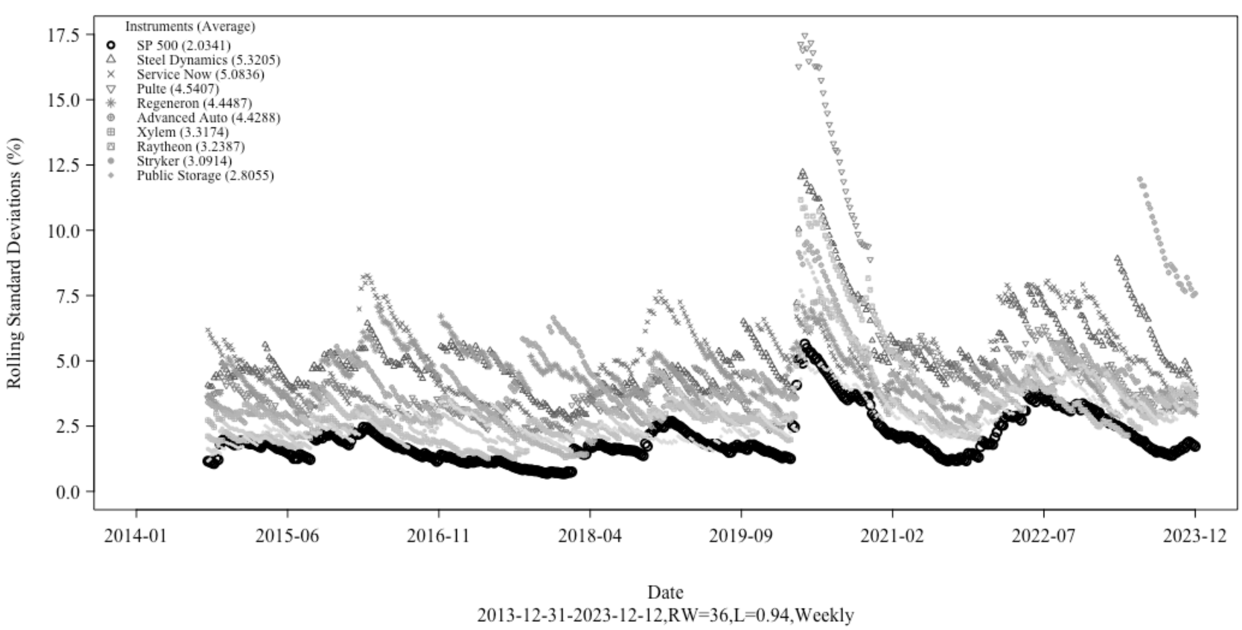


Figure 14.1.4 provides results similar to Figure 14.1.3 except the underlying nine instruments are sector ETFs. Again, this figure maps the rolling standard deviation of returns on SPY and nine randomly selected stocks. The returns are based on weekly holding periods and a EWMA lambda of 0.94. Interestingly, we see

that the standard deviation of SPY often much higher than a particular sector ETF. For example, as expected a portfolio of utility stocks frequently has a lower rolling standard deviation.

Figure 14.1.4. Rolling Standard Deviation Illustration with Nine Sector ETFs

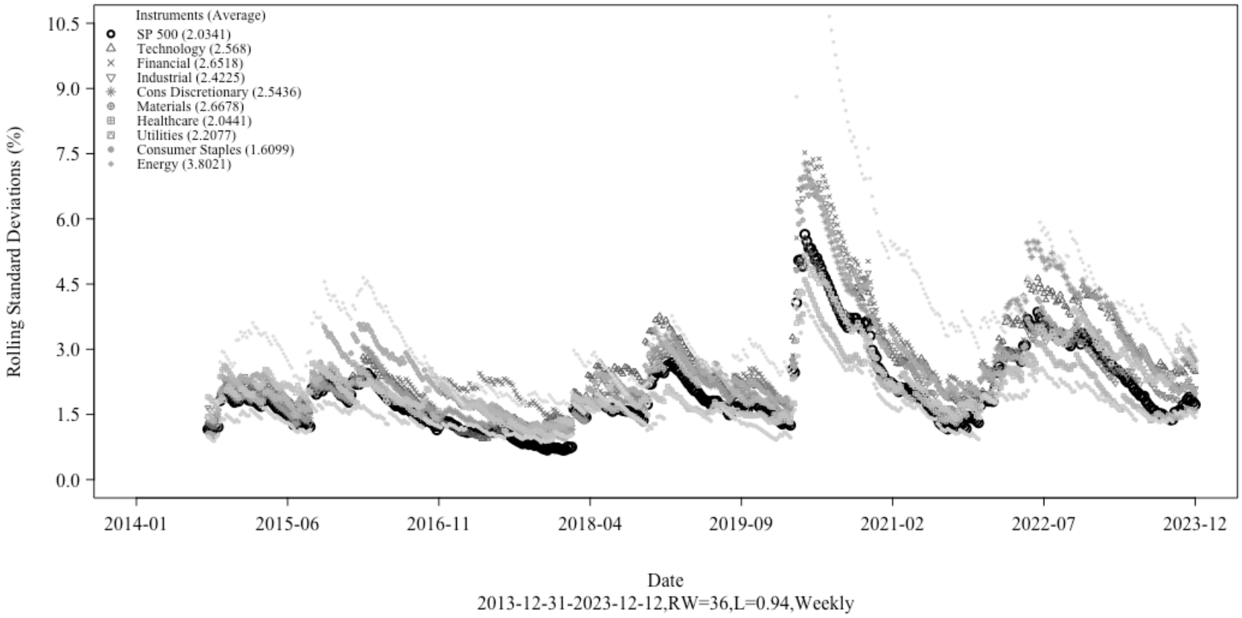


Figure 14.1.5 provides the rolling beta with SPY of nine individual stocks. By definition, the rolling beta of SPY is always 1.0. This figure maps the rolling beta of returns on SPY and nine randomly selected stocks. Again, the returns are based on weekly holding periods and a EWMA lambda of 0.94. One interesting insight is that rolling betas are quite volatile. Thus, historical betas are poor forecasts of future betas.

Figure 14.1.5. Rolling Beta Illustration with Nine Individual Stocks

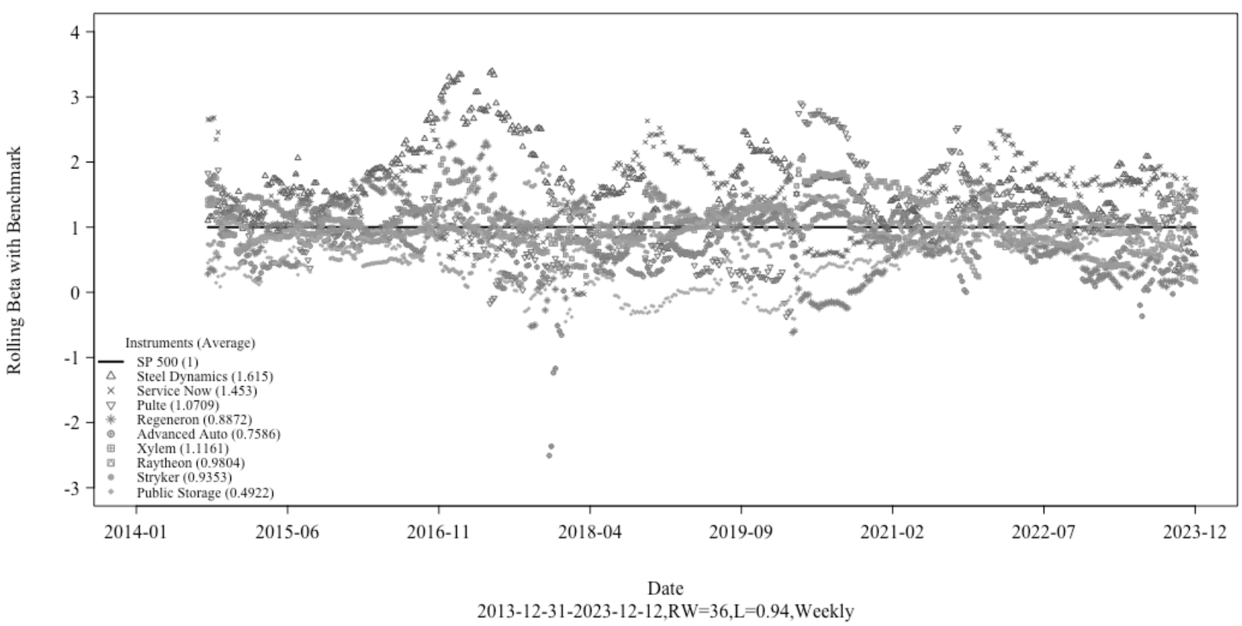
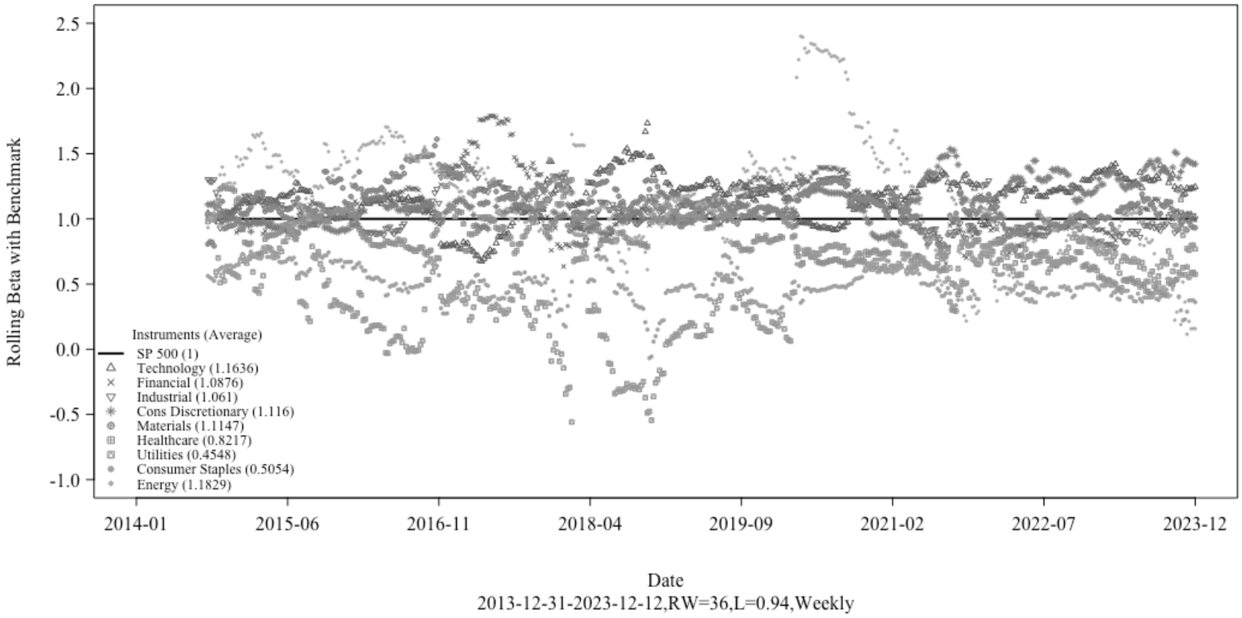


Figure 14.1.6 provides the rolling beta with SPY of nine sector ETFs. Note that the y-axis range has been reduced to –1.0 to 2.5. Sector ETF betas are much more stable and closer to 1.0 than individual stock betas.

Again, by definition, the rolling beta of SPY is always 1.0. Again, one interesting insight is that rolling betas remain quite volatile. Thus, historical betas are poor forecasts of future betas, even for sector ETFs.

Figure 14.1.6. Rolling Beta Illustration with Nine Sector ETFs



As implied by the prior figures, there are benefits to diversification. We will measure these diversification benefits (DBs) in two ways, volatility benefit (VB) and return due to diversification (RDD). The volatility benefit is predicated on the standard deviation of a portfolio being mathematically less than the weighted average of the standard deviations of each position within the portfolio. Thus, one can define the DB in terms of volatility as the *VB* is simply the percentage of the portfolio standard deviation remaining after normalizing by the weighted average of the standard deviations of each position within the portfolio. The detailed mathematical formulation is given later in this chapter. For now, the range of VB is from 0% to 100%, where 0% indicates no VB at all and 100% indicates the maximal VB as the portfolio standard deviation is zero.

Figure 14.1.7 provides the rolling VB with SPY compared with the average of nine individual stocks. This figure maps the rolling VB to SPY for the nine randomly selected stocks. Note that SPY is not the actual portfolio. Again, the returns are based on weekly holding periods and a EWMA lambda of 0.94. One interesting insight is that rolling VB are quite volatile. The sharp drop in the VB with the onset of the pandemic in early 2020 highlights the weakness of relying too heavily on diversification to mitigate portfolio risk. During extreme movements, correlations tend toward one and the VB tends toward zero.

Figure 14.1.7. Rolling Volatility Benefit Illustration with Average of Nine Individual Stocks

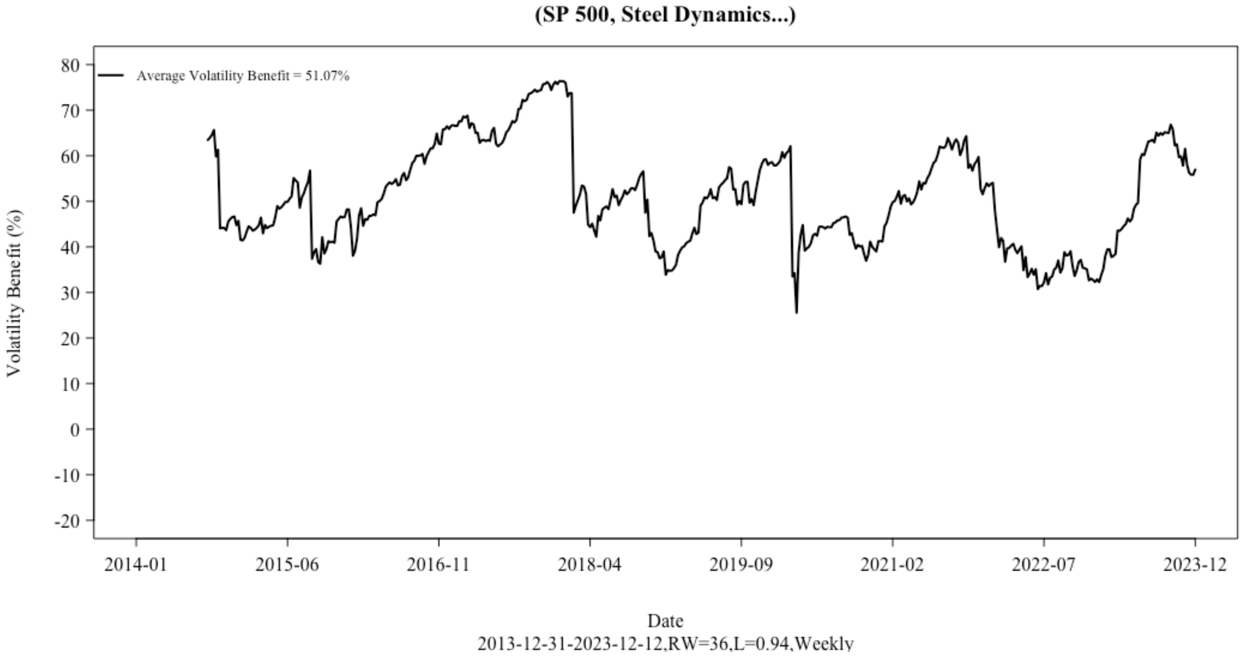
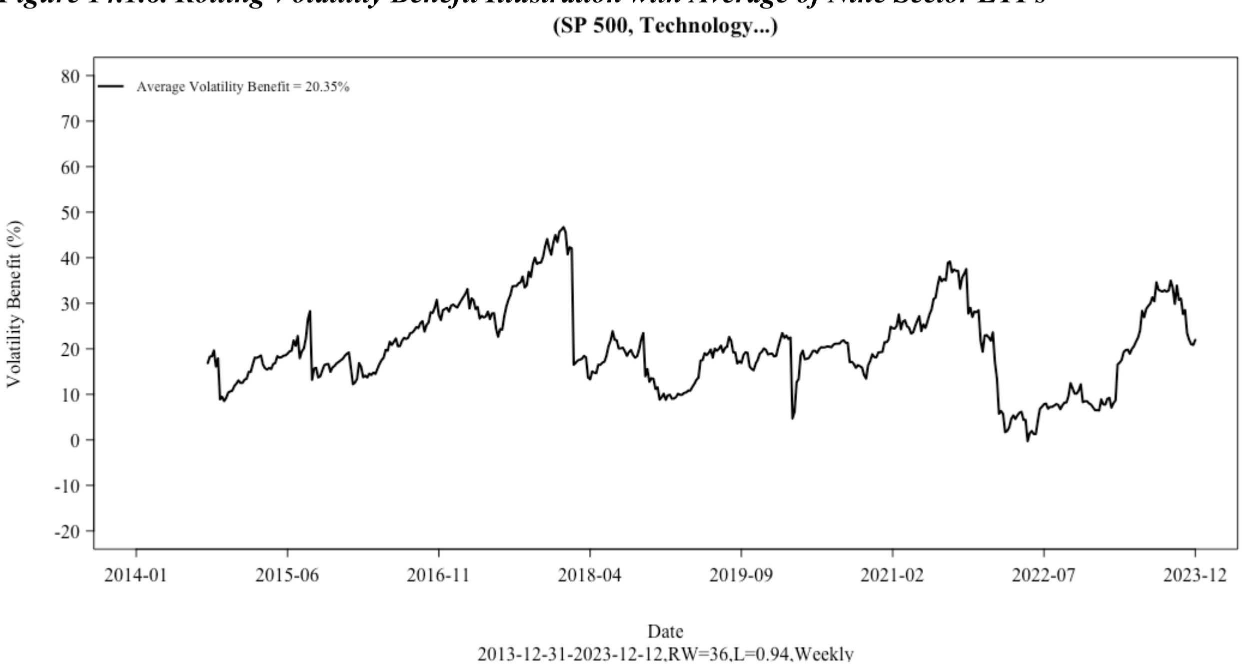


Figure 14.1.8 provides the rolling VB with SPY compared with the average of nine sector ETFs. This figure maps the rolling VB to SPY for these nine sector ETFs. Note again that SPY is not the actual implied portfolio. One clear observation is that the VB is dramatically lower than Figure 14.1.8 with nine randomly selected stocks. As with most financial statistics, the rolling VB are quite volatile.

Figure 14.1.8. Rolling Volatility Benefit Illustration with Average of Nine Sector ETFs



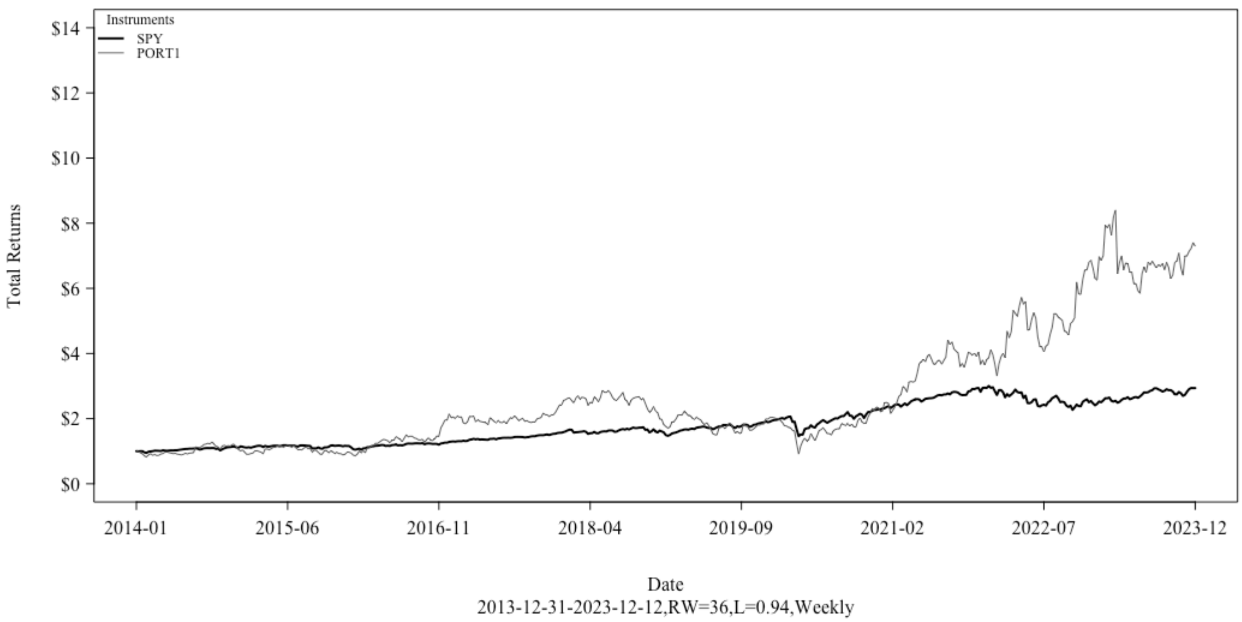
Return due to diversification is covered in the quantitative finance materials below.

We now present numerous figures illustrating the marginal influence of additional financial instruments on various statistical measures.

Marginal influence of additional instrument on statistical measures
We focus here on marginal influence. Specifically, we show the marginal influence on total returns, standard deviations, betas, and volatility benefit.

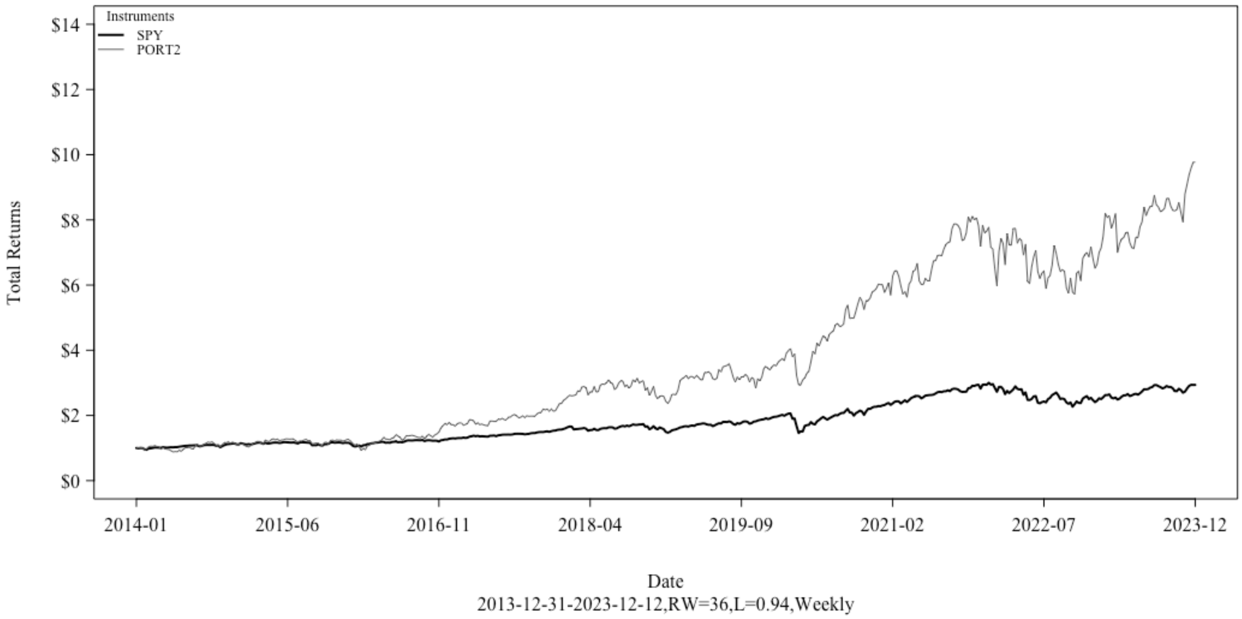
Marginal influence of additional instrument on total return
Figure 14.1.9 illustrates multiple panels of the marginal influence on total returns with adding additional stocks. The stocks are assumed equally weighted as numbers increase through the different panels. As the number of stocks increase, we see the equally weighted portfolio trending toward SPY.

Figure 14.1.9. Marginal Influence on Total Return with Individual Stocks
Panel A: One Stock with SPY



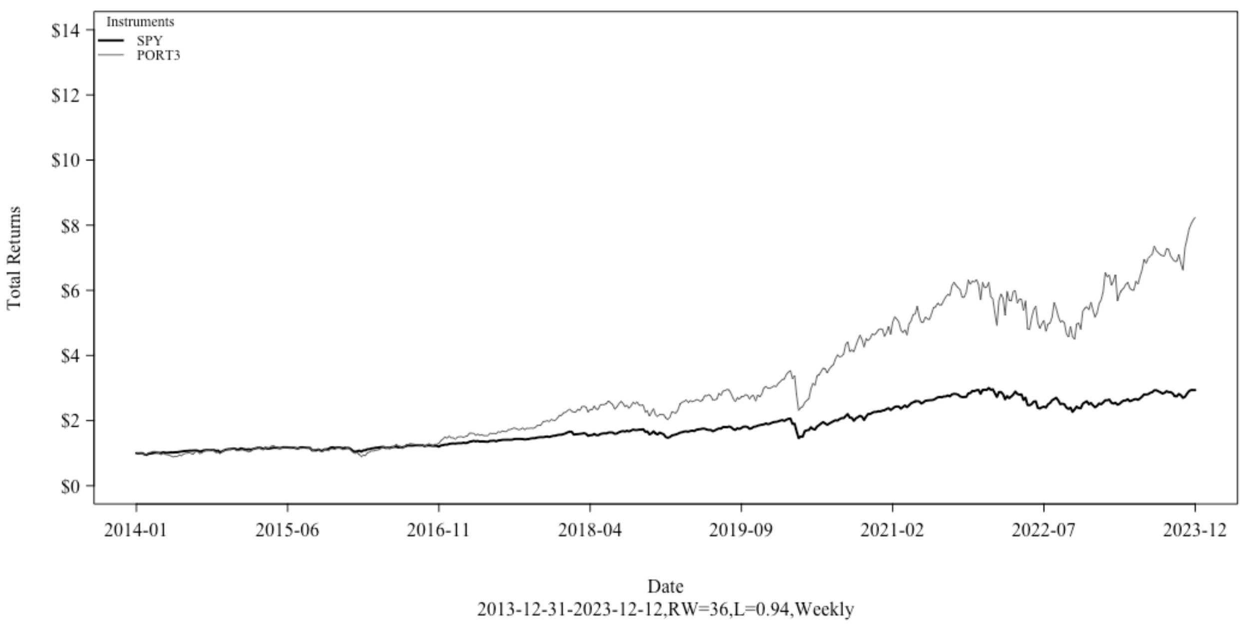
Note: Steel Dynamics (STLD).

Panel B: Two Stocks with SPY



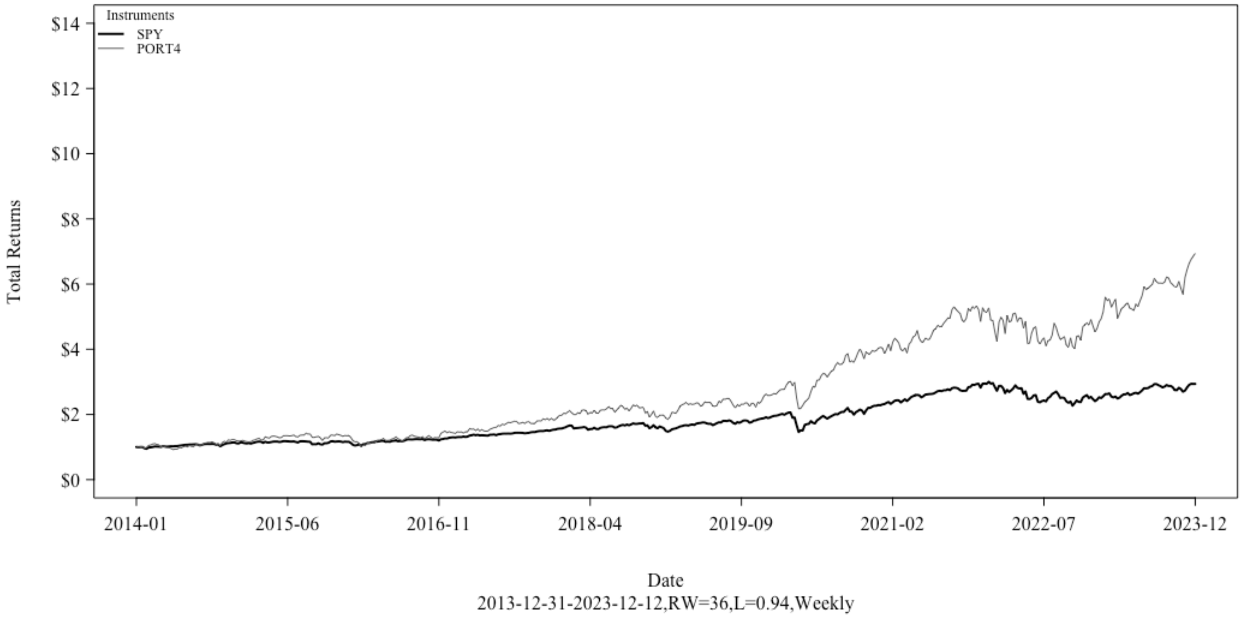
Note: Steel Dynamics (STLD) and ServiceNow (NOW).

Panel C: Three Stocks with SPY



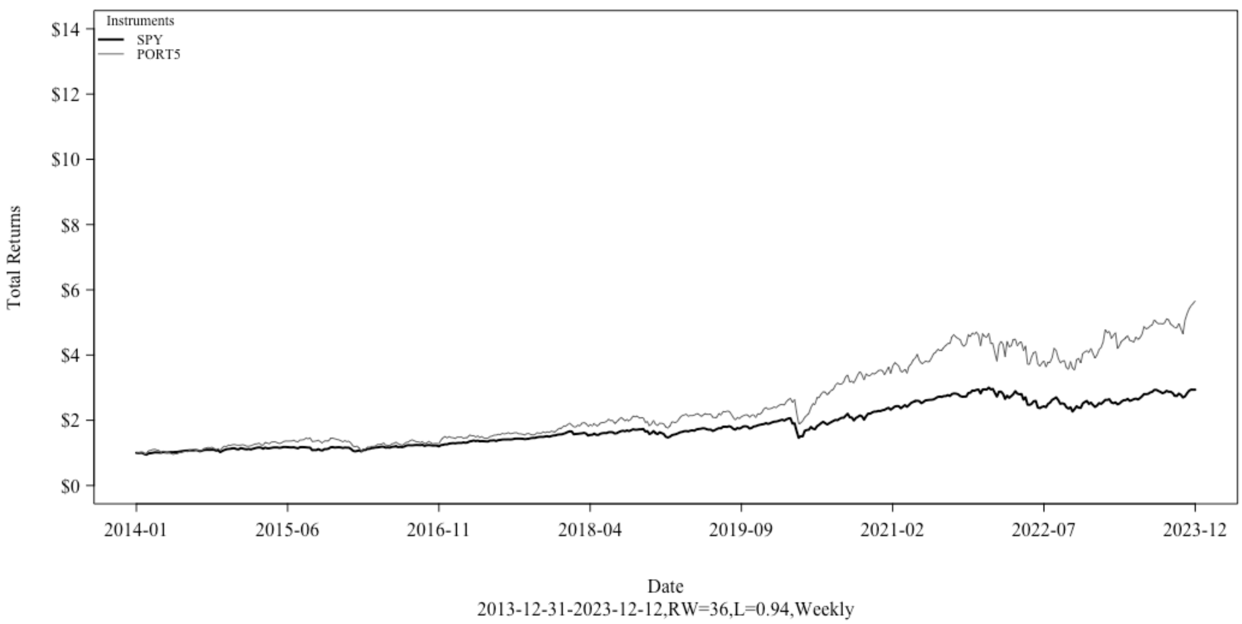
Note: Steel Dynamics (STLD), Service Now (NOW), and Pulte Group (PHM).

Panel D: Four Stocks with SPY



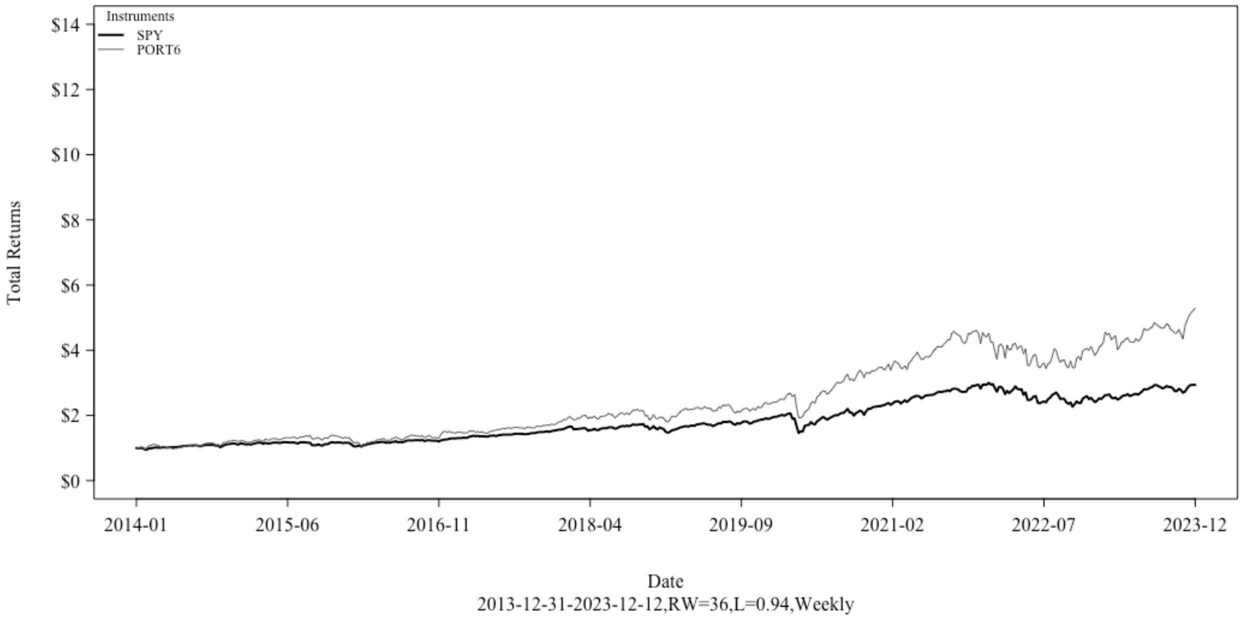
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), and Regeneron (REGN).

Panel E: Five Stocks with SPY



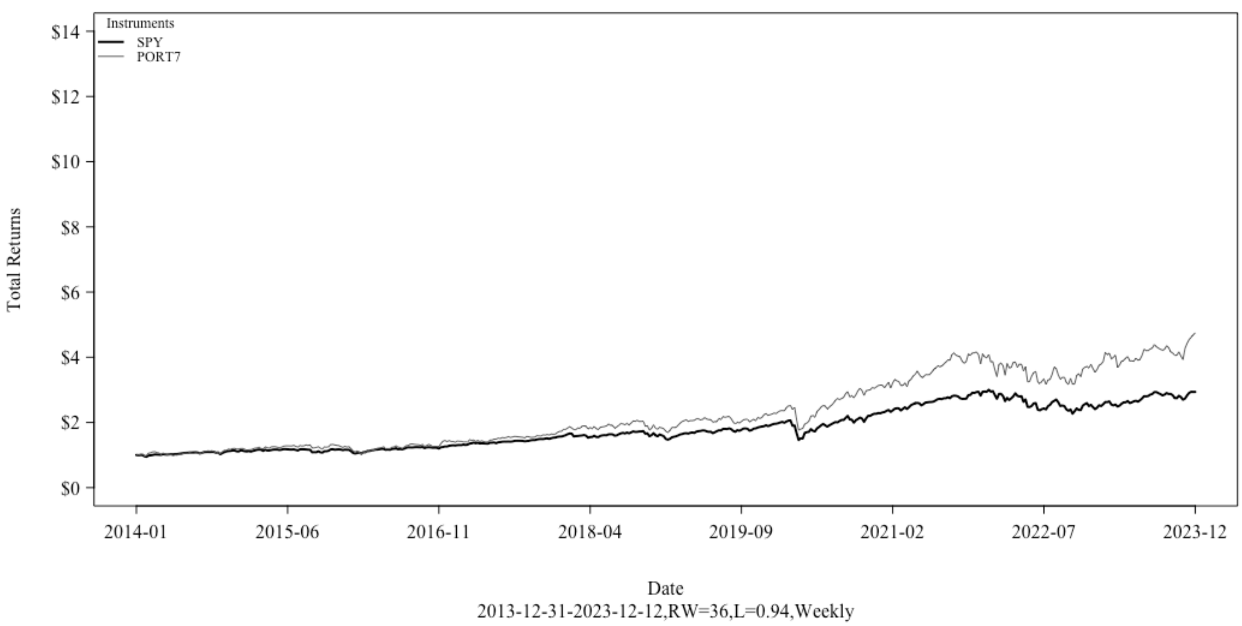
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), and Advance Auto Parts (AAP).

Panel F: Six Stocks with SPY



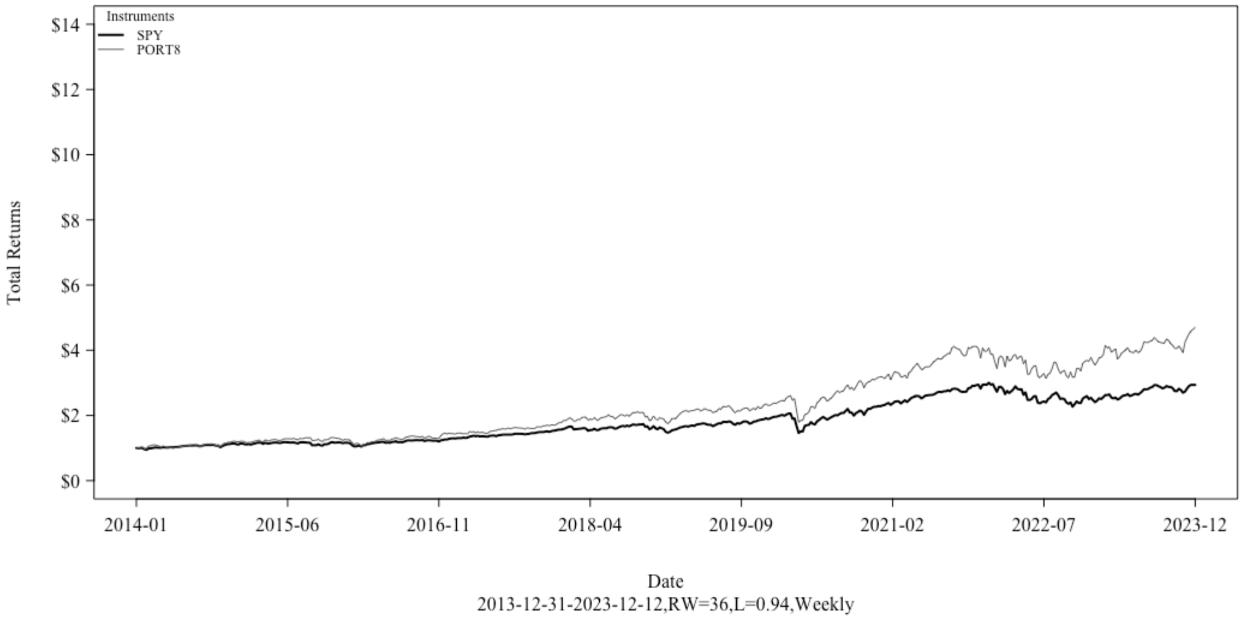
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), and Xylem (XYL).

Panel G: Seven Stocks with SPY



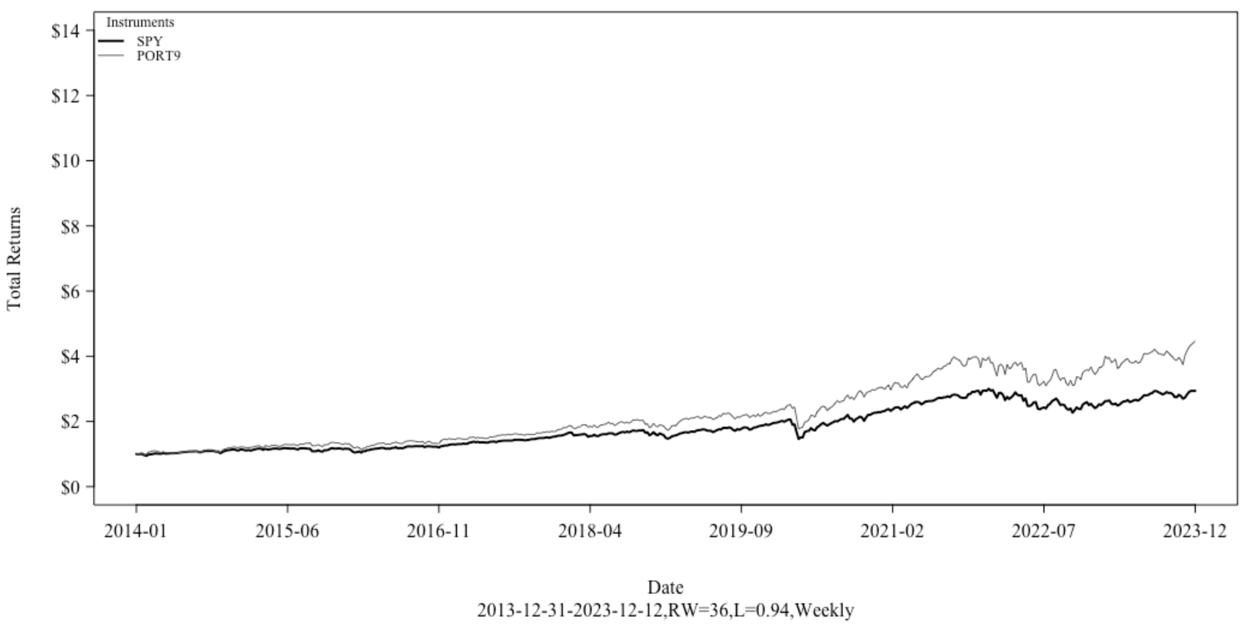
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), and Raytheon (RTX).

Panel H: Eight Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), and Stryker (SYK).

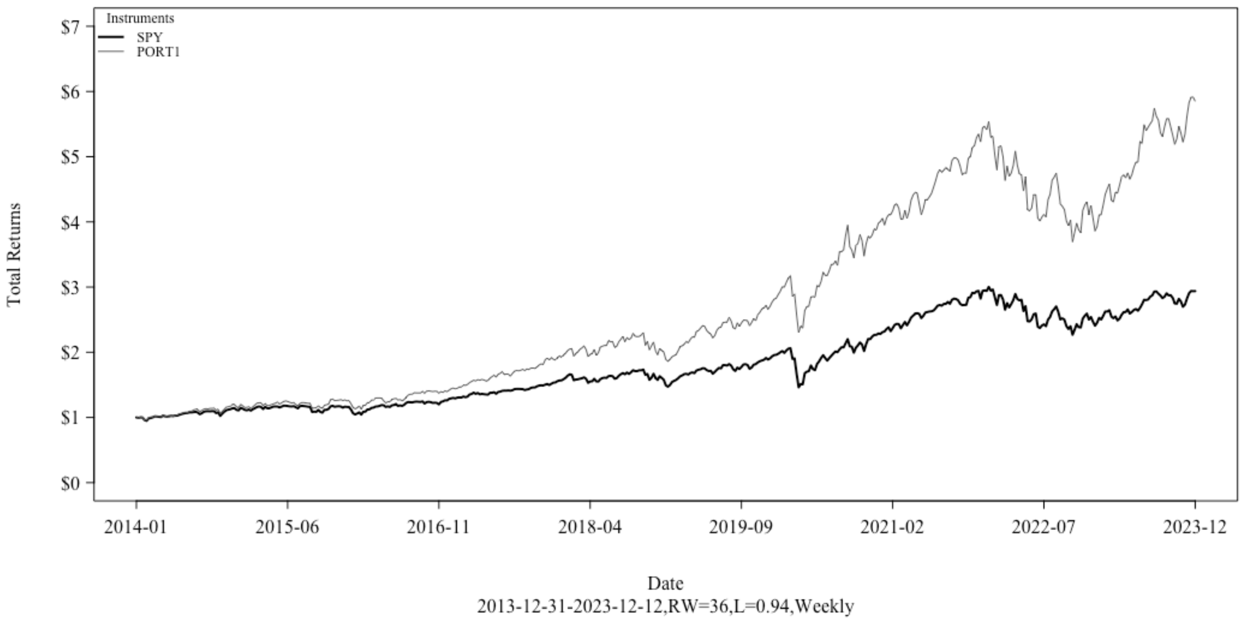
Panel I: Nine Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), Stryker (SYK), Public Storage (PSA).

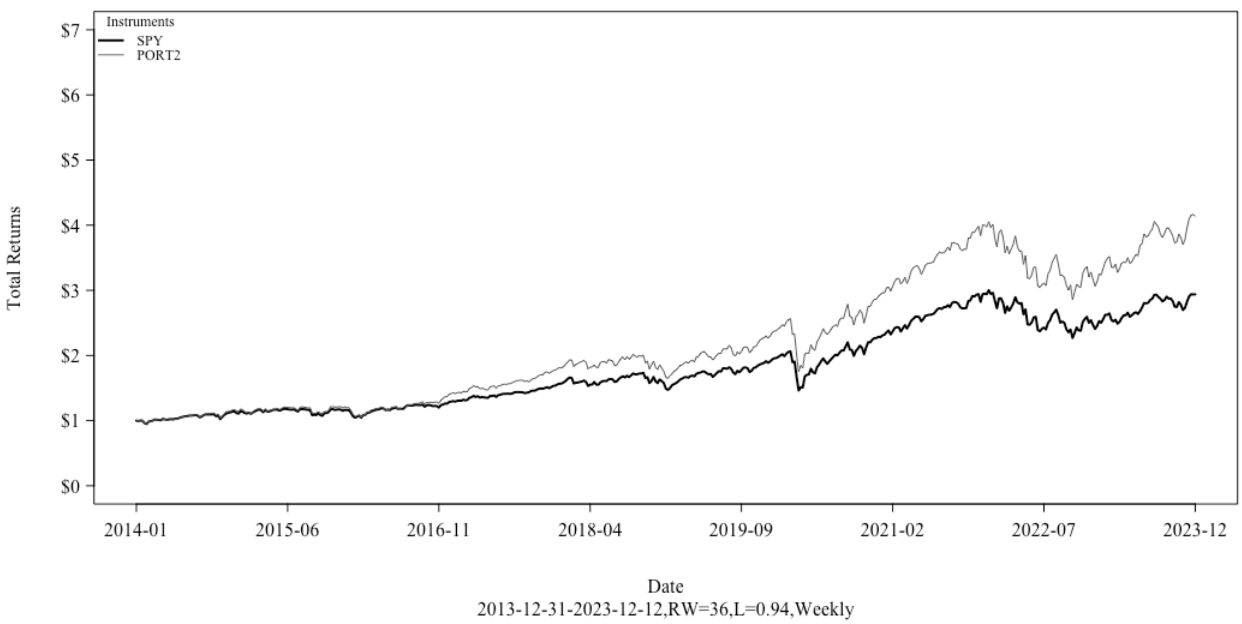
Figure 14.1.10 illustrates multiple panels of the marginal influence on total returns with adding additional sector ETFs. Note the y-axis has been cut in half ranging from \$0 to \$7 rather than \$0 to \$14. Again, the sector ETFs are assumed equally weighted as numbers increase through the different panels. As the number of sector ETFs increase, we see the equally weighted portfolio trending toward SPY. We do not observe exact convergence as SPY is value weighted as well as tech heavy.

Figure 14.1.10. Marginal Influence on Total Return Illustration with Sector ETFs
Panel A: One Sector ETF with SPY



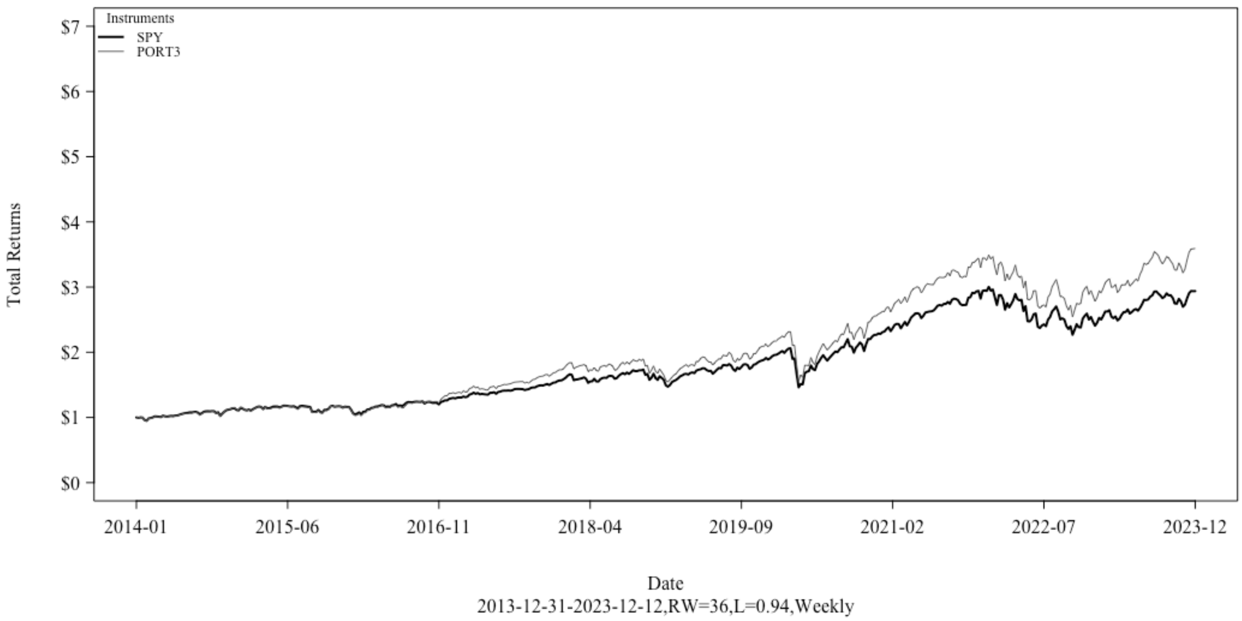
Note: Technology (XLK).

Panel B: Two Sector ETFs with SPY



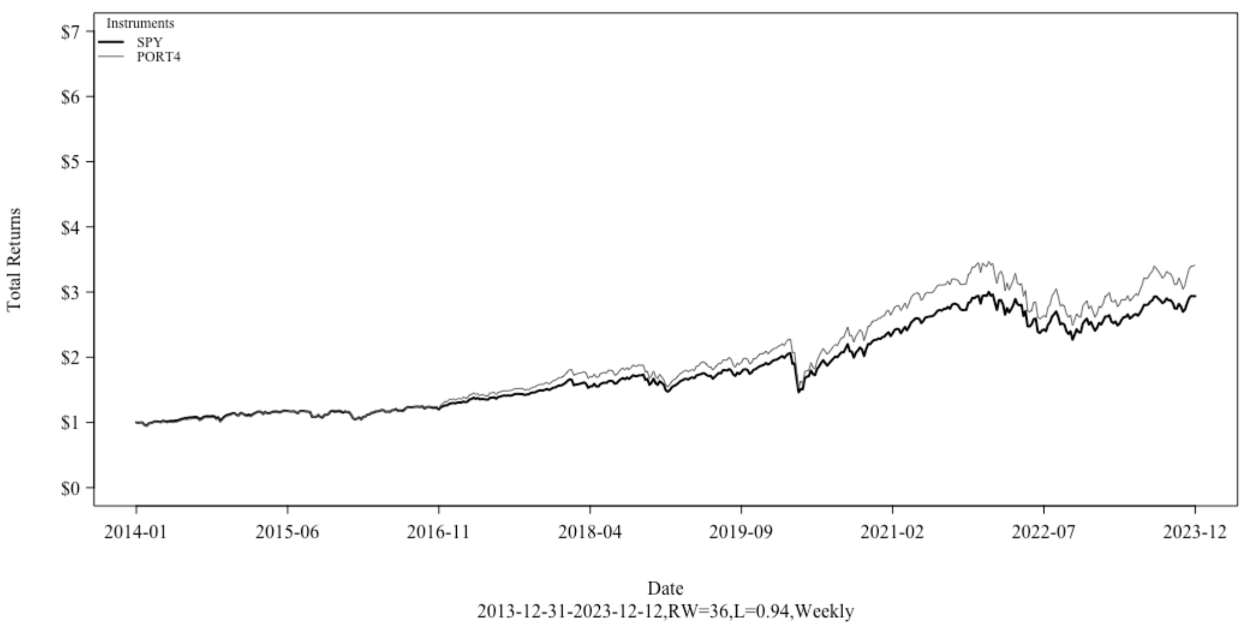
Note: Technology (XLK) and Finance (XLF).

Panel C: Three Sector ETFs with SPY



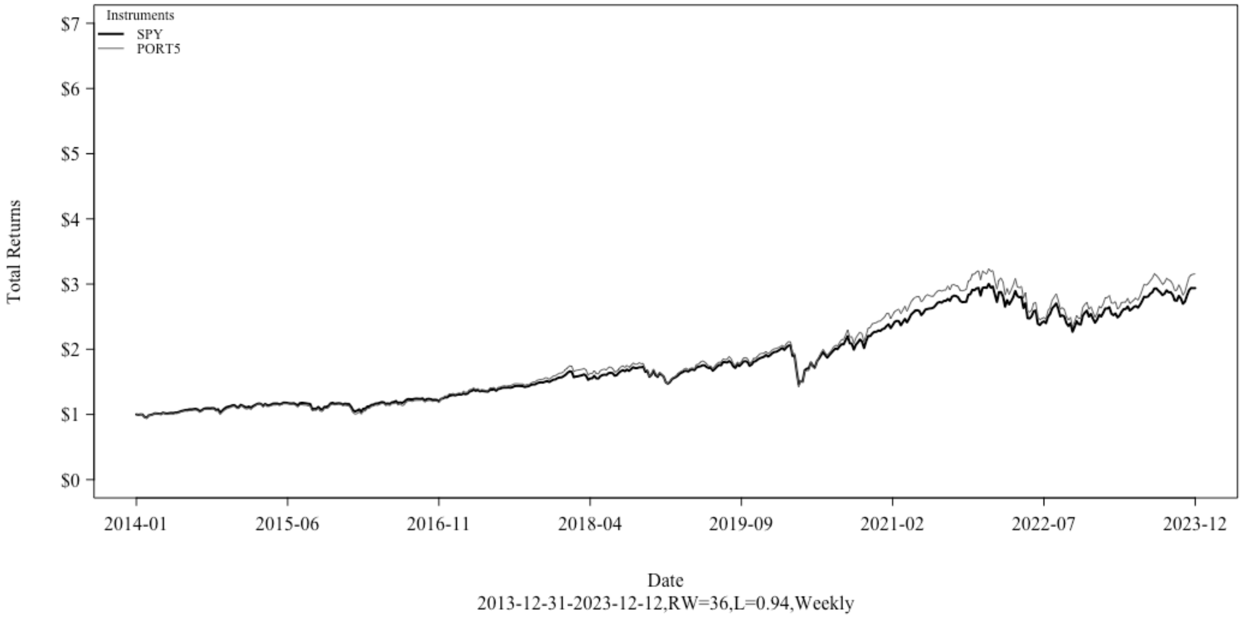
Note: Technology (XLK), Finance (XLF), and Industrial (XLI).

Panel D: Four Sector ETFs with SPY



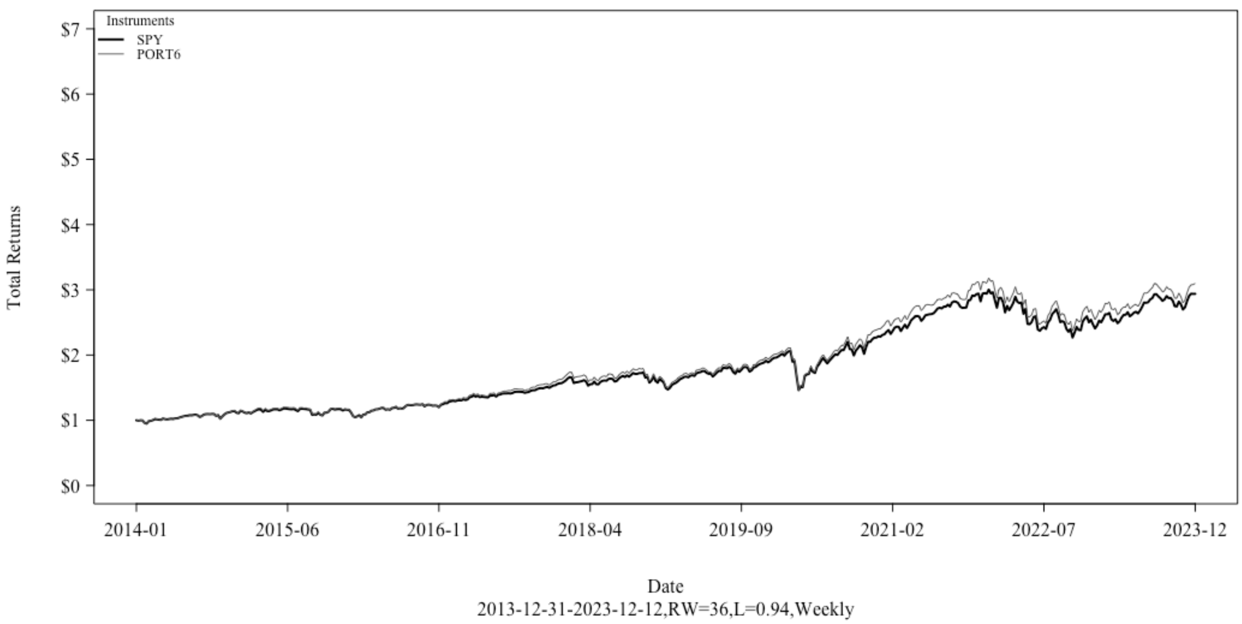
Note: Technology (XLK), Finance (XLF), Industrial (XLI), and Consumer Discretionary (XLY).

Panel E: Five Sector ETFs with SPY



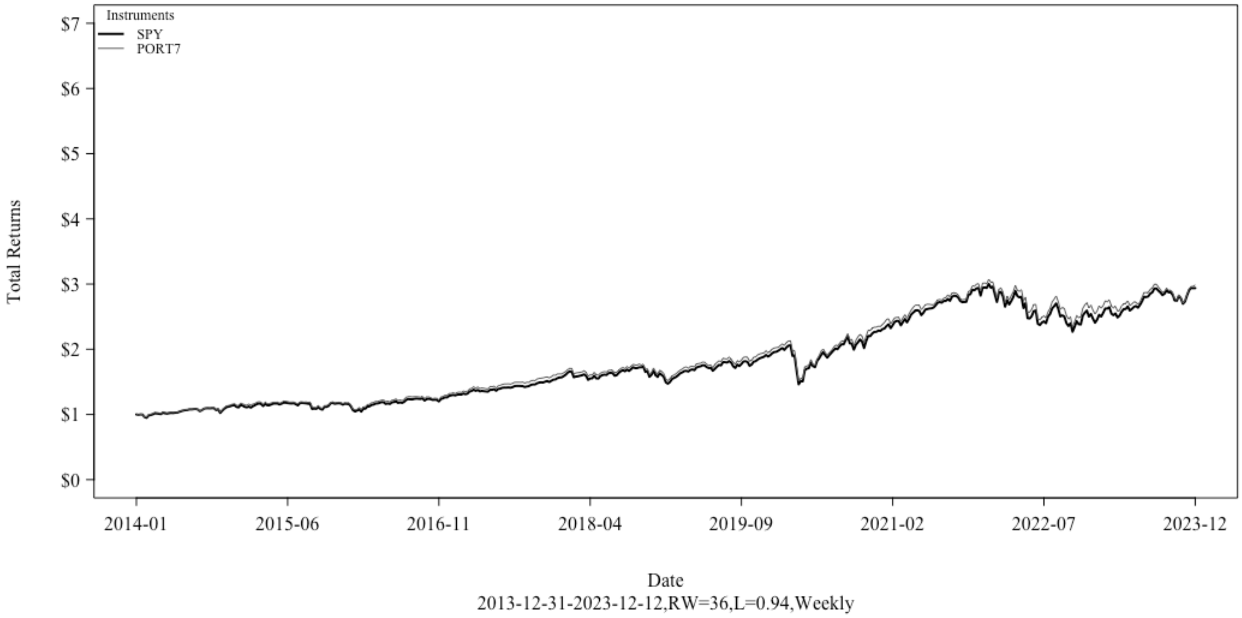
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), and Materials (XLB).

Panel F: Six Sector ETFs with SPY



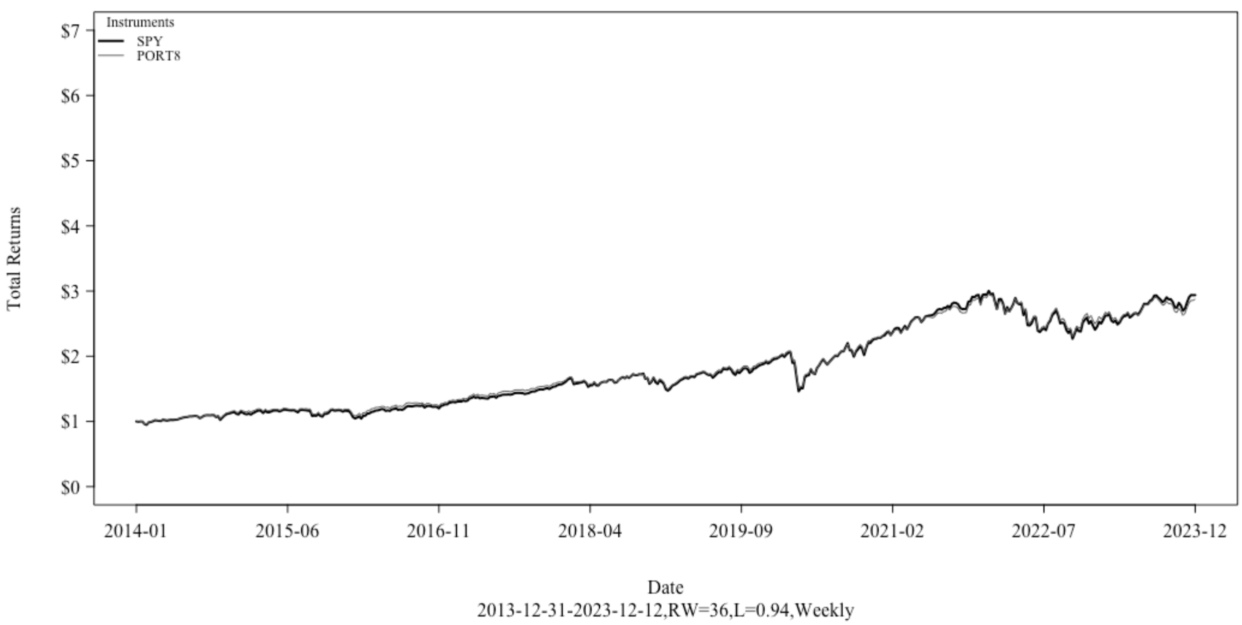
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), and Healthcare (XLV).

Panel G: Seven Sector ETFs with SPY



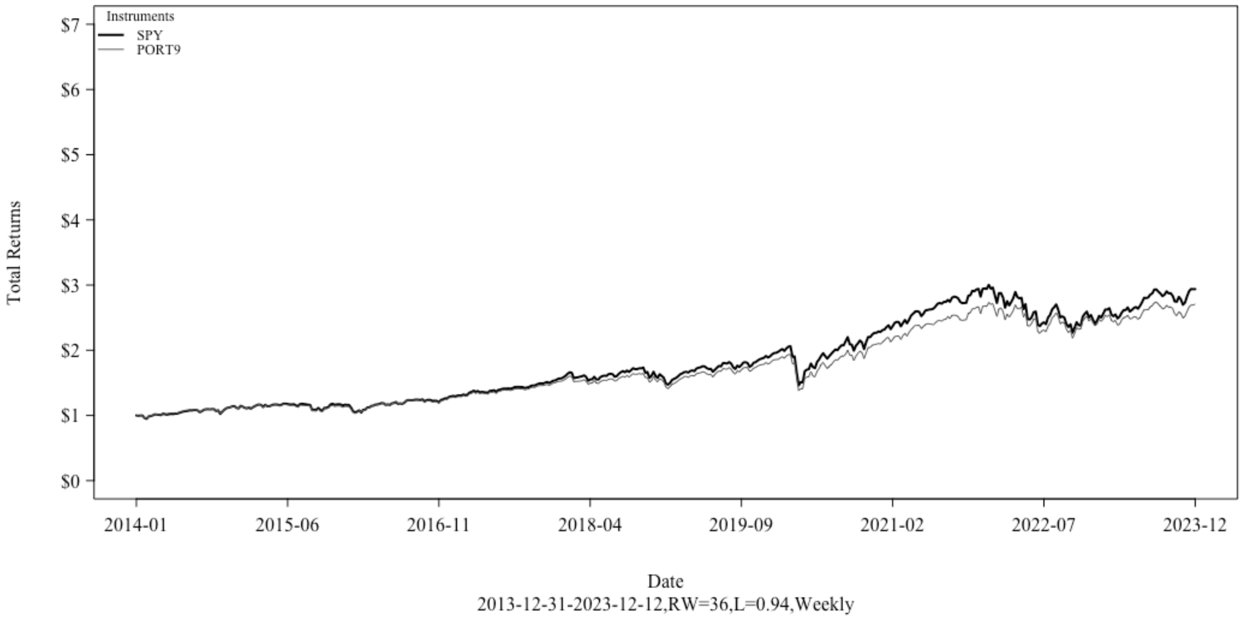
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), and Utilities (XLU).

Panel H: Eight Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), and Consumer Staples (XLP).

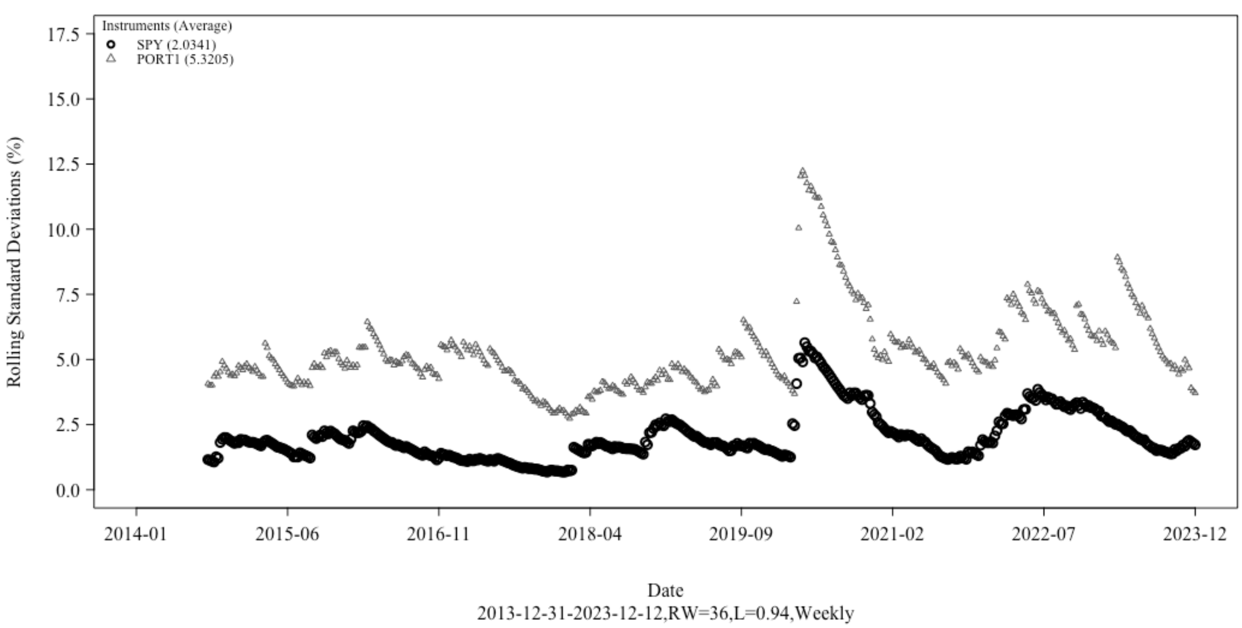
Panel I: Nine Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), Consumer Staples (XLP), and Energy (XLE).

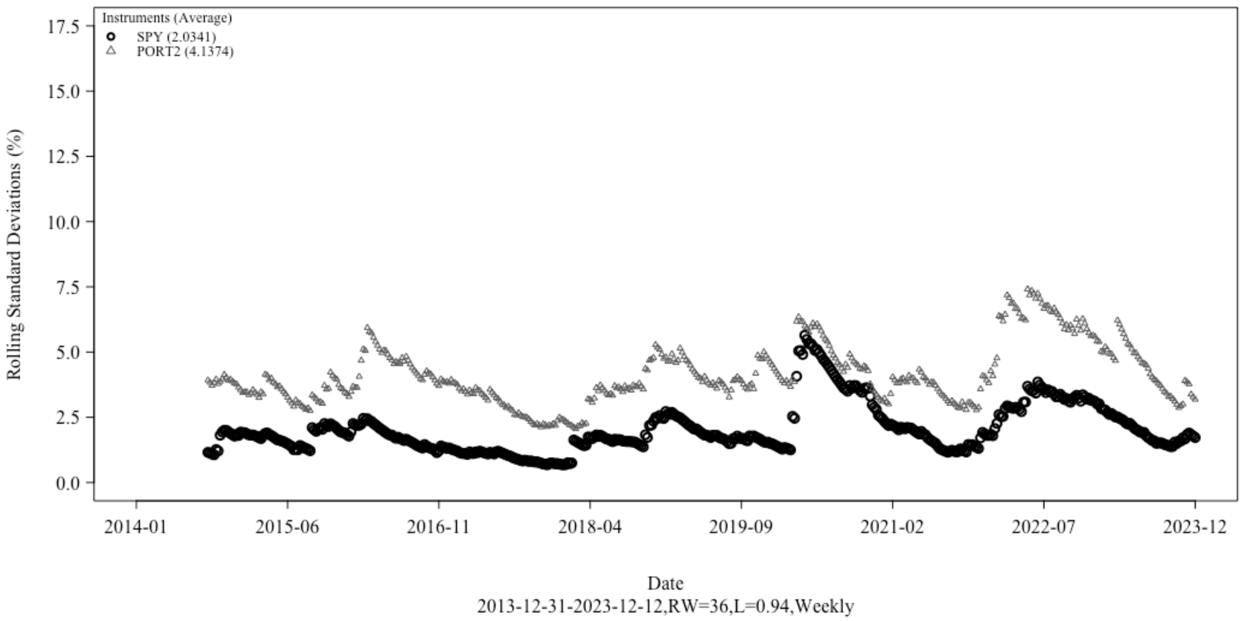
Marginal influence of additional instrument on rolling standard deviation
Figure 14.1.11 illustrates multiple panels of the marginal influence on rolling standard deviations with adding additional stocks. Again, the stocks are assumed equally weighted as numbers increase through the different panels. Further, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of stocks increase, we see the equally weighted portfolio rolling standard deviation trending toward the SPY rolling standard deviation.

Figure 14.1.11. Marginal Influence on Rolling Standard Deviation Illustration with Individual Stocks
Panel A: One Stock with SPY



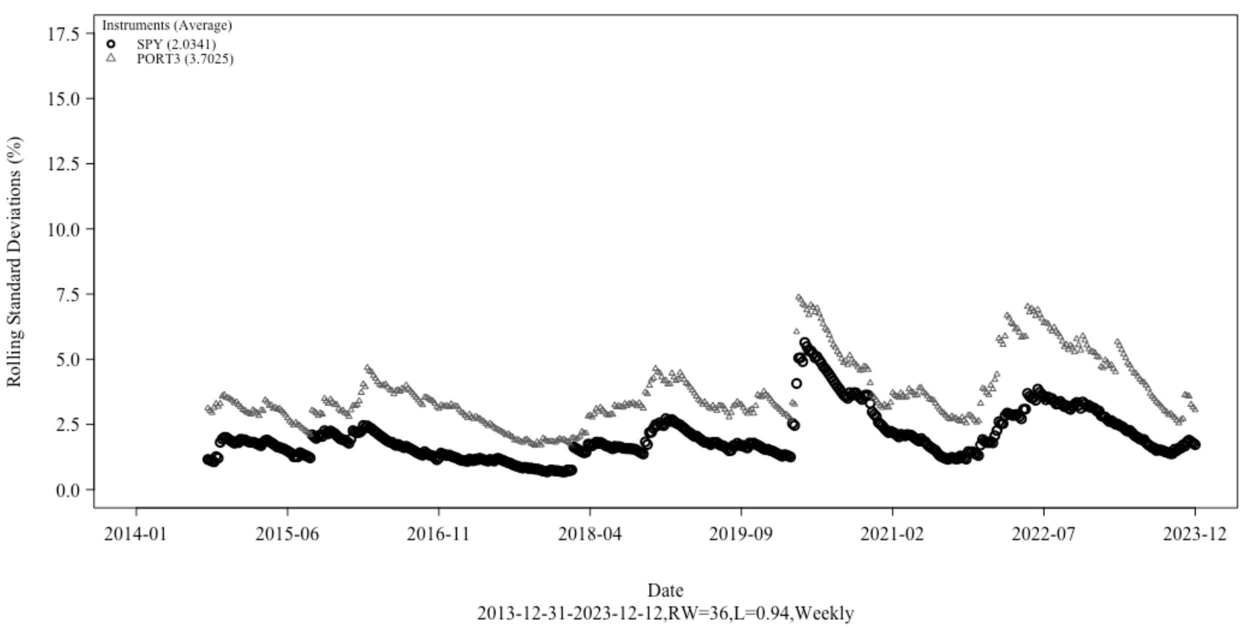
Note: Steel Dynamics (STLD).

Panel B: Two Stocks with SPY



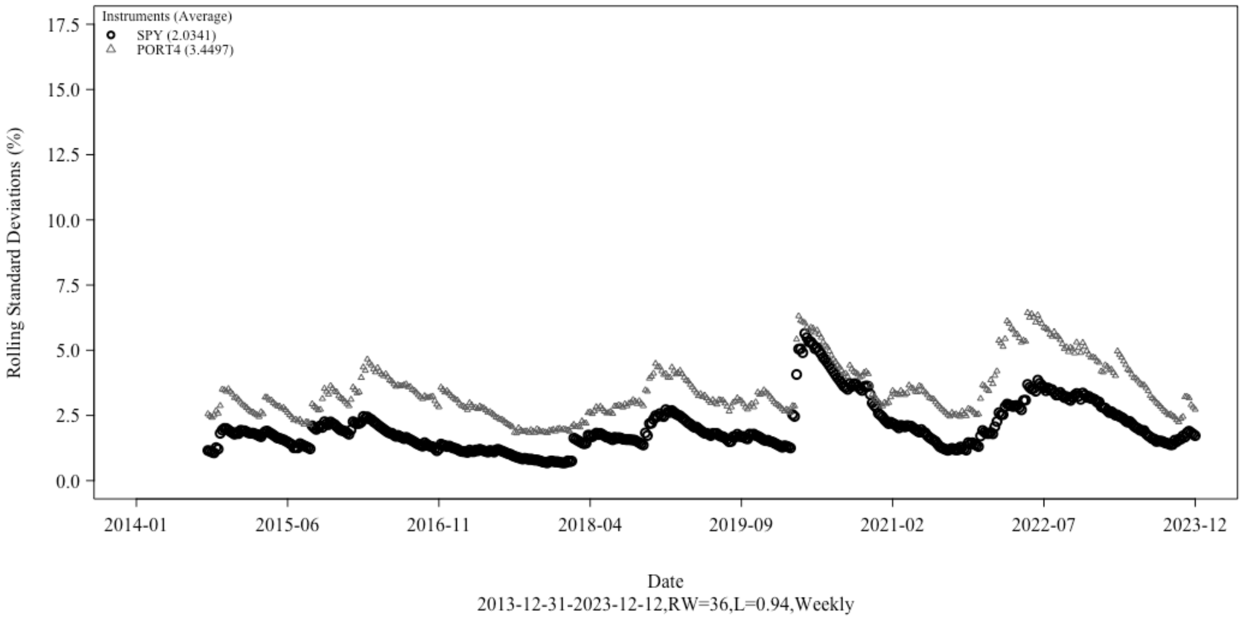
Note: Steel Dynamics (STLD) and ServiceNow (NOW).

Panel C: Three Stocks with SPY



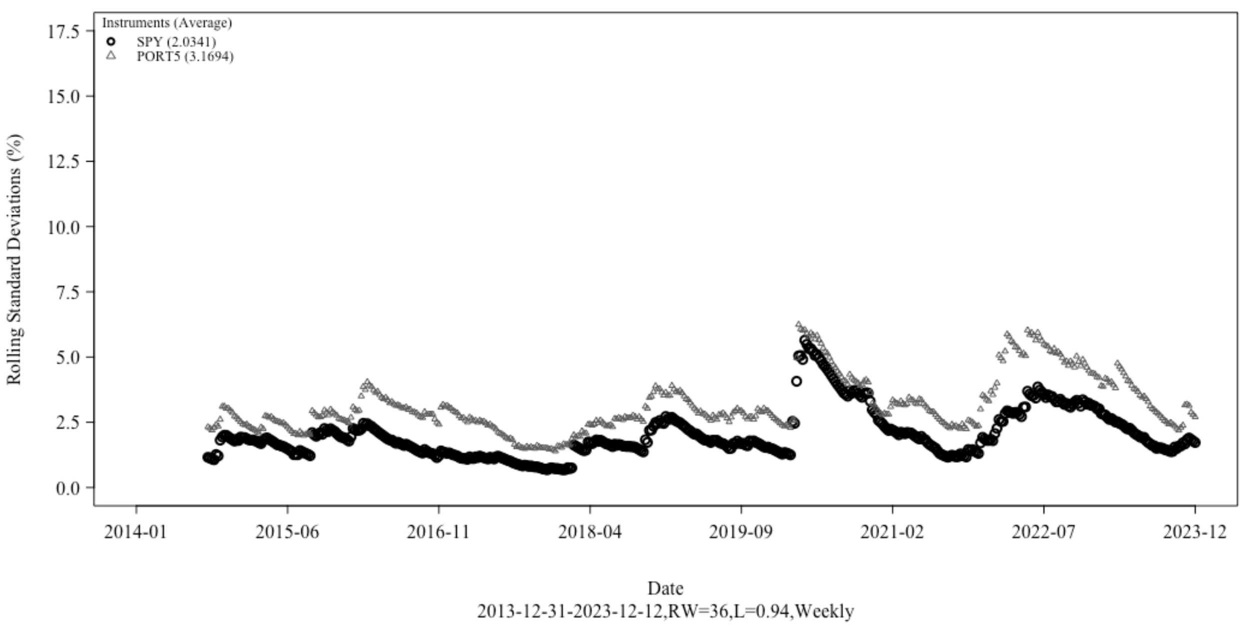
Note: Steel Dynamics (STLD), Service Now (NOW), and Pulte Group (PHM).

Panel D: Four Stocks with SPY



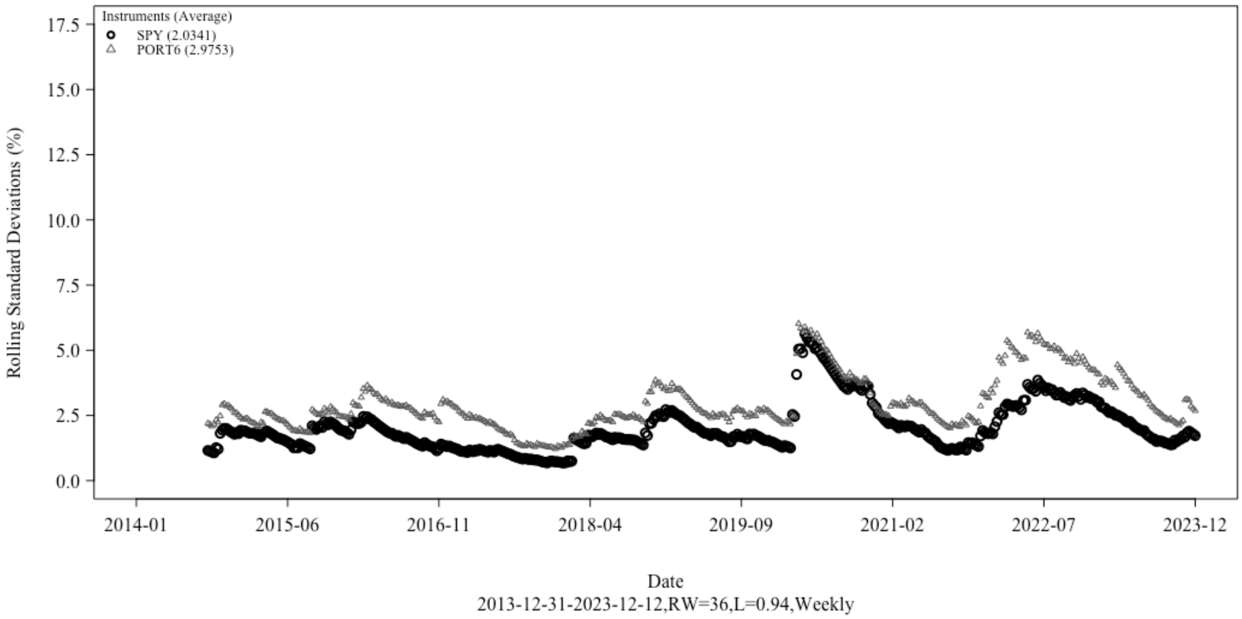
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), and Regeneron (REGN).

Panel E: Five Stocks with SPY



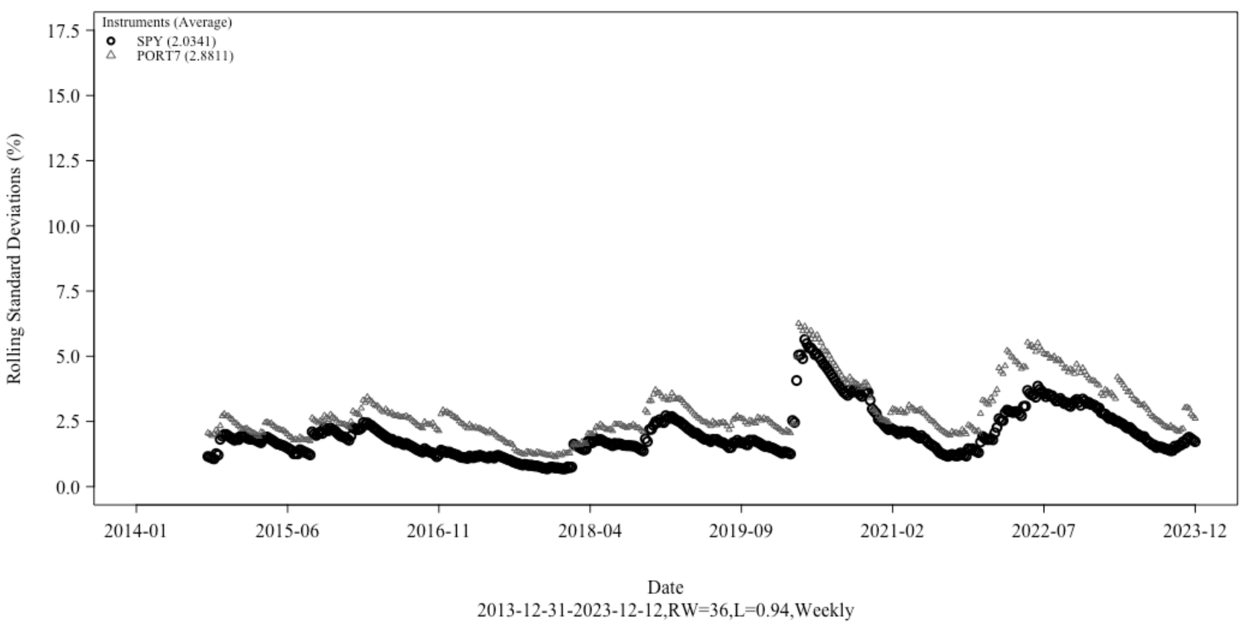
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), and Advance Auto Parts (AAP).

Panel F: Six Stocks with SPY



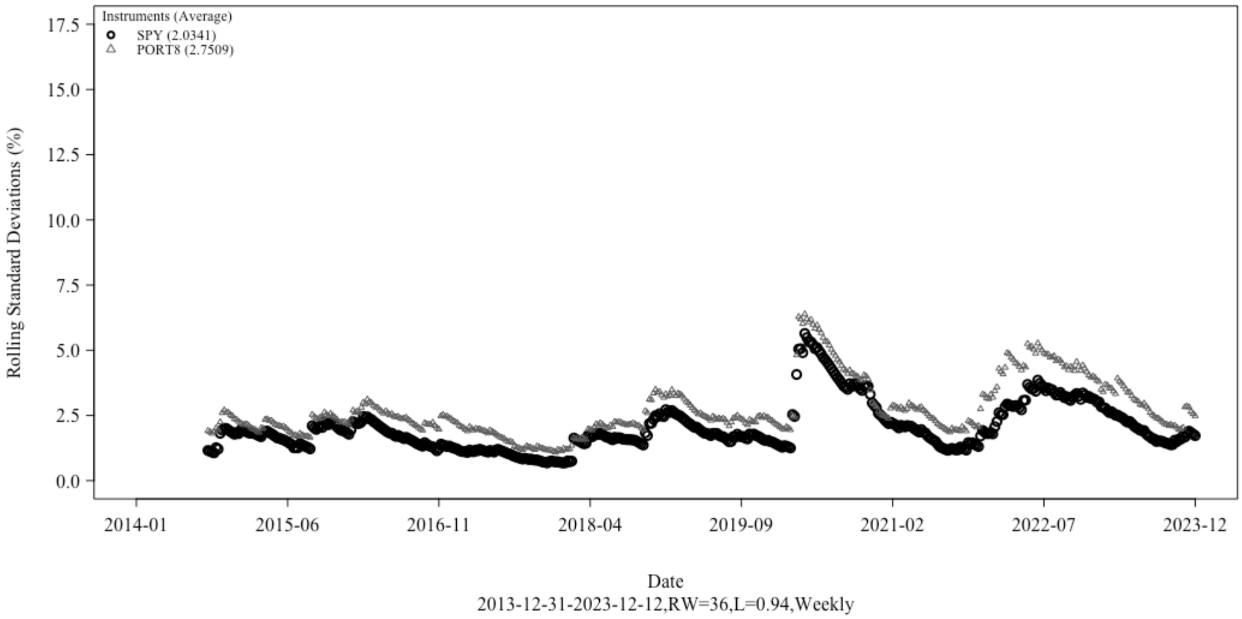
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), and Xylem (XYL).

Panel G: Seven Stocks with SPY



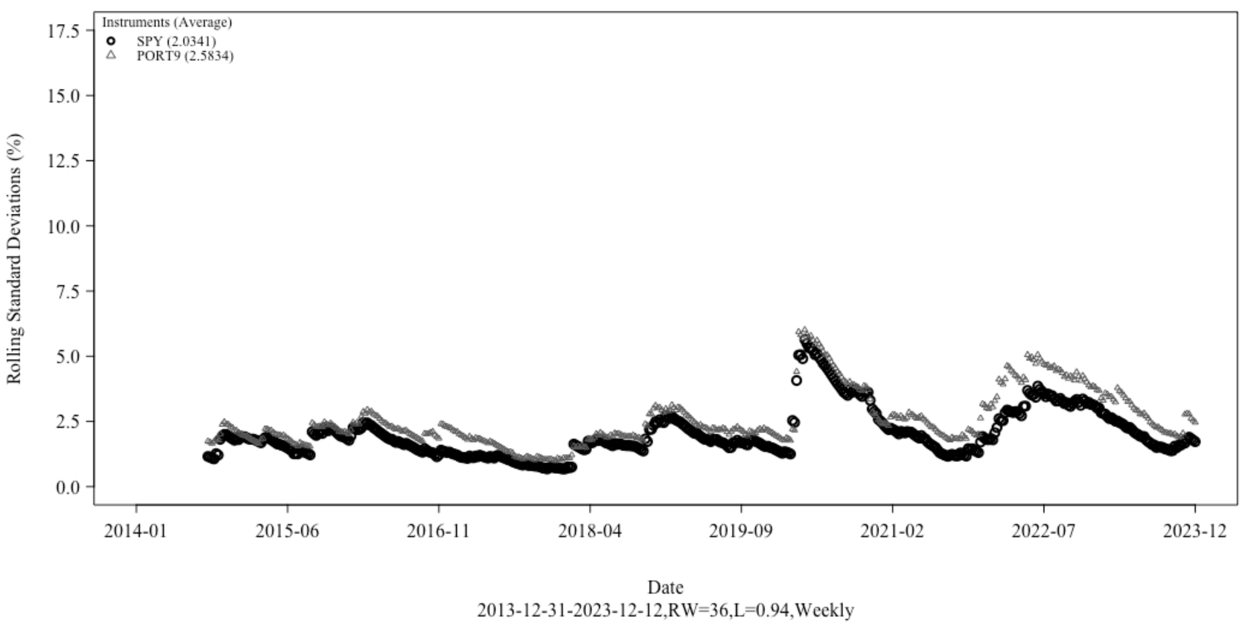
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), and Raytheon (RTX).

Panel H: Eight Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), and Stryker (SYK).

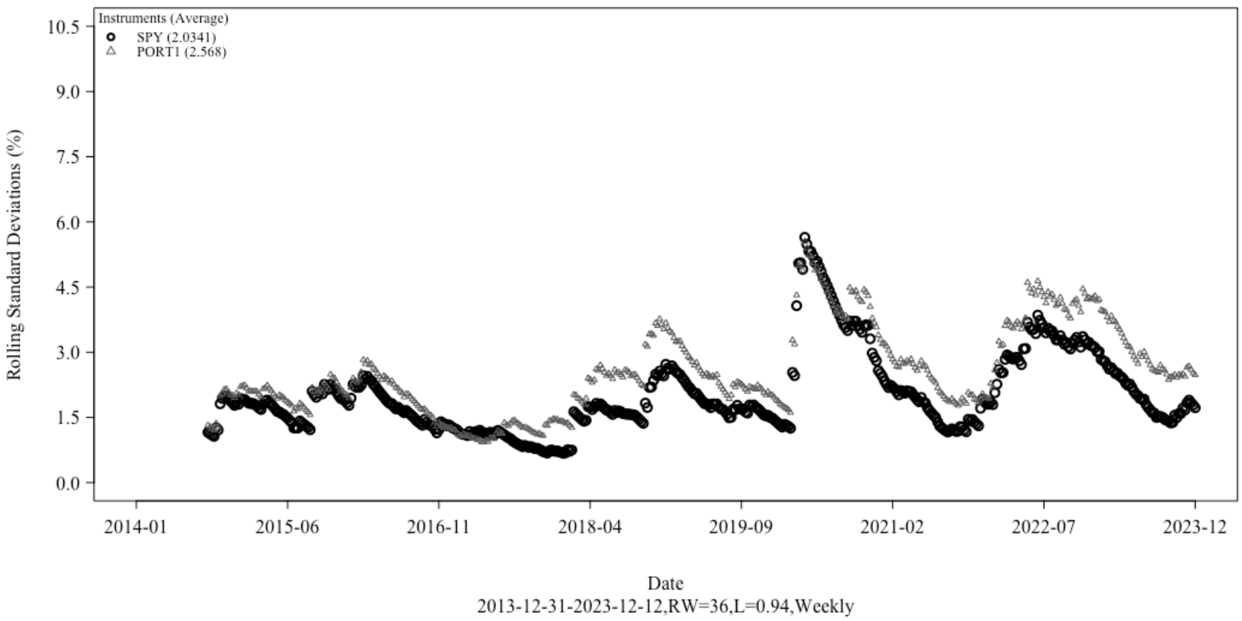
Panel I: Nine Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), Stryker (SYK), Public Storage (PSA).

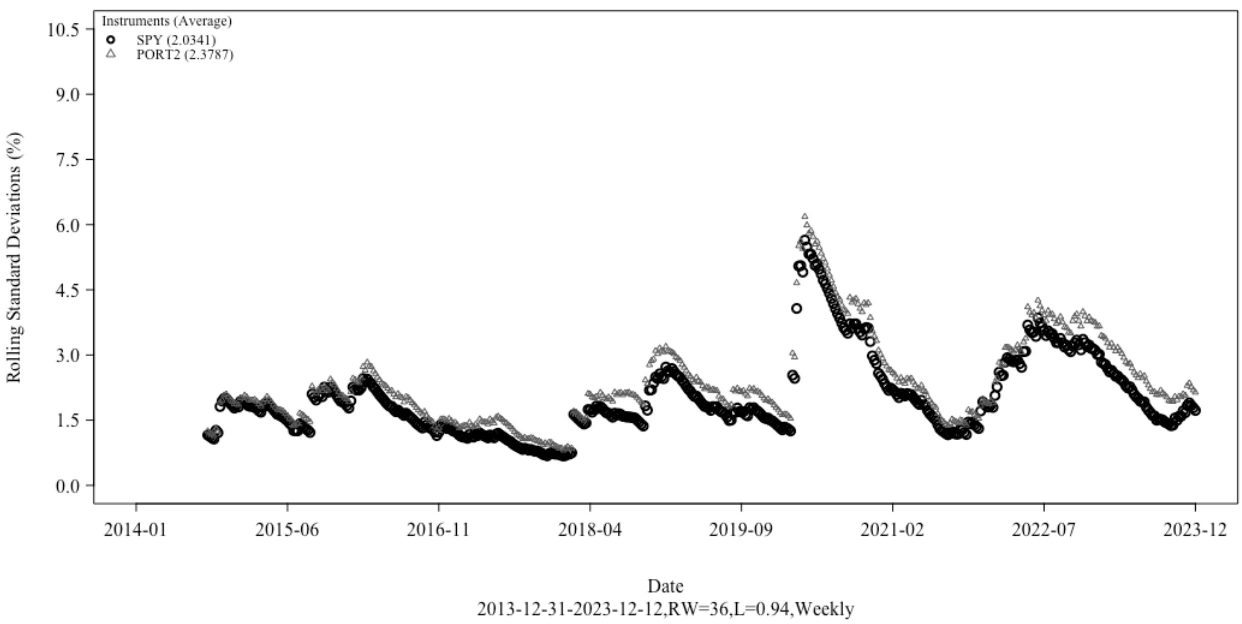
Figure 14.1.12 illustrates multiple panels of the marginal influence on rolling standard deviations with adding additional sector ETFs. Again, the sector ETFs are assumed equally weighted as numbers increase through the different panels. Note to highlight differences, the y-axis is reduced to 0% to 10.5% compared with 0% to 17.5% with individual stocks. As before, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of sector ETFs increase, we see the equally weighted portfolio rolling standard deviation trending toward the SPY rolling standard deviation.

Figure 14.1.12. Marginal Influence on Rolling Standard Deviation Illustration with Sector ETFs
Panel A: One Sector ETF with SPY



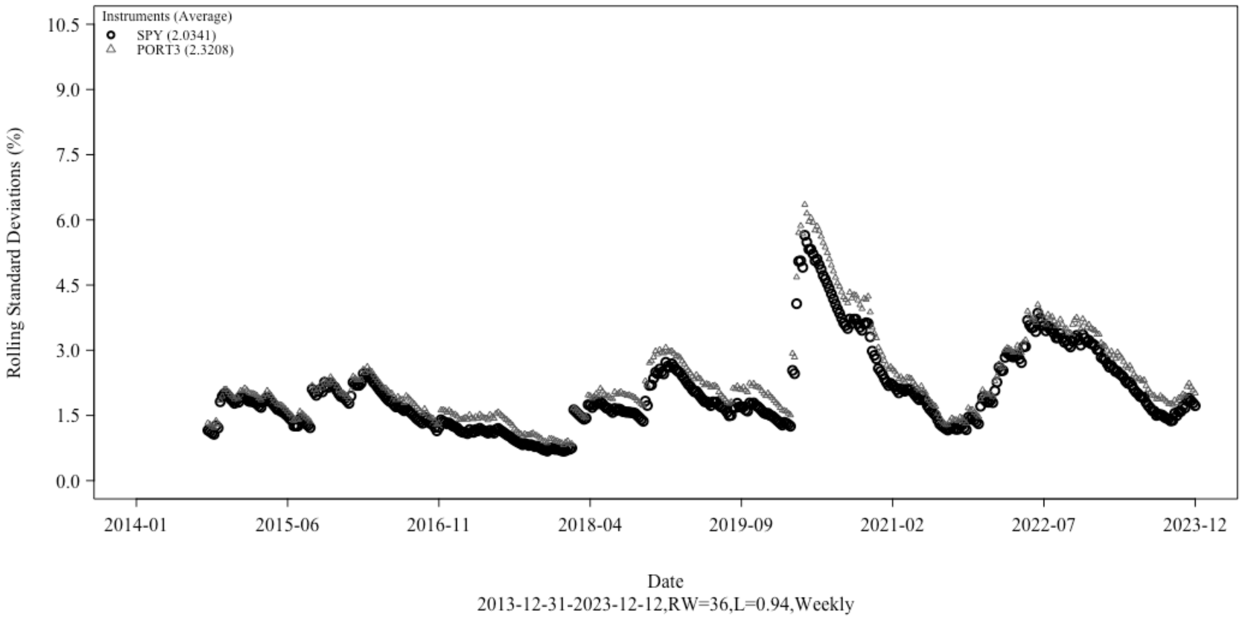
Note: Technology (XLK).

Panel B: Two Sector ETFs with SPY



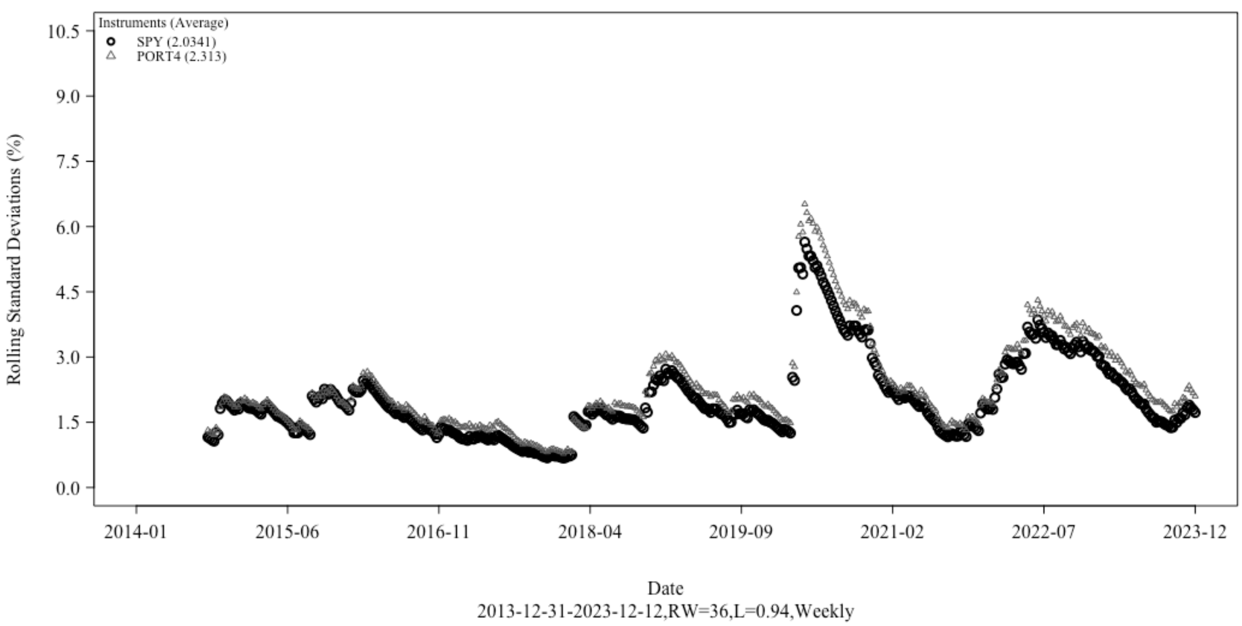
Note: Technology (XLK) and Finance (XLF).

Panel C: Three Sector ETFs with SPY



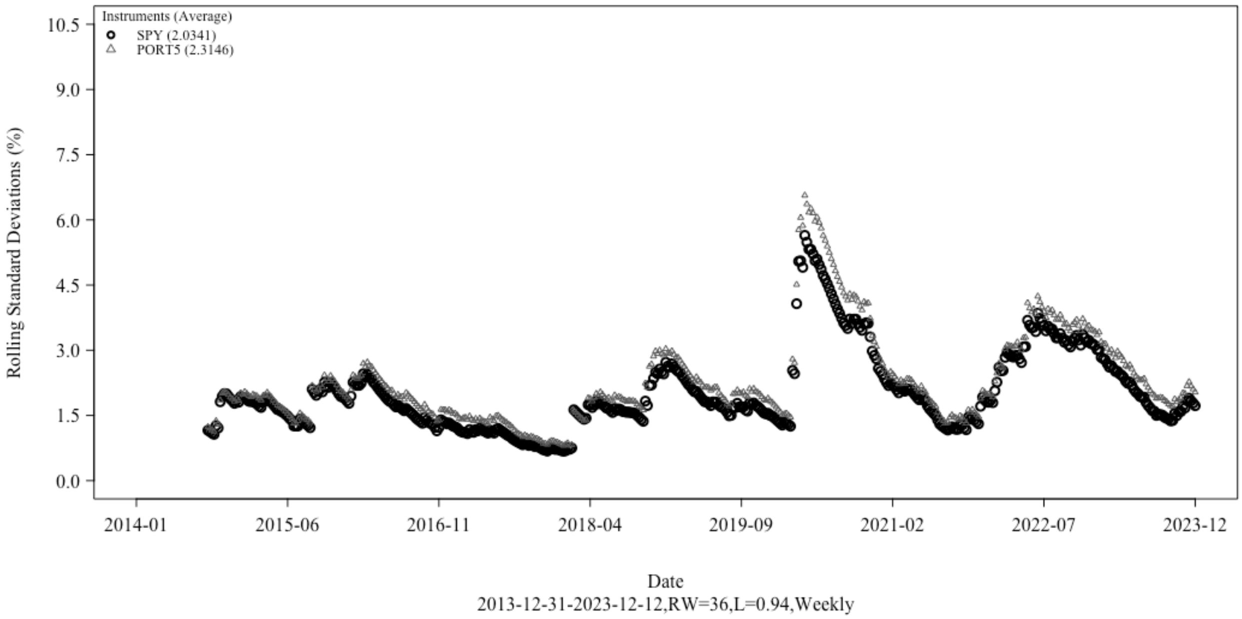
Note: Technology (XLK), Finance (XLF), and Industrial (XLI).

Panel D: Four Sector ETFs with SPY



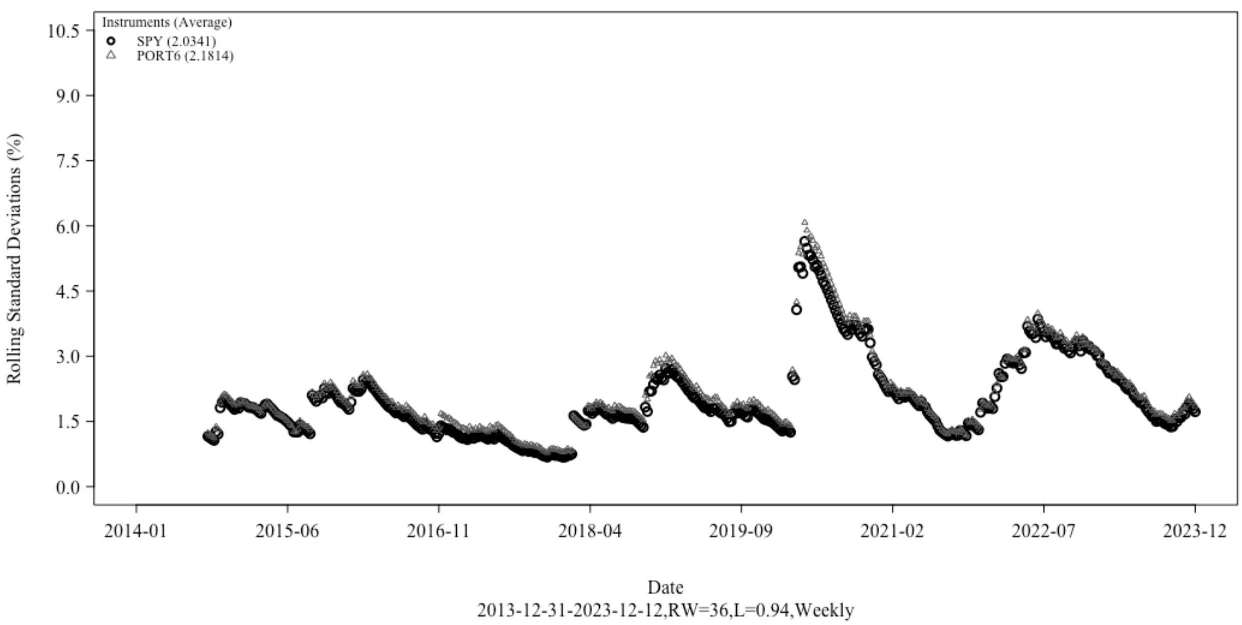
Note: Technology (XLK), Finance (XLF), Industrial (XLI), and Consumer Discretionary (XLY).

Panel E: Five Sector ETFs with SPY



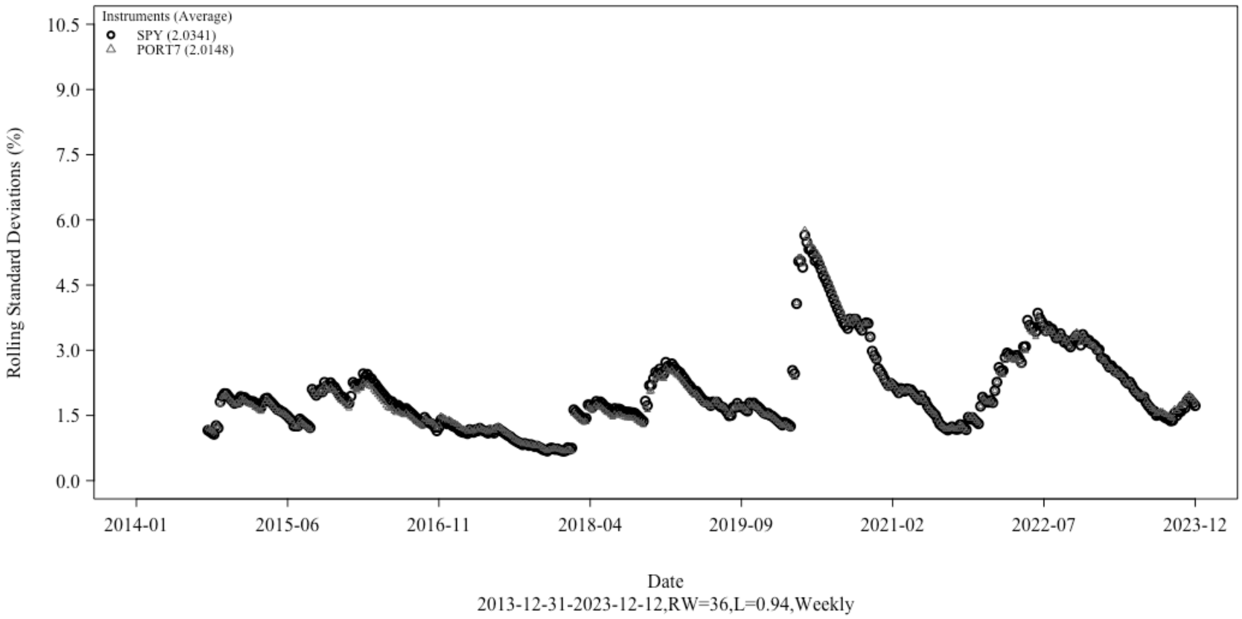
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), and Materials (XLB).

Panel F: Six Sector ETFs with SPY



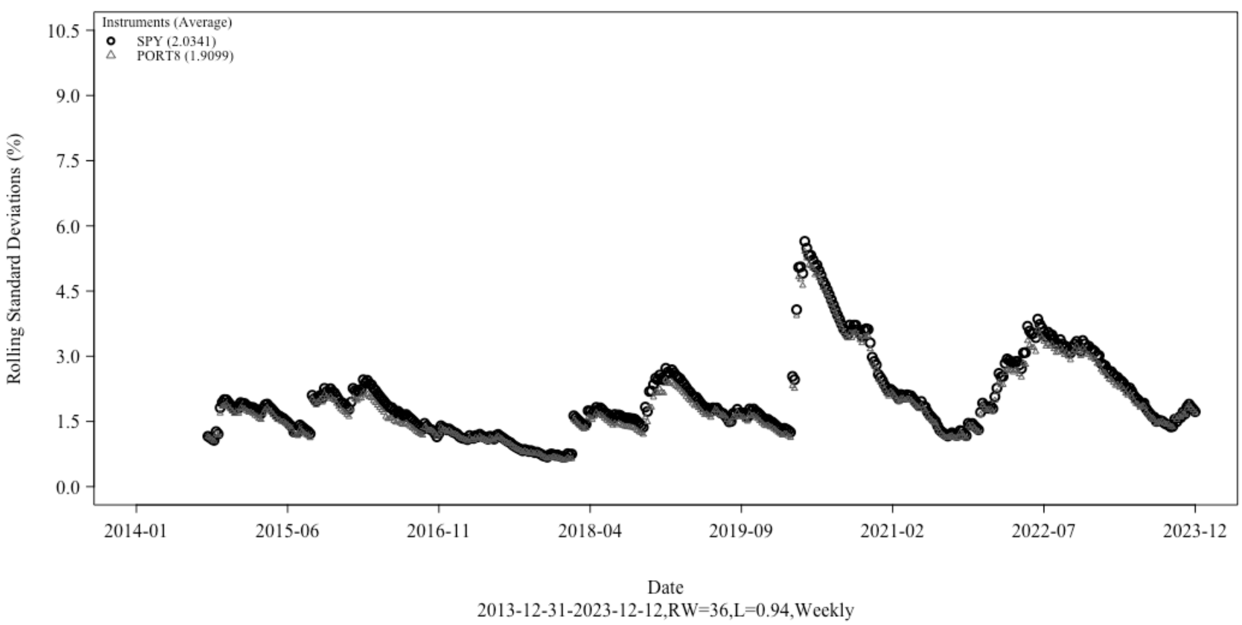
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), and Healthcare (XLV).

Panel G: Seven Sector ETFs with SPY



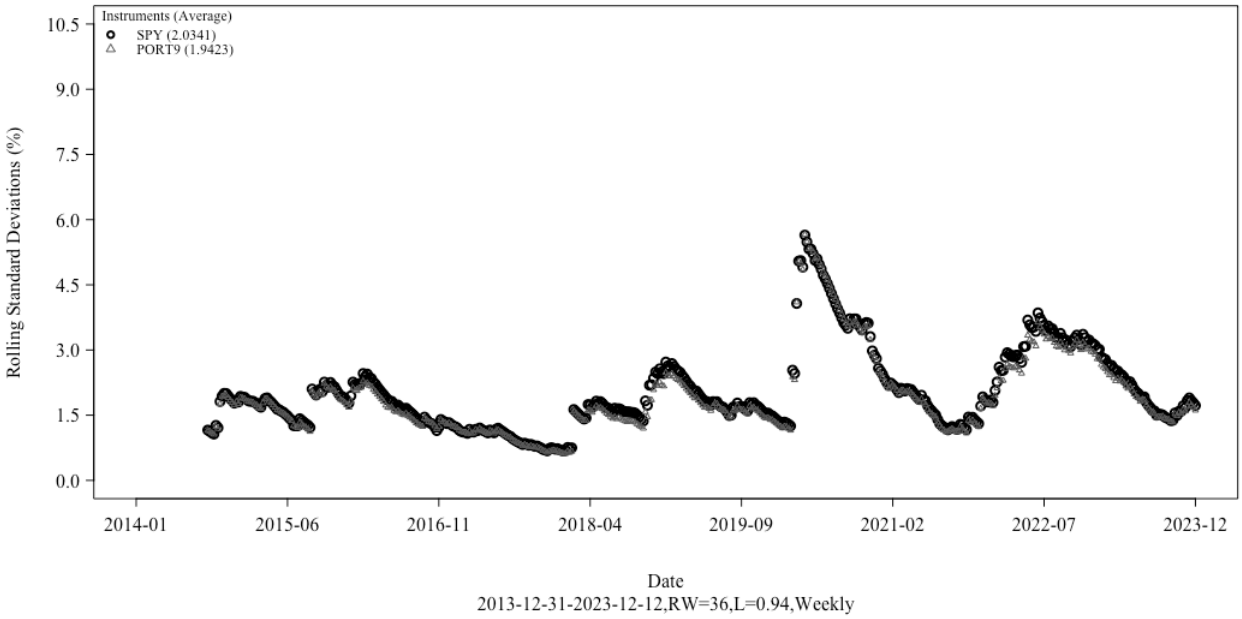
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), and Utilities (XLU).

Panel H: Eight Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), and Consumer Staples (XLP).

Panel I: Nine Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), Consumer Staples (XLP), and Energy (XLE).

Figure 14.1.13 summarizes the marginal influence on rolling standard deviations with adding additional stocks in one bar chart. As the number of stocks increase, we see the average of equally weighted portfolio rolling standard deviations trending toward the SPY average rolling standard deviation. Note the considerable drop in average standard deviation from 9 stocks to SPY (with 500 stocks).

Figure 14.1.13. Individual Stock Average Rolling Portfolio Standard Deviations

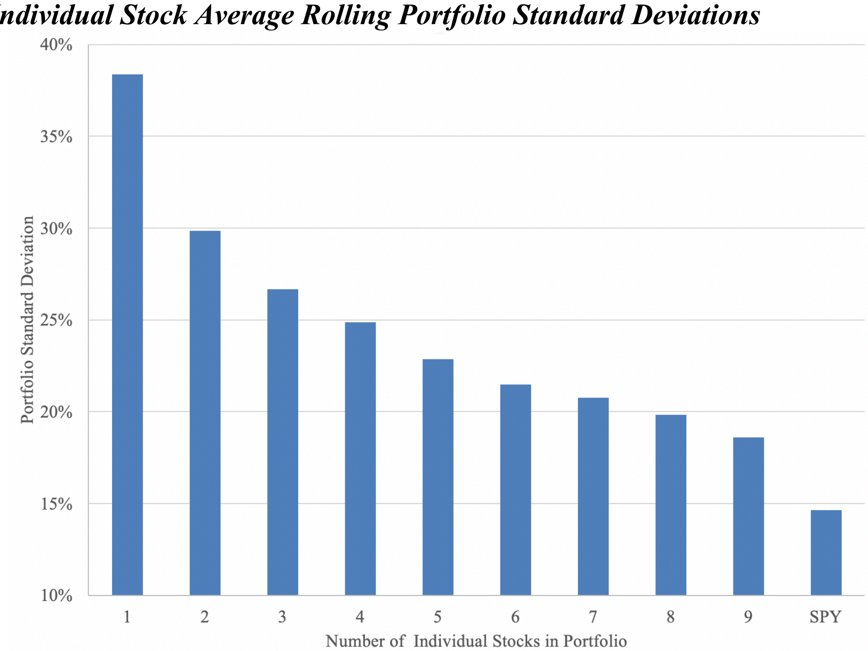
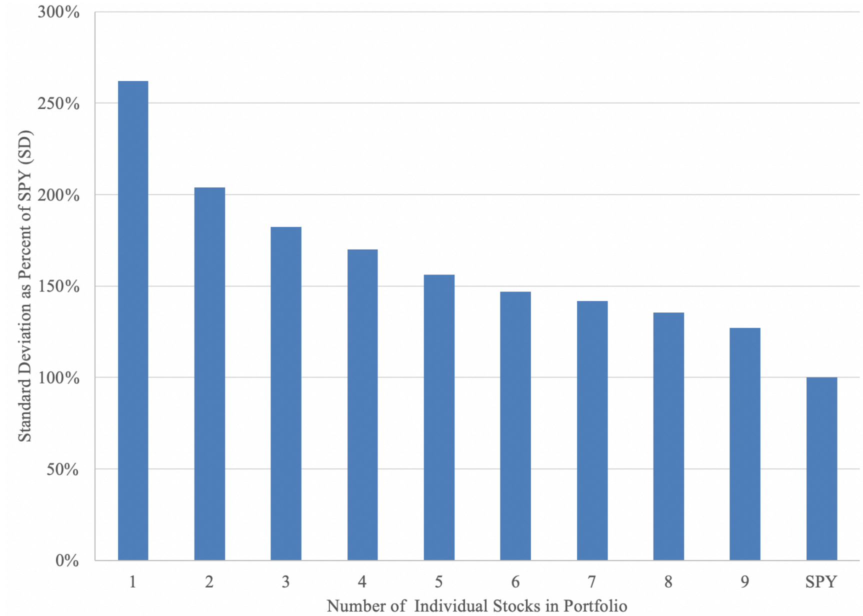


Figure 14.1.14 again summarizes the prior results as a percentage of SPY of the marginal influence on rolling standard deviations with adding additional stocks in one bar chart. Again, as the number of stocks increase, we see this percentage initially drop quickly and then begin to taper.

Figure 14.1.14. Individual Stock Percent of SPY Average Rolling Portfolio Standard Deviations



Thus, we have four key insights for individual stocks: First, individual stock risk or even average of individual stock risk overstates portfolio uncertainty. Second, risk reduction is initially dramatic as additional stocks are added. Third, marginal diversification benefit declines as number of stocks increases. Finally, even with diversification, significant risk remains in portfolios.

Figure 14.1.15 summarizes the marginal influence on rolling standard deviations with adding additional sector ETFs in one bar chart. As the number of sector ETFs increase, we see the average of equally weighted portfolio rolling standard deviations trending toward the SPY average rolling standard deviation and even drops below it. SPY is overweighted in risky technology when compared to an equally-weighted portfolio. By keeping the y-axis scale chosen for individual stocks (Figure 14.1.13), we see that sector ETFs have dramatically lower standard deviations as they are already diversified within each sector.

Figure 14.1.15. Sector ETFs Average Rolling Portfolio Standard Deviations

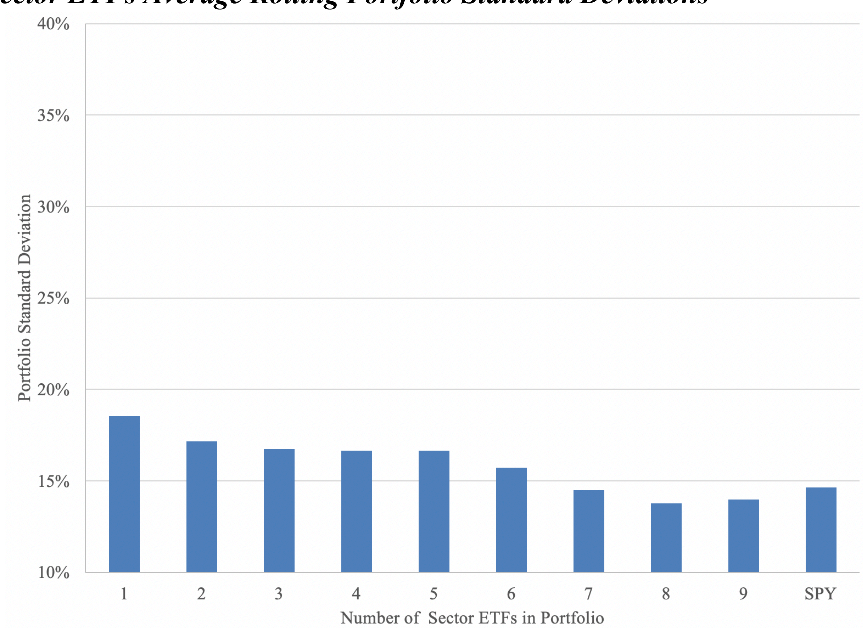
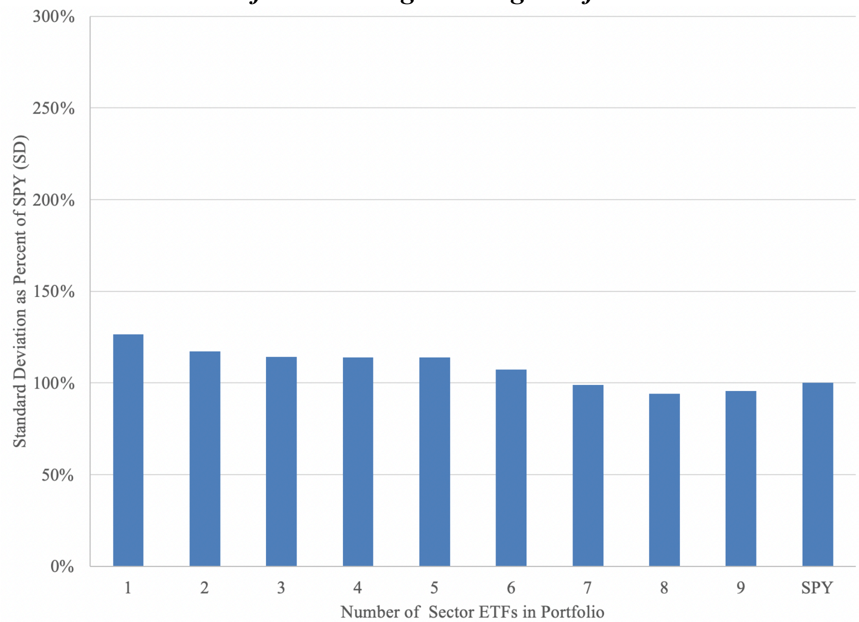


Figure 14.1.16 again summarizes the prior results as a percentage of SPY of the marginal influence on rolling standard deviations with adding additional sector ETFs in one bar chart. Again, as the number of sector ETFs increase, we see this percentage slowly and erratically decline and then actually rise.

Figure 14.1.16. Sector ETFs Percent of SPY Average Rolling Portfolio Standard Deviations



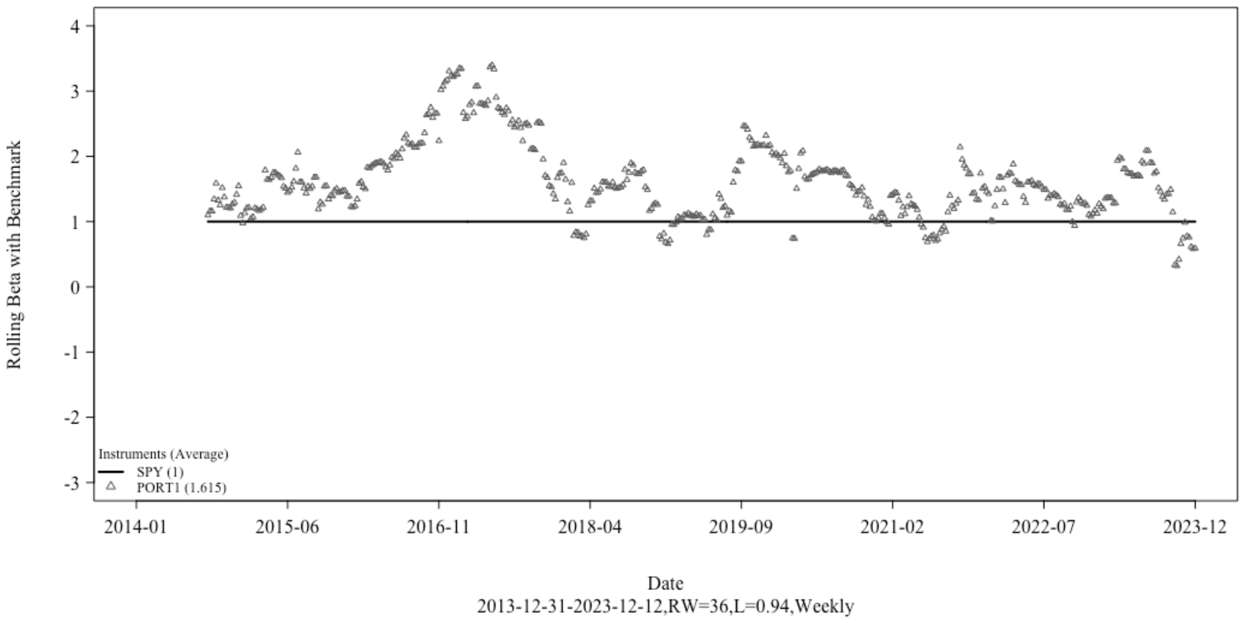
Again, we have four key insights for sector ETFs: First, sector ETF risk or even average of sector ETF risk overstates portfolio uncertainty. Second, risk reduction with ETFs is marginal. Third, marginal diversification benefit declines as number of sector ETFs increases and eventually increases. Finally, even with sector ETF diversification, significant risk remains in portfolios.

We now turn to examine rolling betas when measured with SPY serving as the market portfolio.

Marginal influence of additional instrument on rolling beta

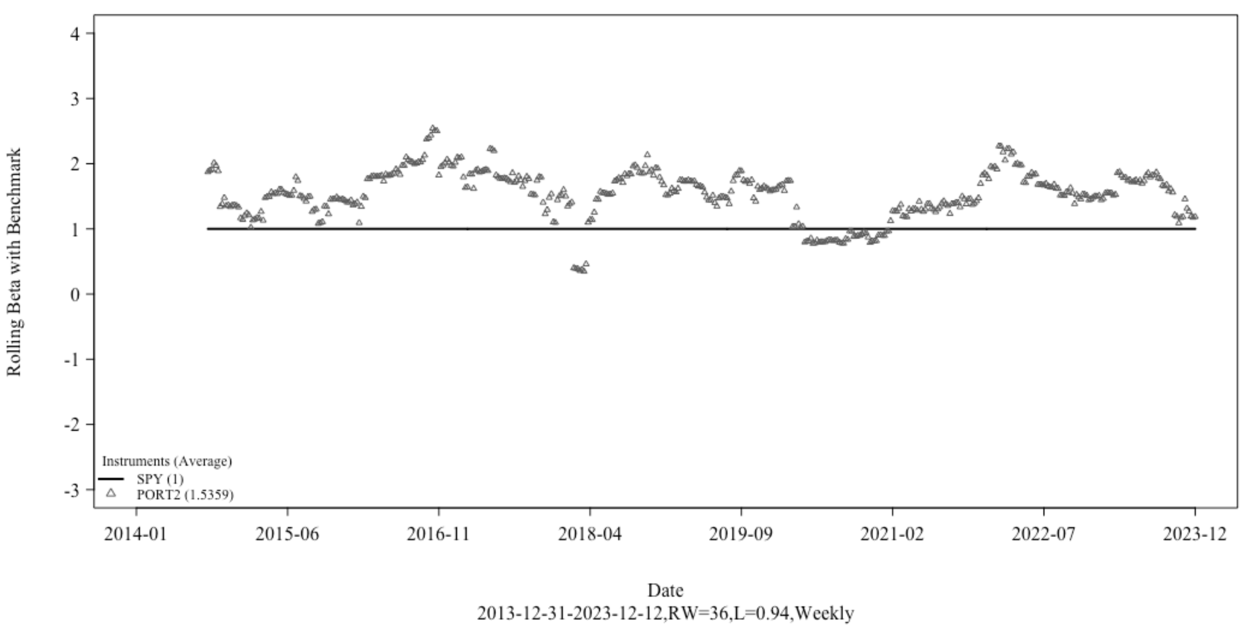
Figure 14.1.17 illustrates multiple panels of the marginal influence on rolling beta with adding additional stocks. Again, the stocks are assumed equally weighted as numbers increase through the different panels. Further, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of stocks increase, we see the equally weighted portfolio rolling beta trending toward 1.0, the SPY rolling beta with itself.

Figure 14.1.17. Marginal Influence on Rolling Beta Illustration with Individual Stocks
Panel A: One Stock with SPY



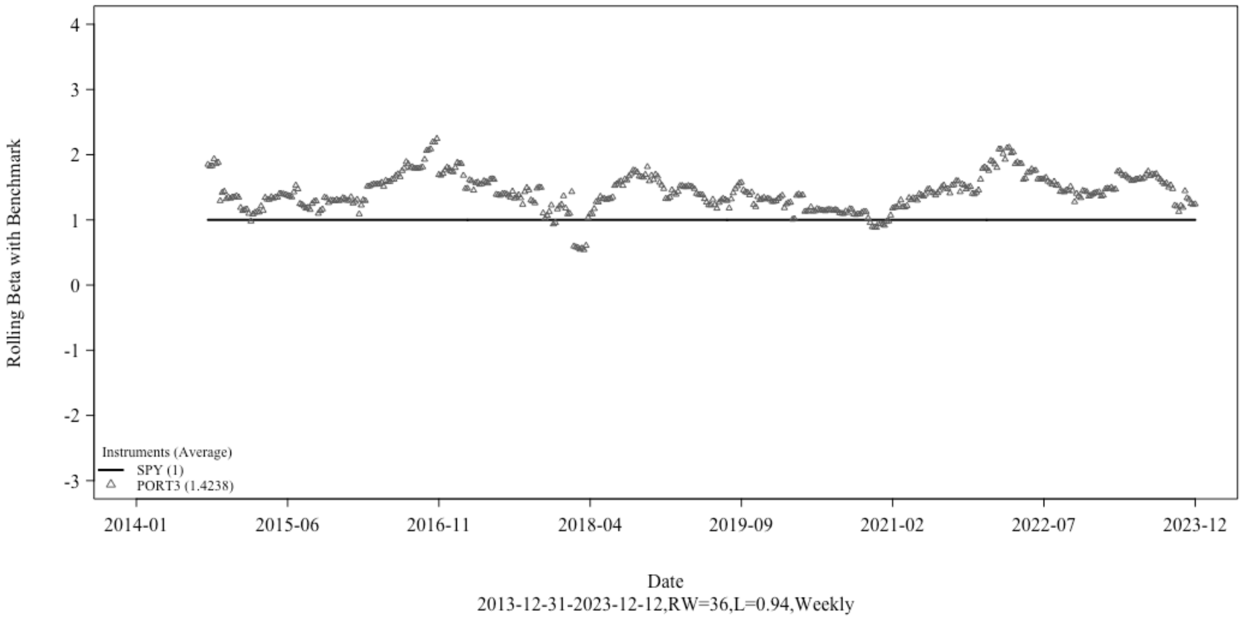
Note: Steel Dynamics (STLD).

Panel B: Two Stocks with SPY



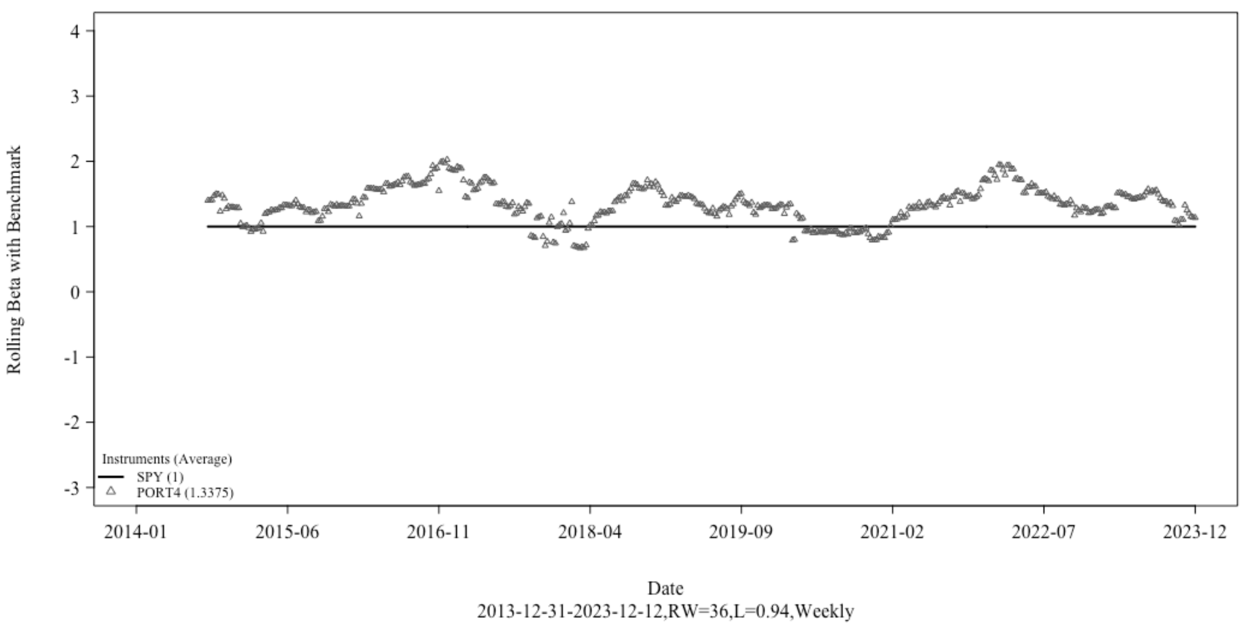
Note: Steel Dynamics (STLD) and ServiceNow (NOW).

Panel C: Three Stocks with SPY



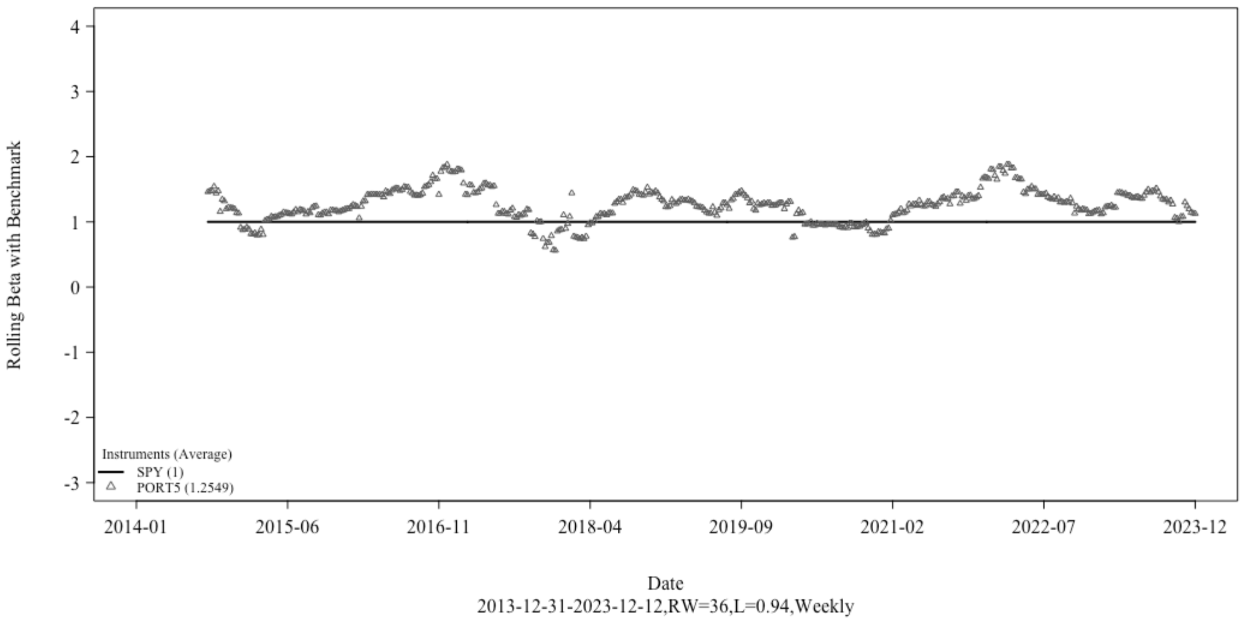
Note: Steel Dynamics (STLD), Service Now (NOW), and Pulte Group (PHM).

Panel D: Four Stocks with SPY



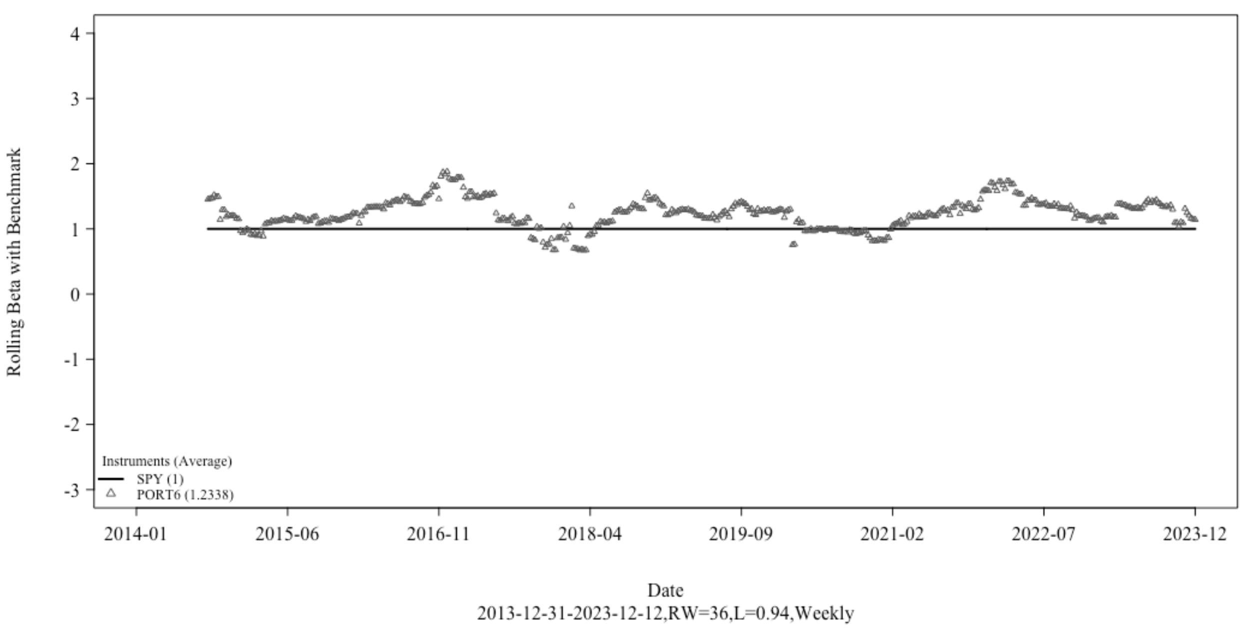
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), and Regeneron (REGN).

Panel E: Five Stocks with SPY



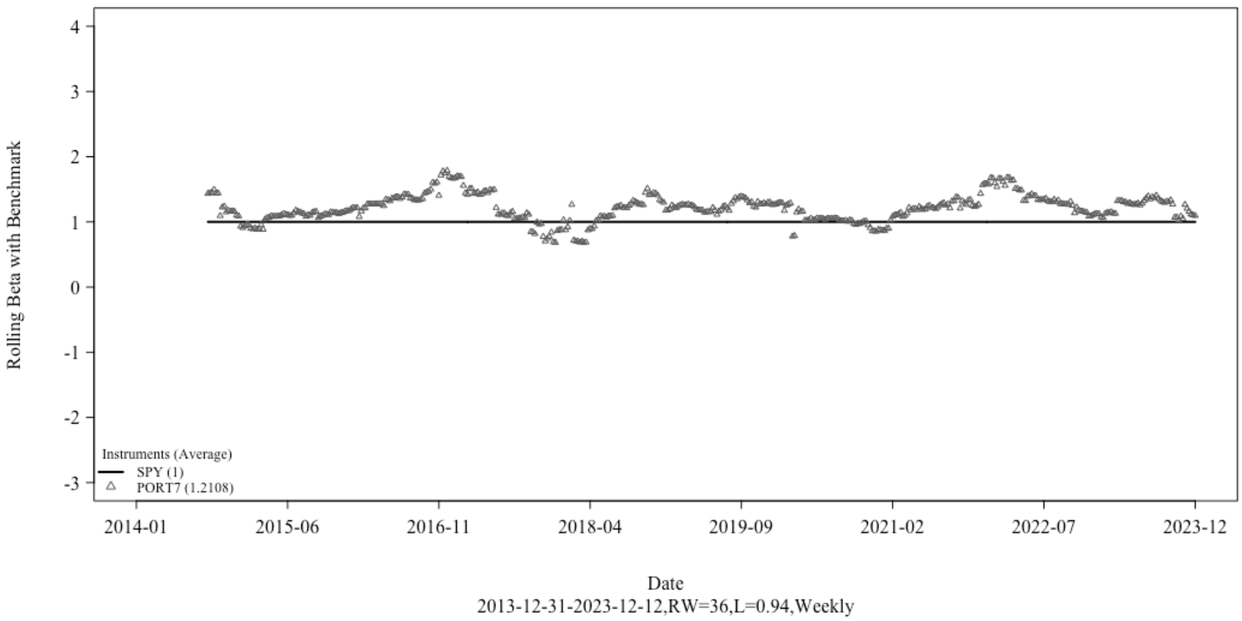
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), and Advance Auto Parts (AAP).

Panel F: Six Stocks with SPY



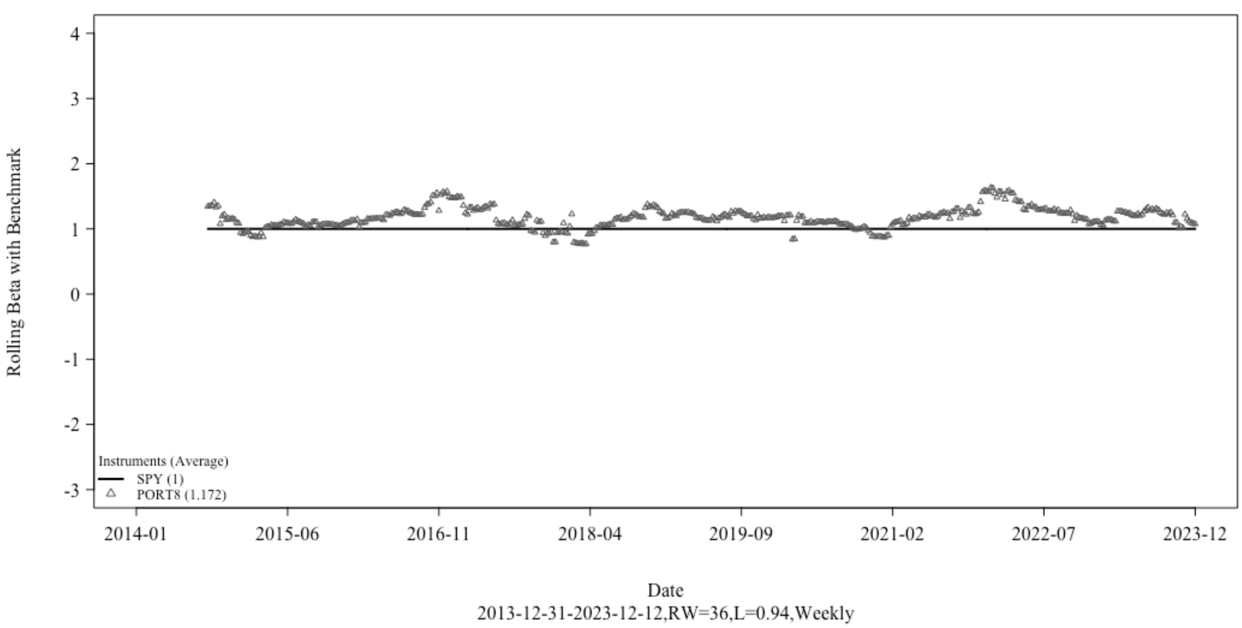
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), and Xylem (XYL).

Panel G: Seven Stocks with SPY



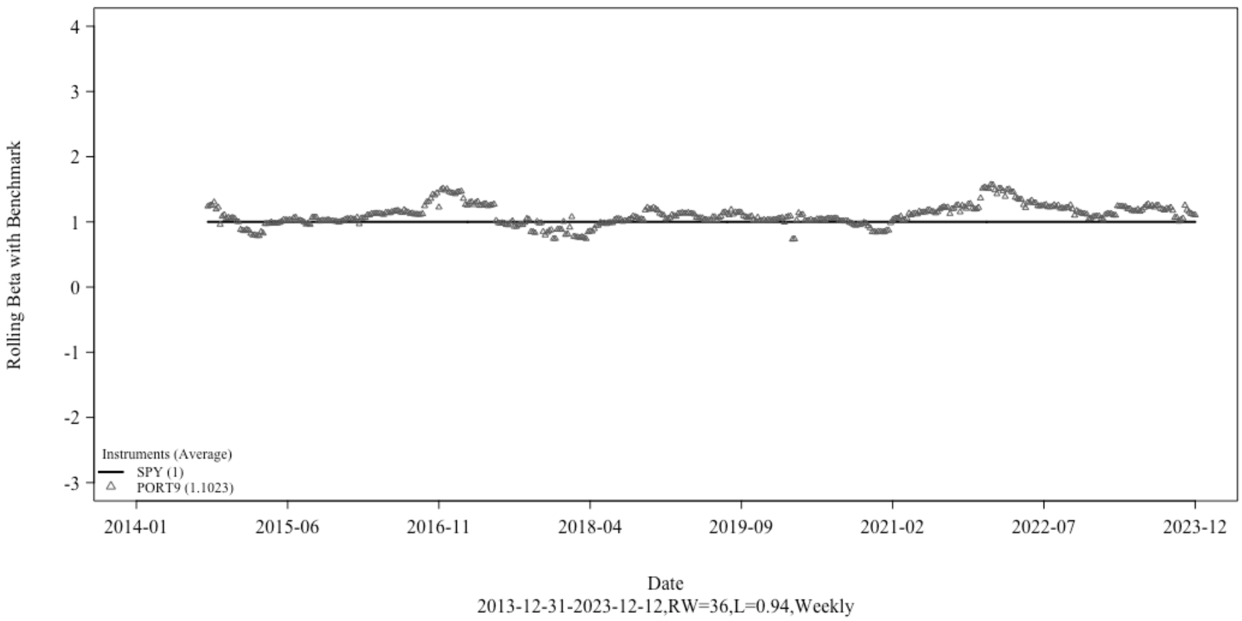
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), and Raytheon (RTX).

Panel H: Eight Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), and Stryker (SYK).

Panel I: Nine Stocks with SPY

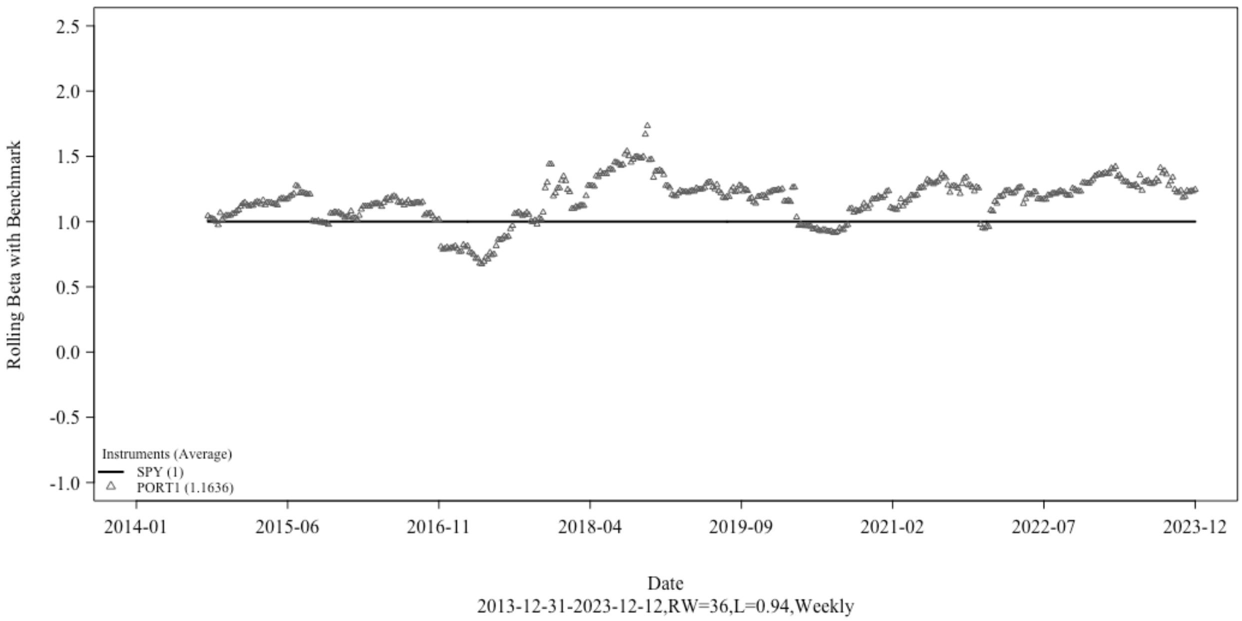


Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), Stryker (SYK), Public Storage (PSA).

The key insights of this analysis of rolling betas with individual stocks are as follows: First, individual rolling betas are quite volatile over time. Second, reducing beta volatility is dramatic as additional stocks are added. Third, the marginal reduction in beta volatility declines as the number of stocks increases. Finally, portfolio beta tends to 1.0 as the number of stocks increases.

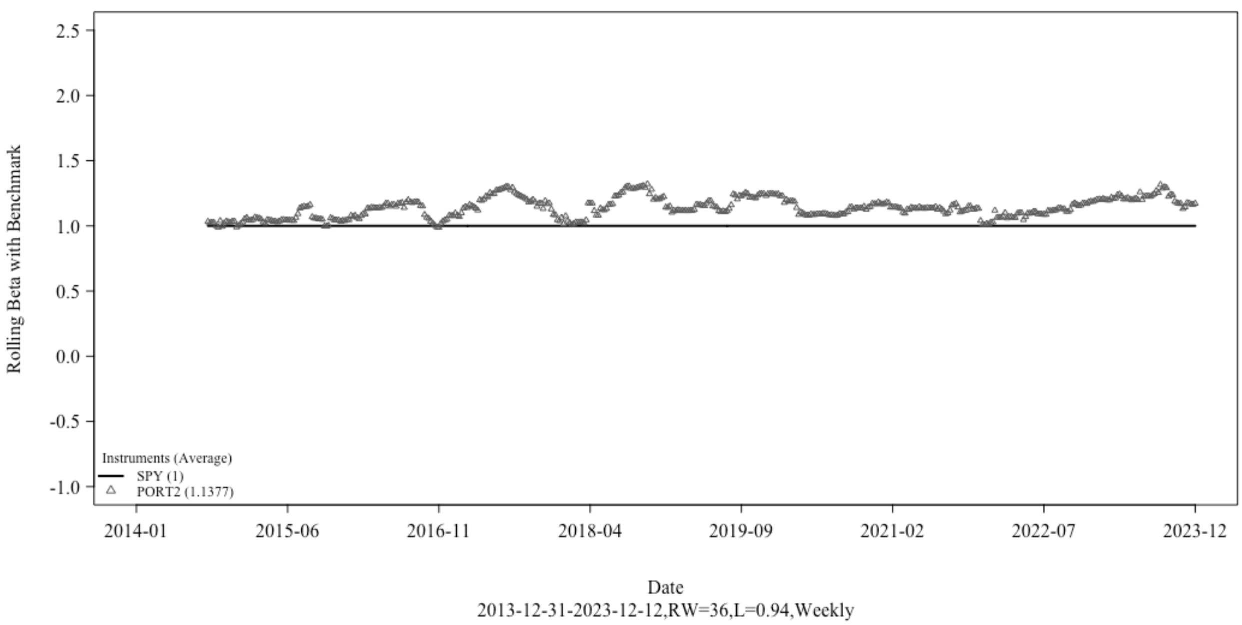
Figure 14.1.18 illustrates multiple panels of the marginal influence on rolling betas with adding additional sector ETFs. Again, the sector ETFs are assumed equally weighted as numbers increase through the different panels. Note to highlight differences, the y-axis is reduced to –1.0 to 2.5 compared with –3.0 to 4.0 with individual stocks. As before, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of sector ETFs increase, we see the equally weighted portfolio rolling betas trending toward 1.0, the SPY rolling beta.

Figure 14.1.18. Marginal Influence on Rolling Beta Illustration with Sector ETFs
Panel A: One Sector ETF with SPY



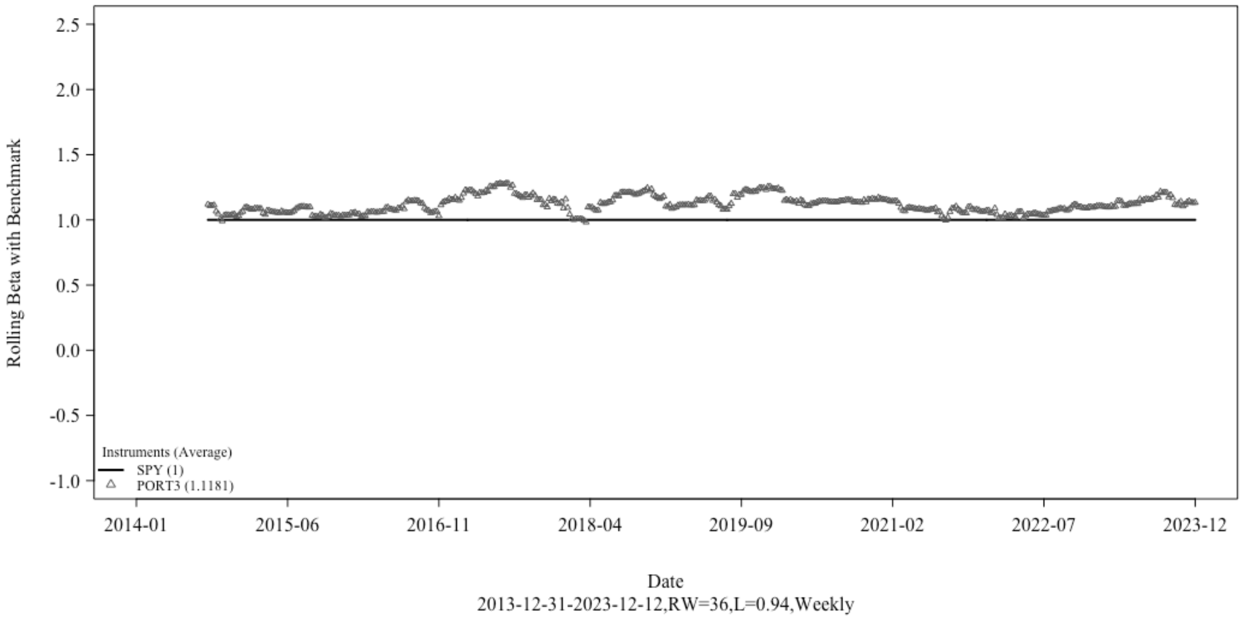
Note: Technology (XLK).

Panel B: Two Sector ETFs with SPY



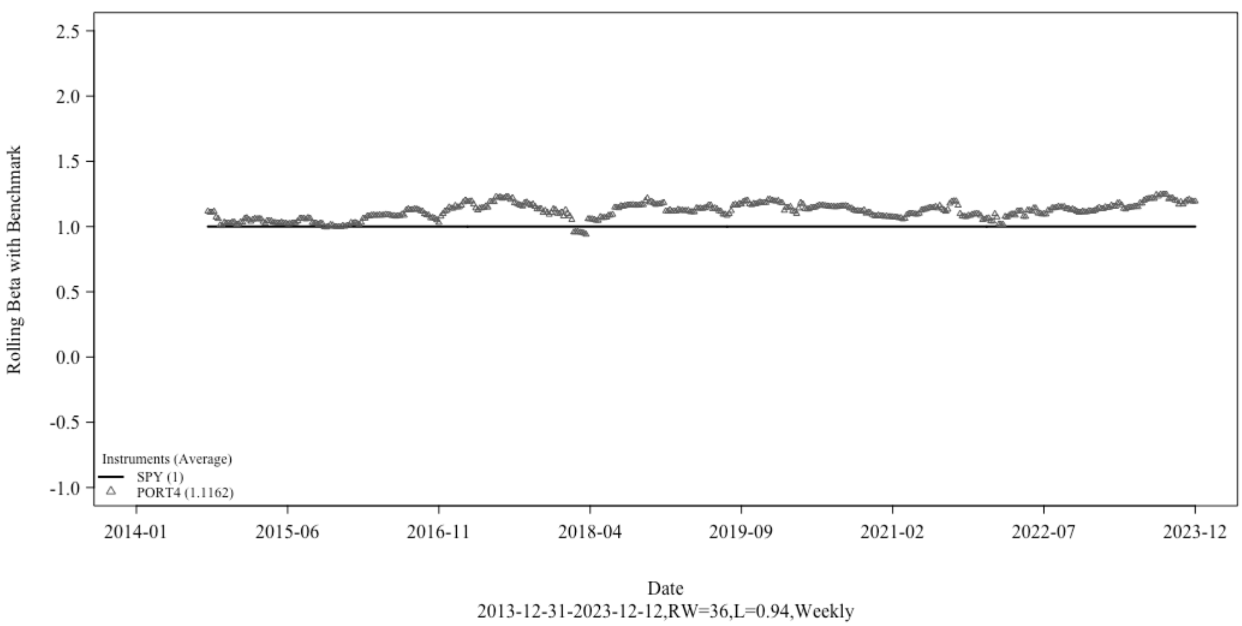
Note: Technology (XLK) and Finance (XLF).

Panel C: Three Sector ETFs with SPY



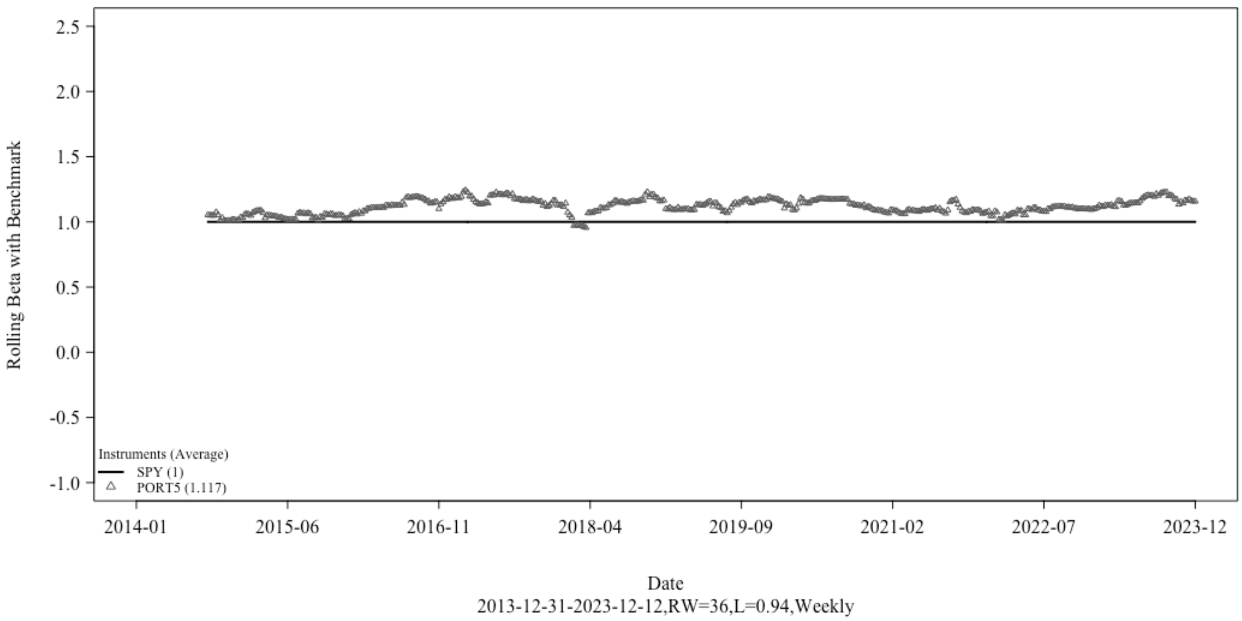
Note: Technology (XLK), Finance (XLF), and Industrial (XLI).

Panel D: Four Sector ETFs with SPY



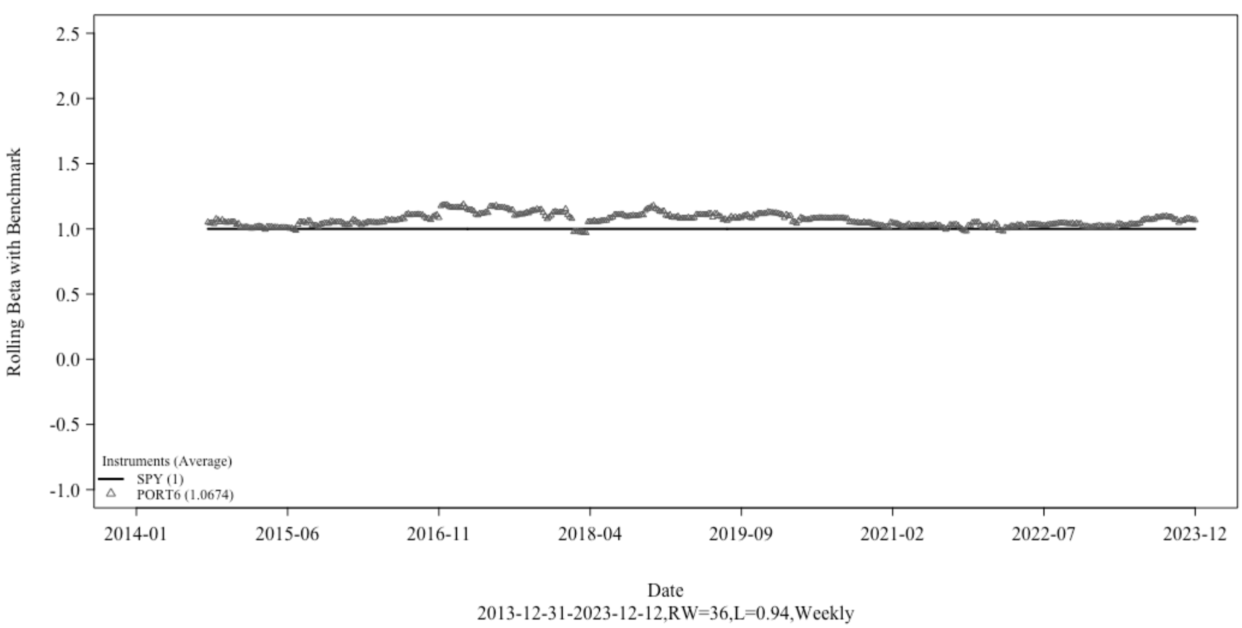
Note: Technology (XLK), Finance (XLF), Industrial (XLI), and Consumer Discretionary (XLY).

Panel E: Five Sector ETFs with SPY



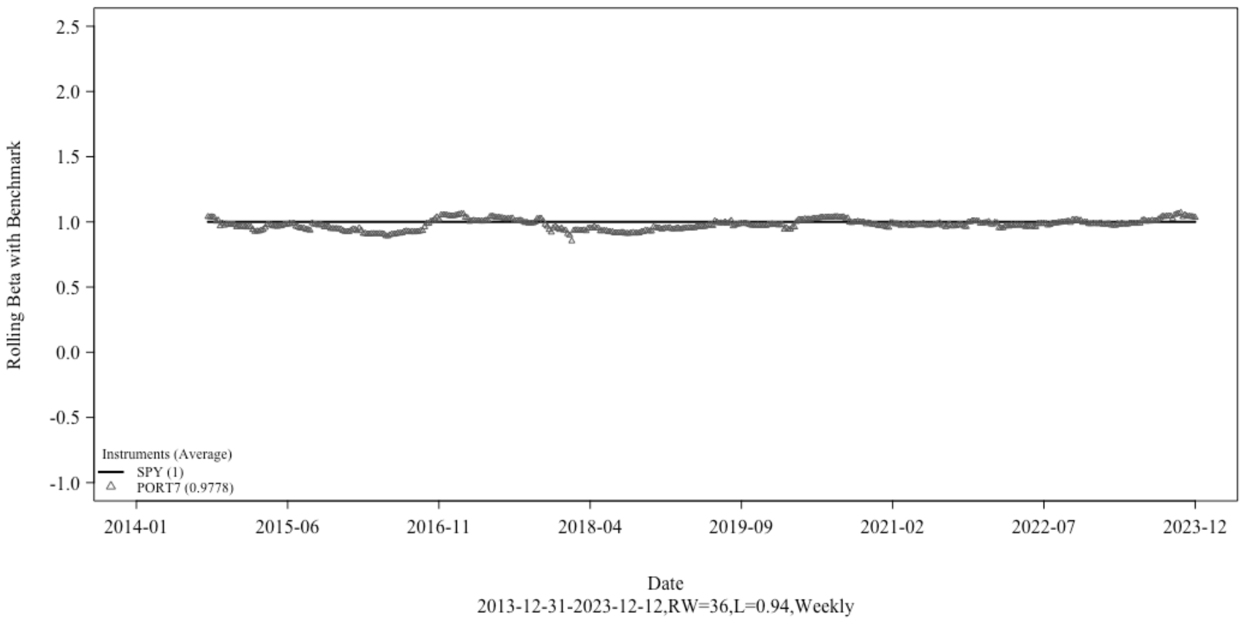
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), and Materials (XLB).

Panel F: Six Sector ETFs with SPY



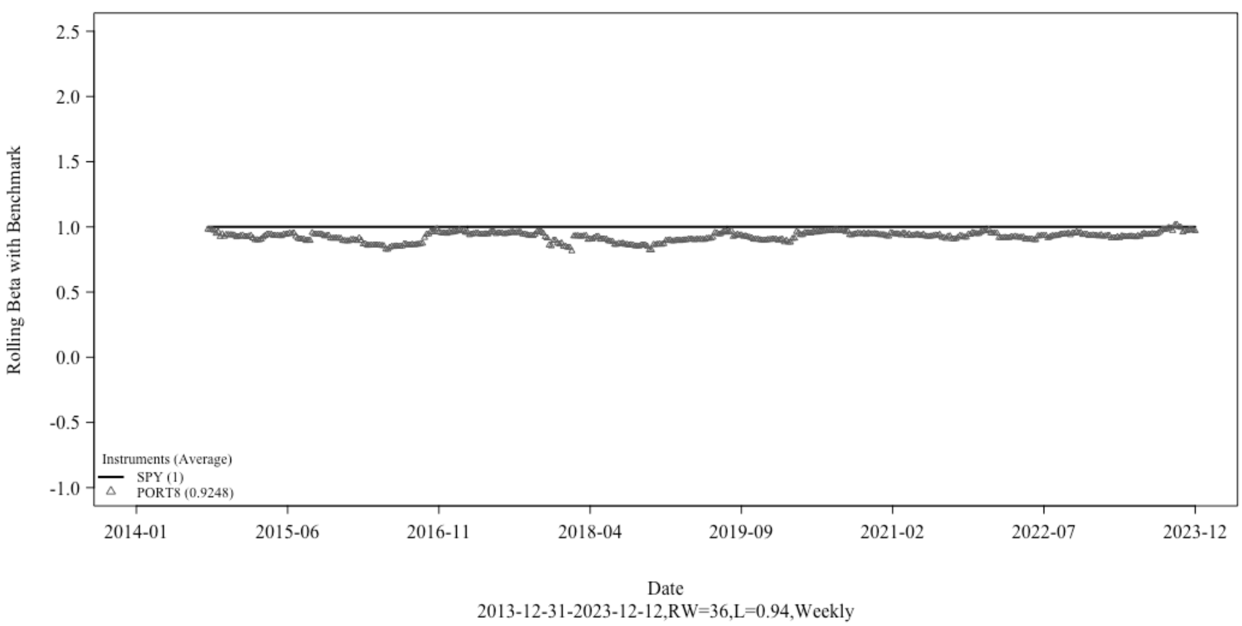
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), and Healthcare (XLV).

Panel G: Seven Sector ETFs with SPY



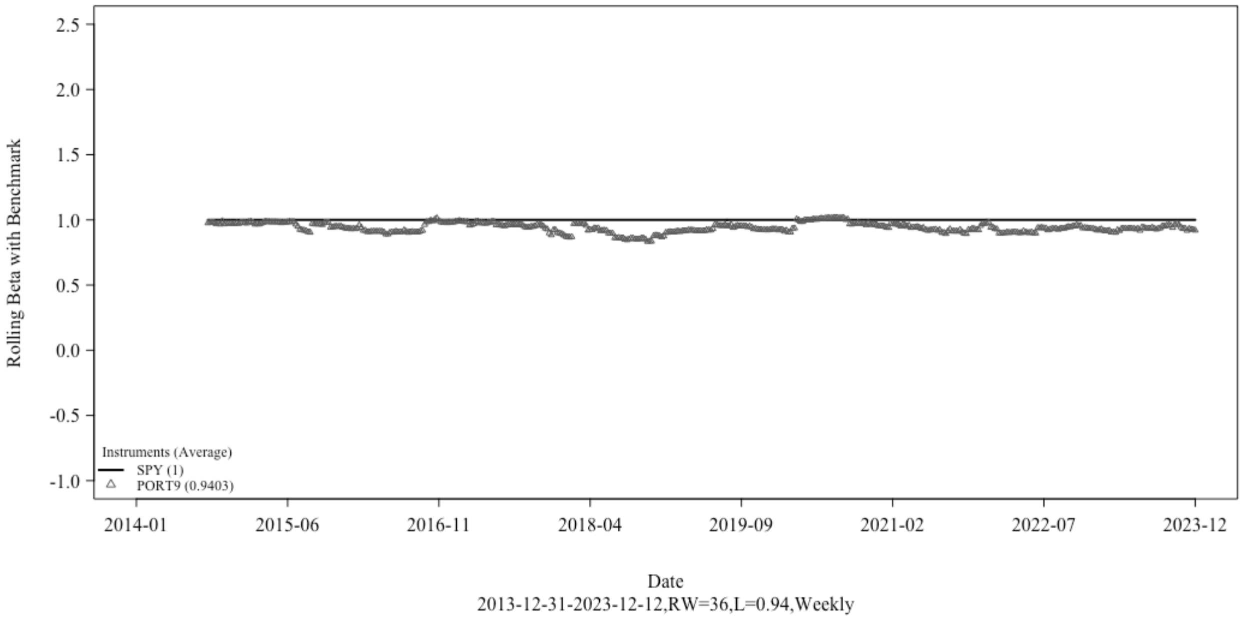
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), and Utilities (XLU).

Panel H: Eight Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), and Consumer Staples (XLP).

Panel I: Nine Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), Consumer Staples (XLP), and Energy (XLE).

The key insights of this analysis of rolling betas with sector ETFs are as follows: First, sector ETF rolling betas are volatile over time. Second, reducing beta volatility is meaningful as additional stocks are added. Third, the marginal reduction in beta volatility declines as the number of sector ETFs increases up to a point. Finally, portfolio beta tends to 1.0 as the number of stocks increases.

Diversification Benefit

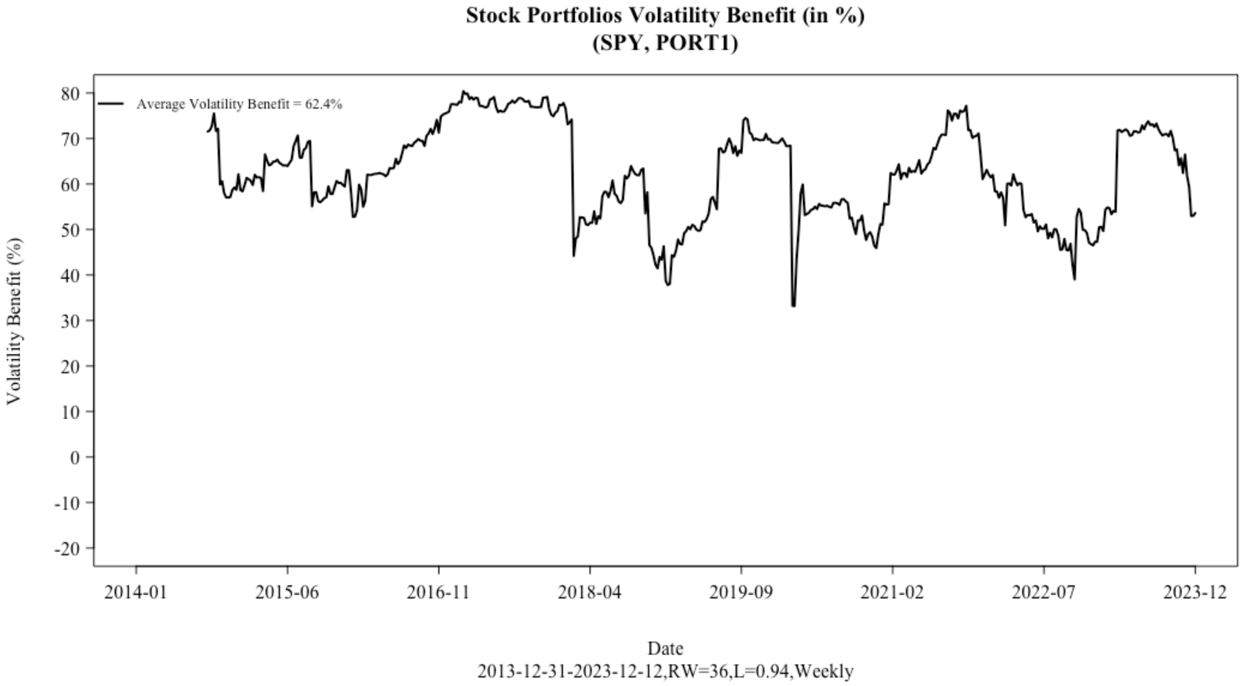
Asset allocation involves partitioning investment funds among different categories of assets to achieve the desired level of diversification. First, the diversification benefits (DB) will be expressed in terms of reduction of risk. We will define this benefit perspective as the volatility benefit (VB). Alternatively, we will illustrate how the DB can be viewed in terms of contribution to the average or expected rate of return. We will define this benefit perspective as the return due to diversification (RDD).

Marginal influence of additional instrument on rolling volatility benefit

We now examine the VB applied to the nine ETFs reflecting the sectors of the S&P 500 index when compared with an equally-weighted portfolio of the nine sector ETFs (denoted Benchmark or BMK, a close proxy to SPY).

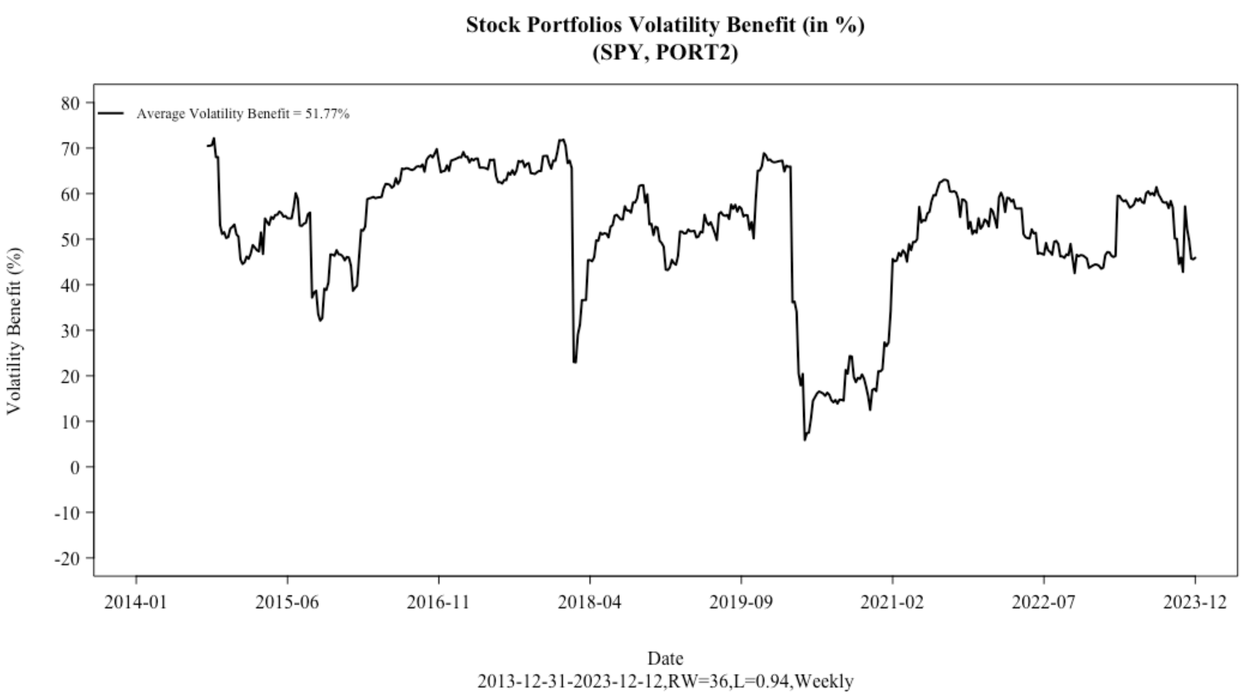
Figure 14.1.19 illustrates multiple panels of the marginal influence on the rolling volatility benefit with adding additional stocks. Again, the stocks are assumed equally weighted as numbers increase through the different panels. Further, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of stocks increase, we see the equally weighted portfolio rolling volatility benefit decreasing significantly.

Figure 14.1.19. Marginal Influence on Rolling Volatility Benefit Illustration with Individual Stocks
Panel A: One Stock with SPY



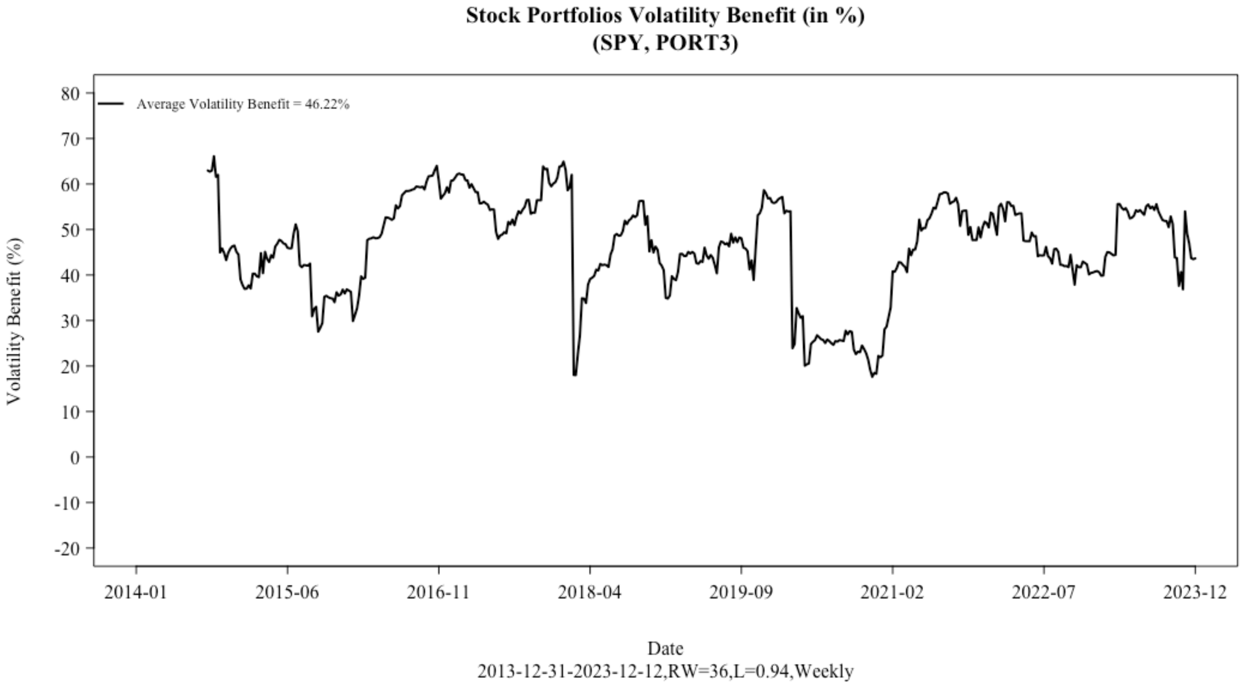
Note: Steel Dynamics (STLD).

Panel B: Two Stocks with SPY



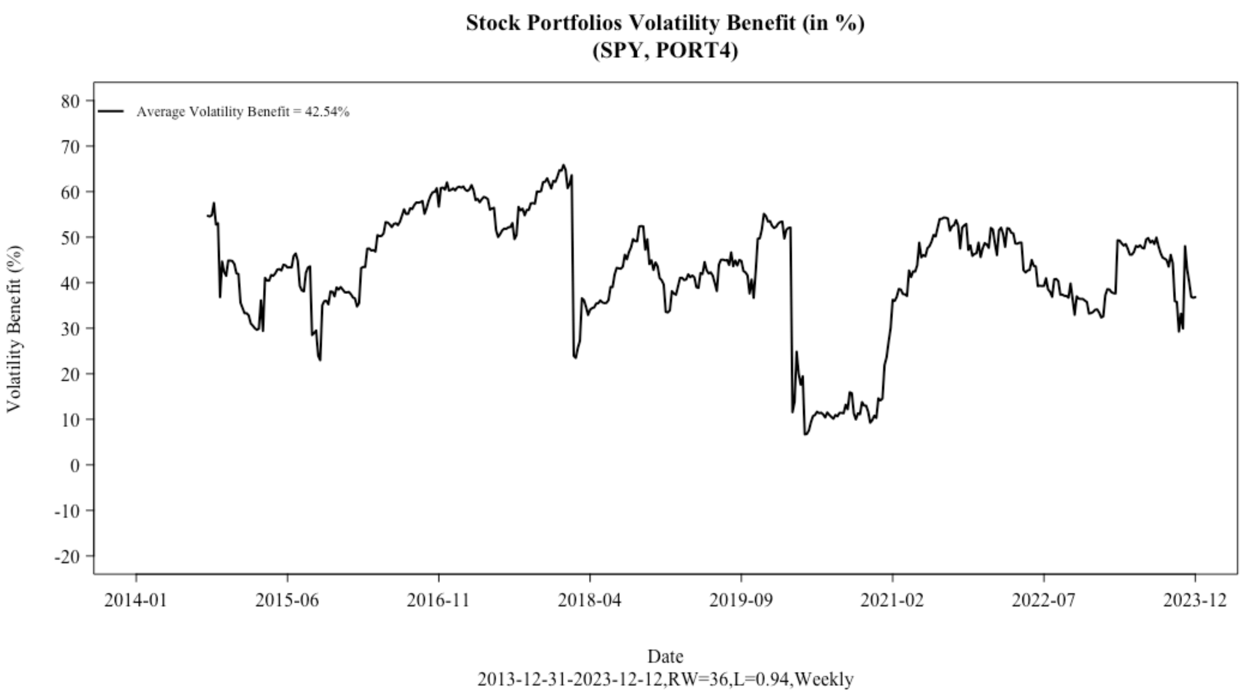
Note: Steel Dynamics (STLD) and ServiceNow (NOW).

Panel C: Three Stocks with SPY



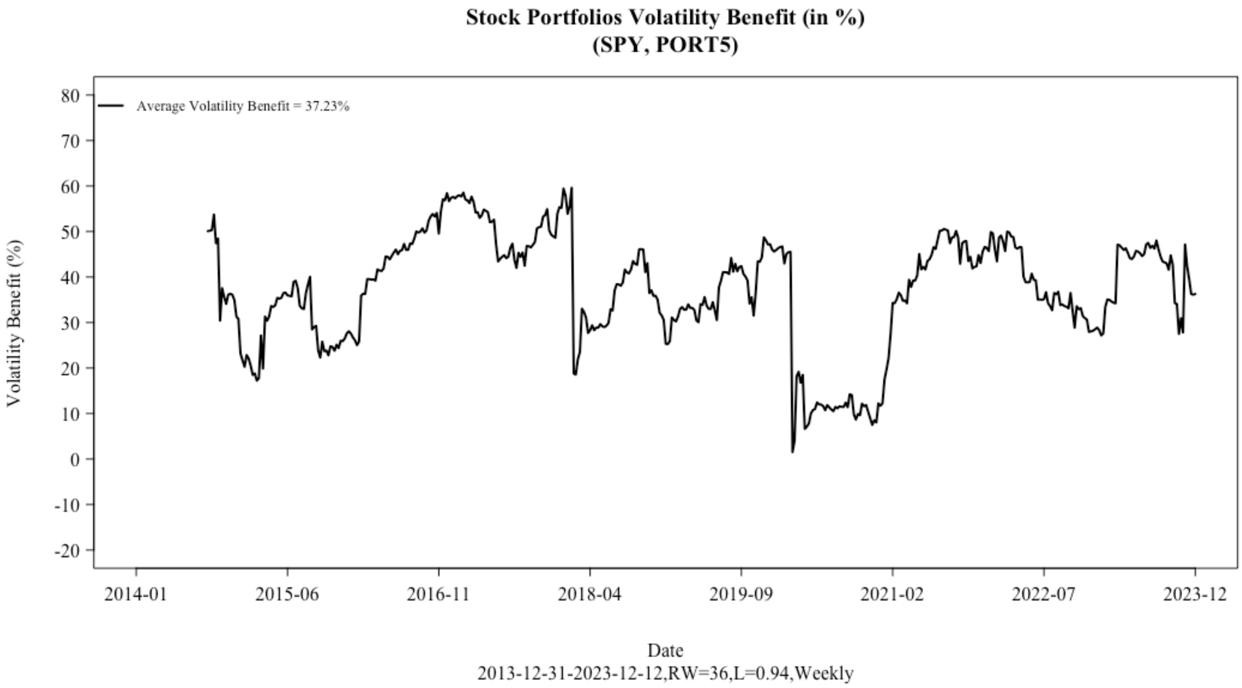
Note: Steel Dynamics (STLD), Service Now (NOW), and Pulte Group (PHM).

Panel D: Four Stocks with SPY



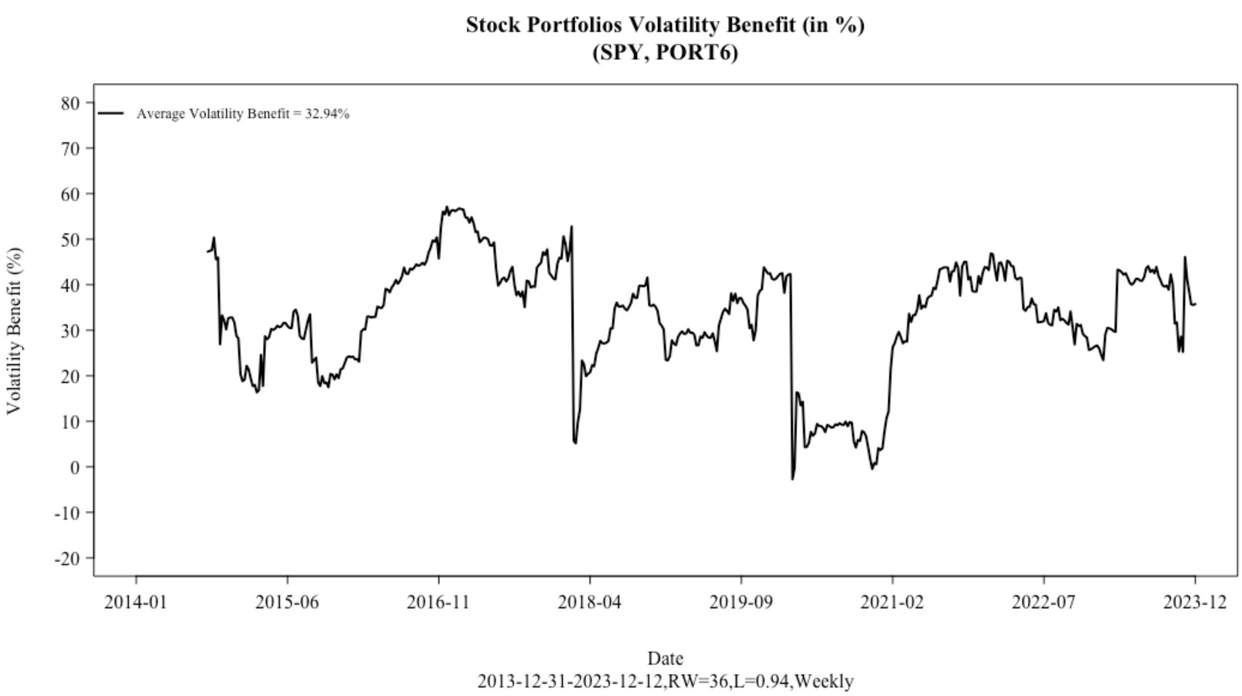
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), and Regeneron (REGN).

Panel E: Five Stocks with SPY



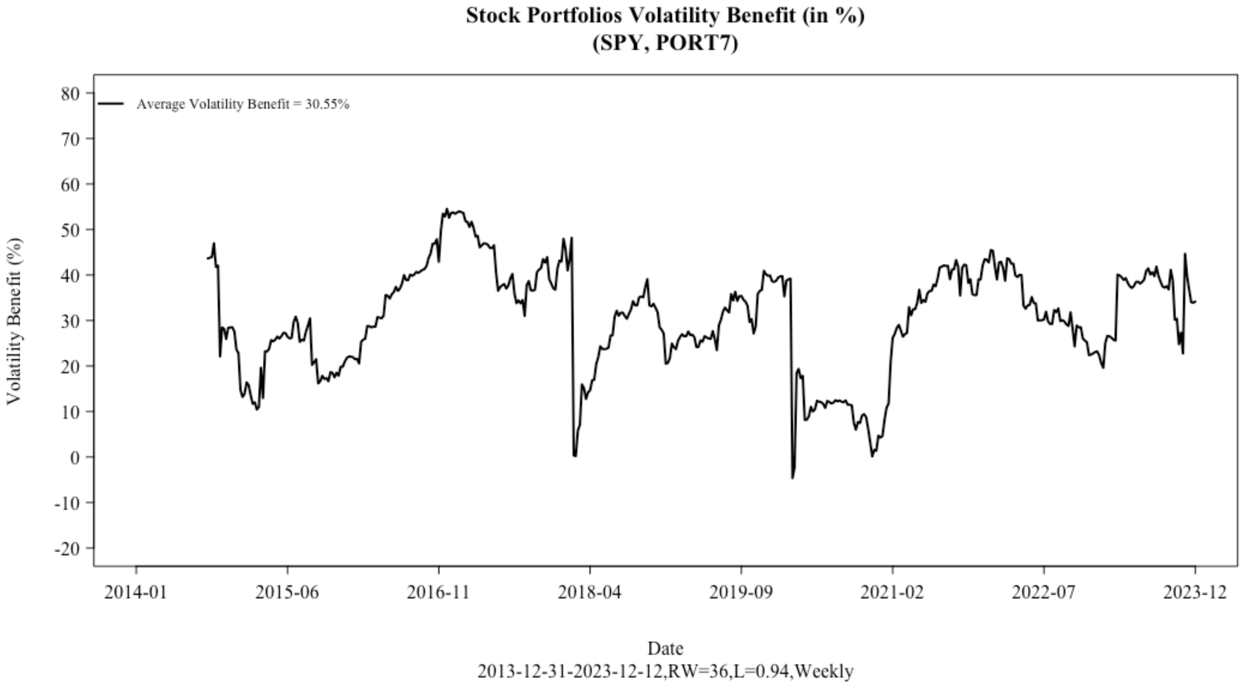
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), and Advance Auto Parts (AAP).

Panel F: Six Stocks with SPY



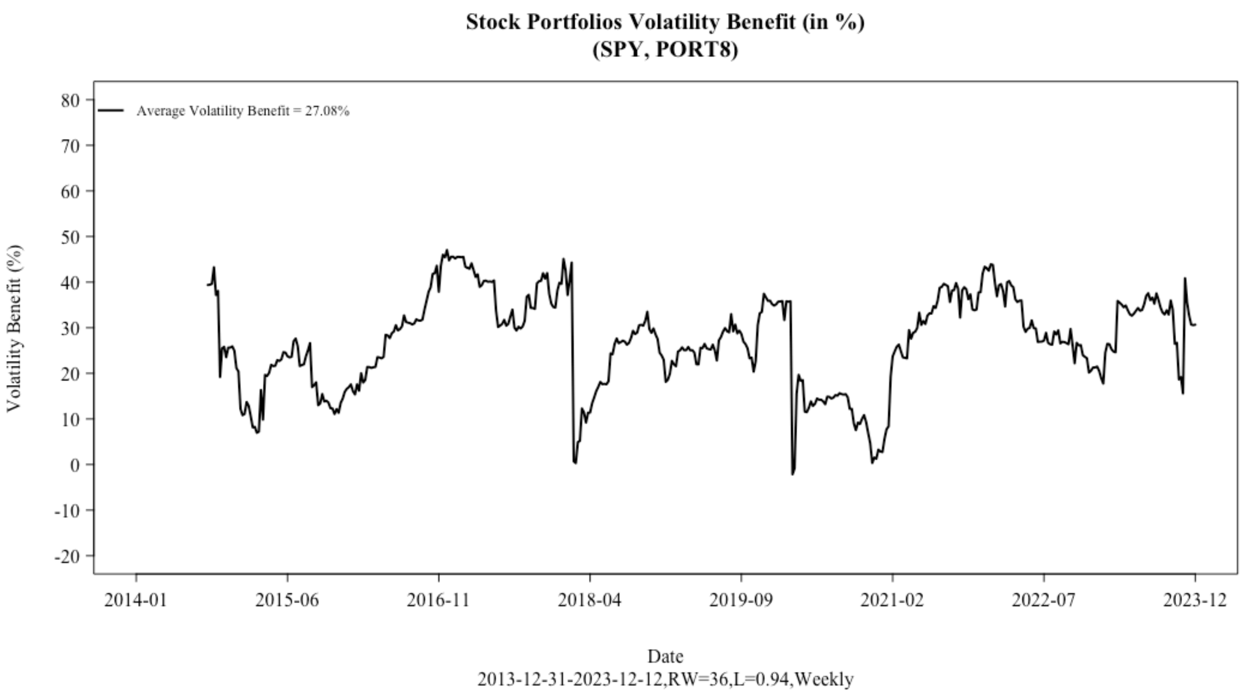
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), and Xylem (XYL).

Panel G: Seven Stocks with SPY



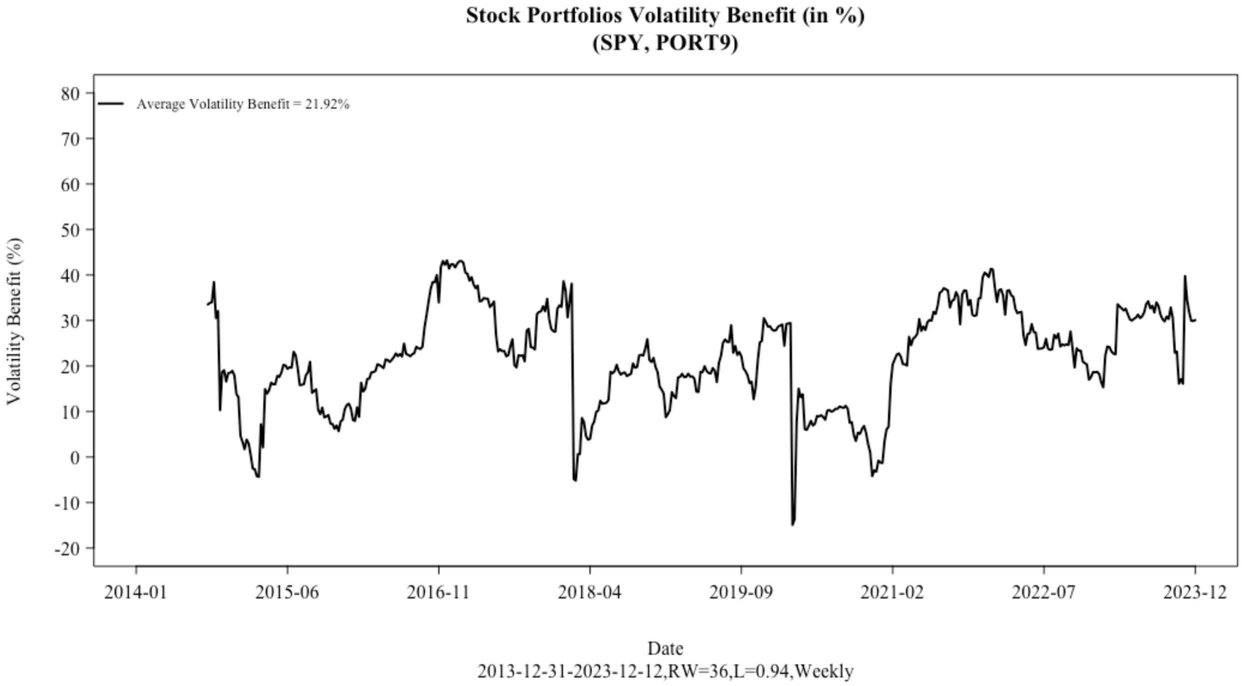
Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), and Raytheon (RTX).

Panel H: Eight Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), and Stryker (SYK).

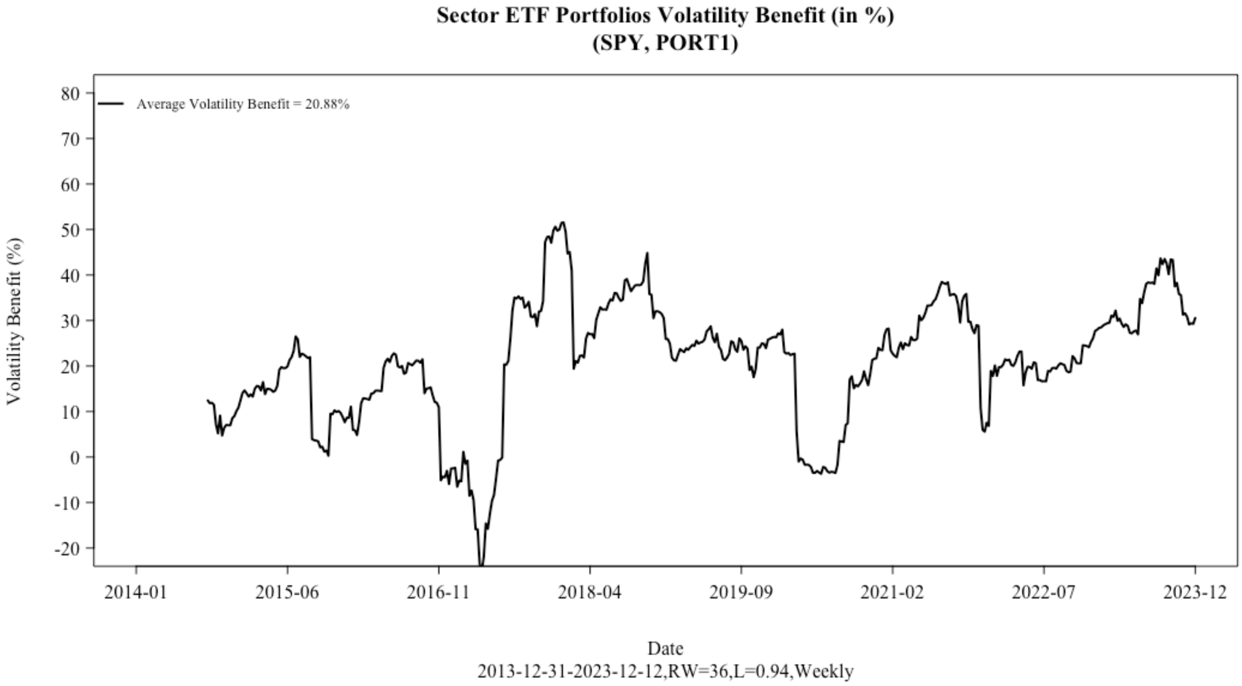
Panel I: Nine Stocks with SPY



Note: Steel Dynamics (STLD), Service Now (NOW), Pulte Group (PHM), Regeneron (REGN), Advance Auto Parts (AAP), Xylem (XYL), Raytheon (RTX), Stryker (SYK), Public Storage (PSA).

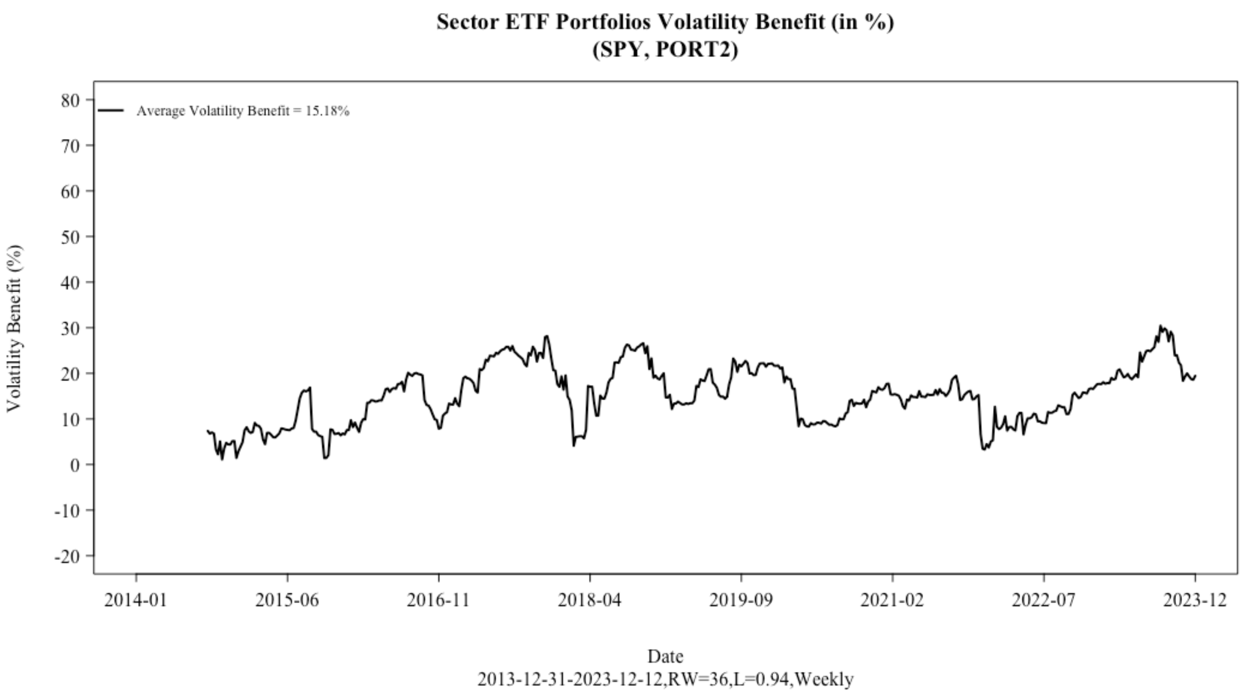
Figure 14.1.20 illustrates multiple panels of the marginal influence on the rolling volatility benefit with adding additional sector ETFs. Again, the sector ETFs are assumed equally weighted as numbers increase through the different panels. Further, the EWMA lambda is 0.94 and the rolling window is 36 weeks. As the number of sector ETFs increase, we see the equally weighted portfolio rolling volatility benefit decreasing significantly and even turning negative as SPY over weights technology.

Figure 14.1.20. Marginal Influence on Rolling Volatility Benefit Illustration with Sector ETFs
Panel A: One Sector ETF with SPY



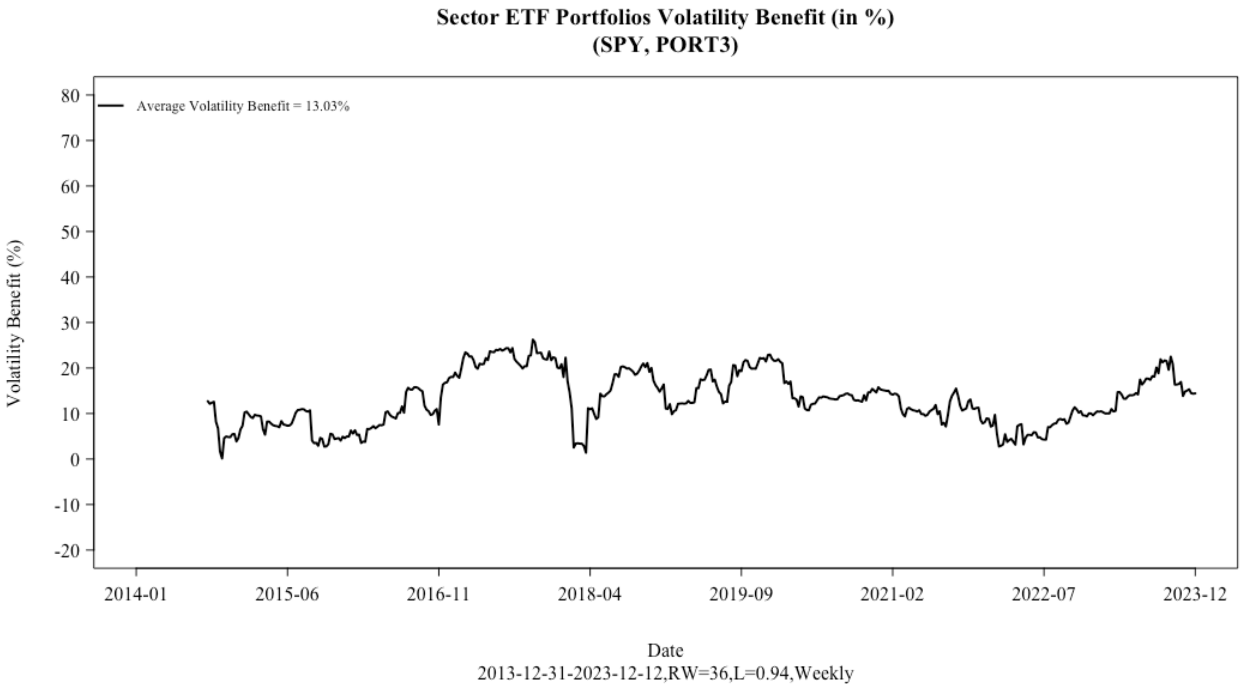
Note: Technology (XLK).

Panel B: Two Sector ETFs with SPY



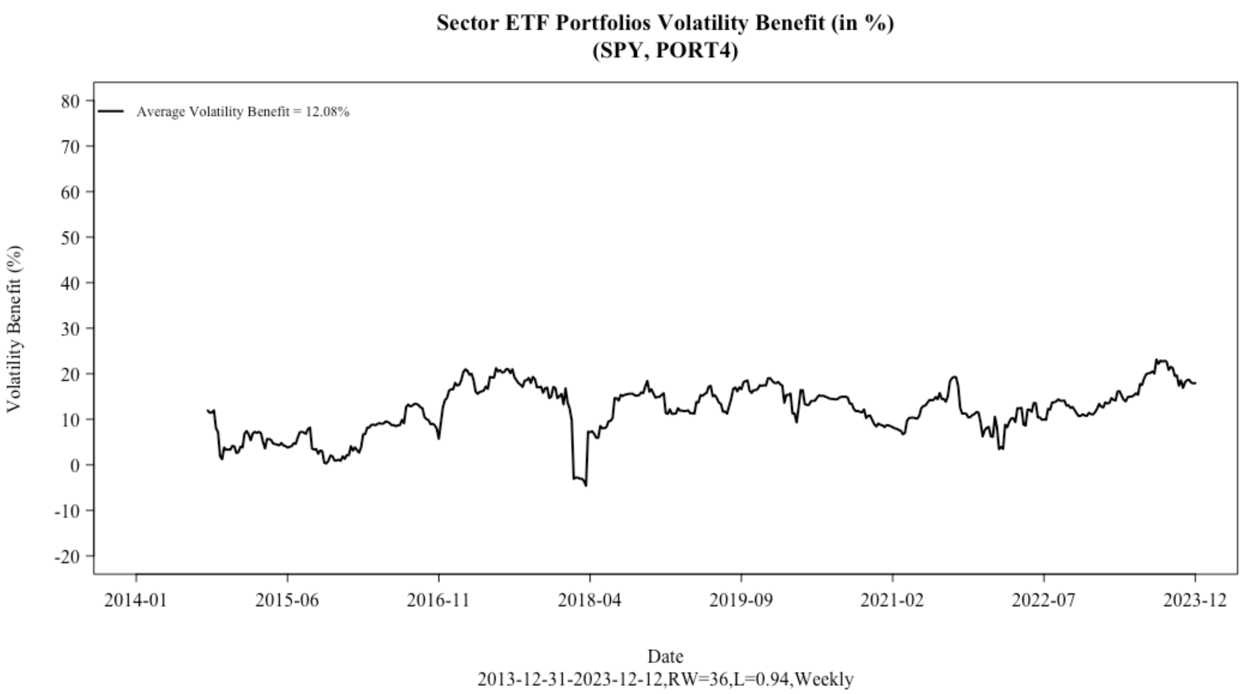
Note: Technology (XLK) and Finance (XLF).

Panel C: Three Sector ETFs with SPY



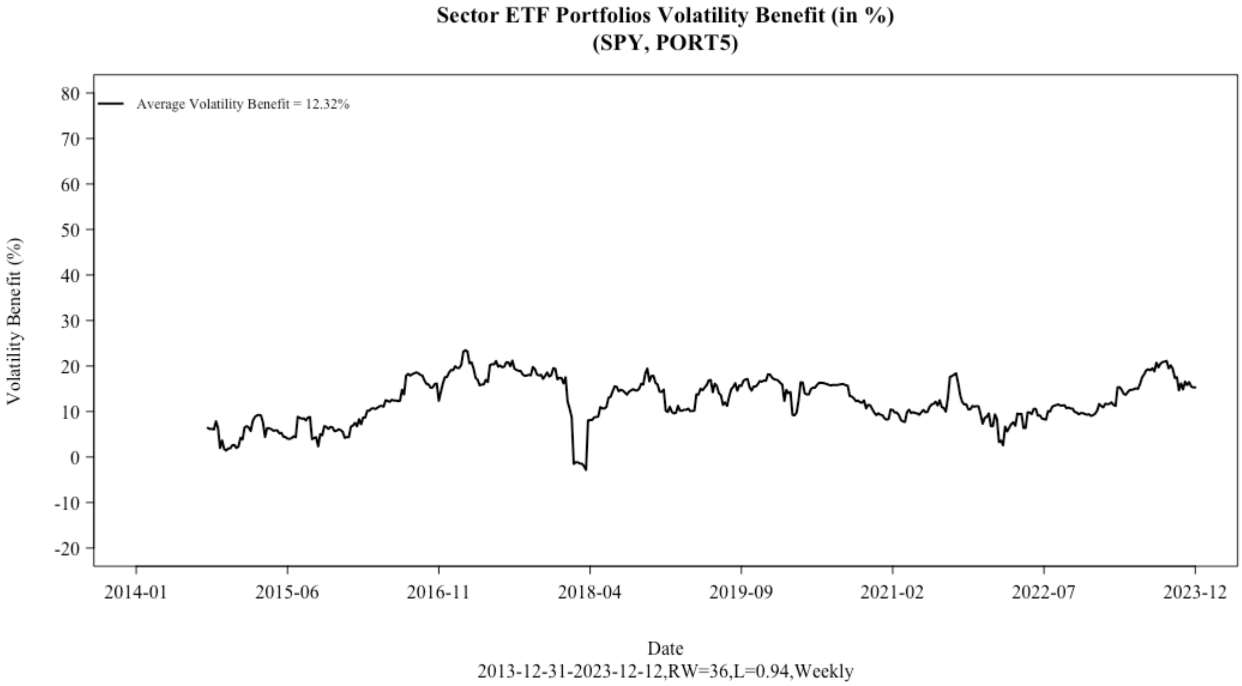
Note: Technology (XLK), Finance (XLF), and Industrial (XLI).

Panel D: Four Sector ETFs with SPY



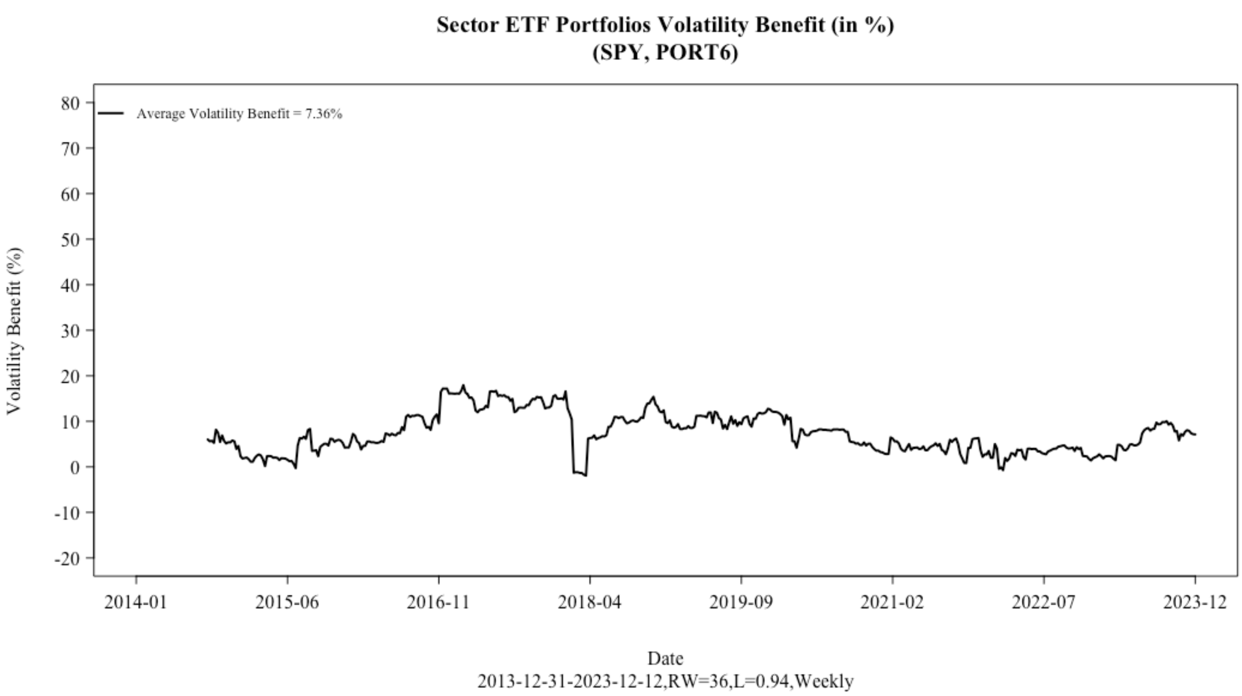
Note: Technology (XLK), Finance (XLF), Industrial (XLI), and Consumer Discretionary (XLY).

Panel E: Five Sector ETFs with SPY



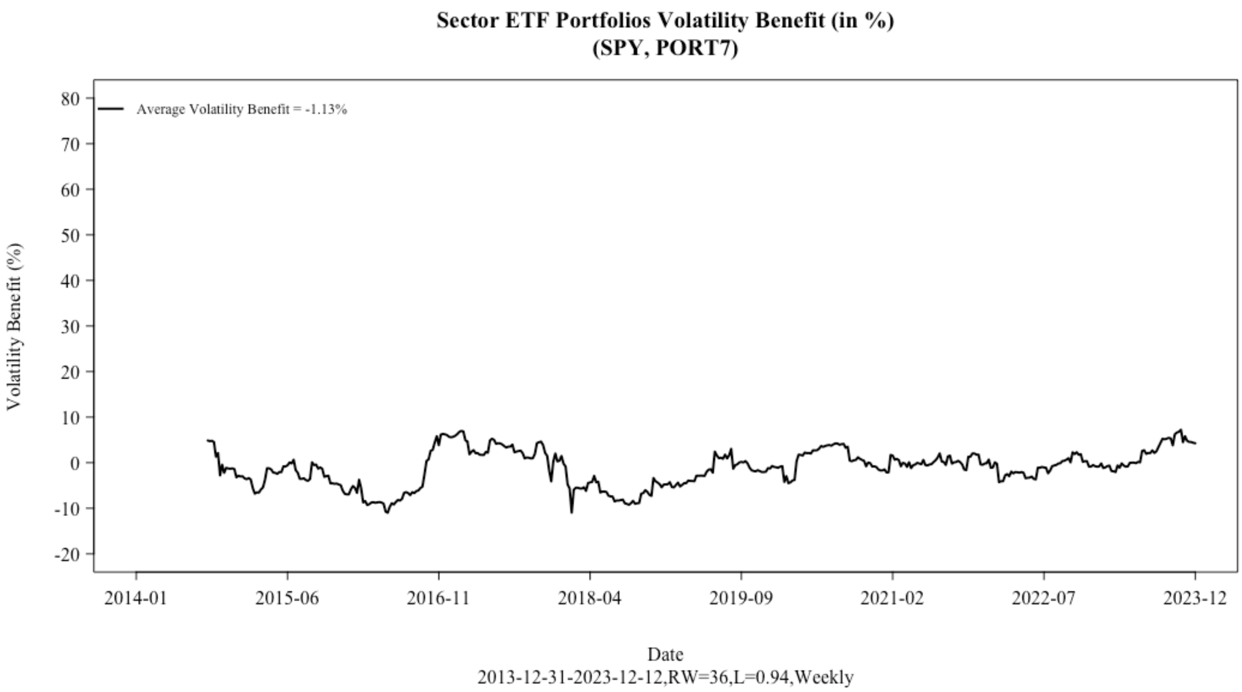
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), and Materials (XLB).

Panel F: Six Sector ETFs with SPY



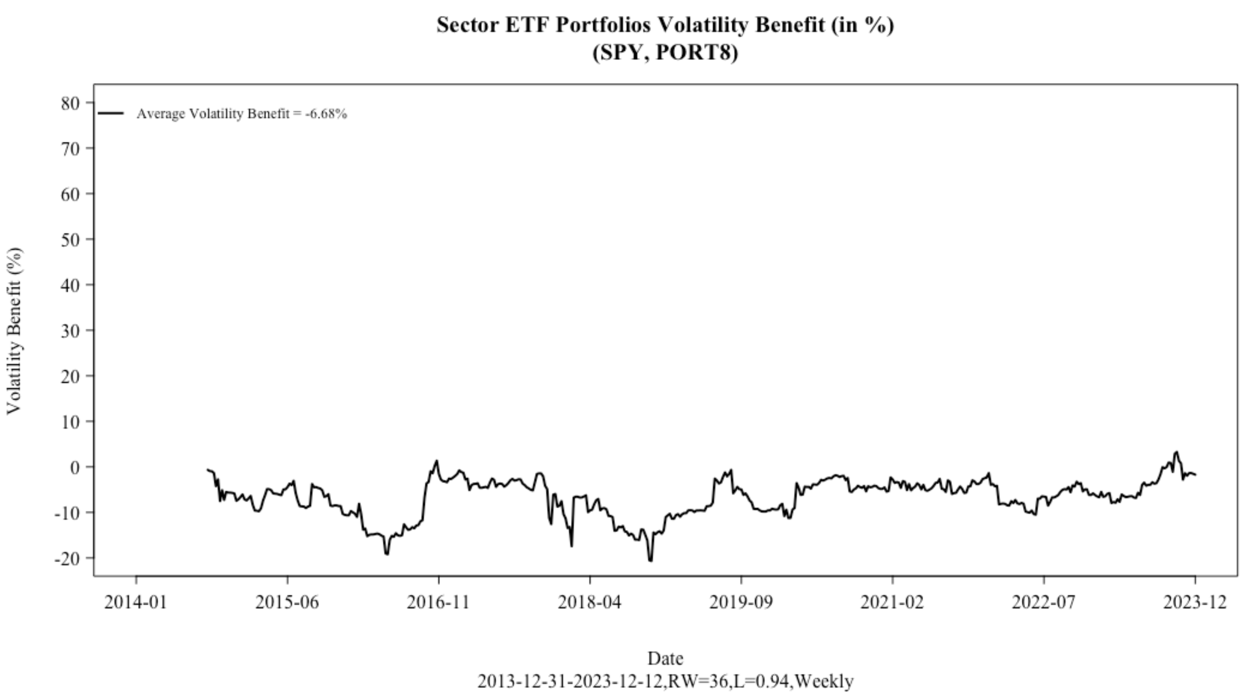
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), and Healthcare (XLV).

Panel G: Seven Sector ETFs with SPY



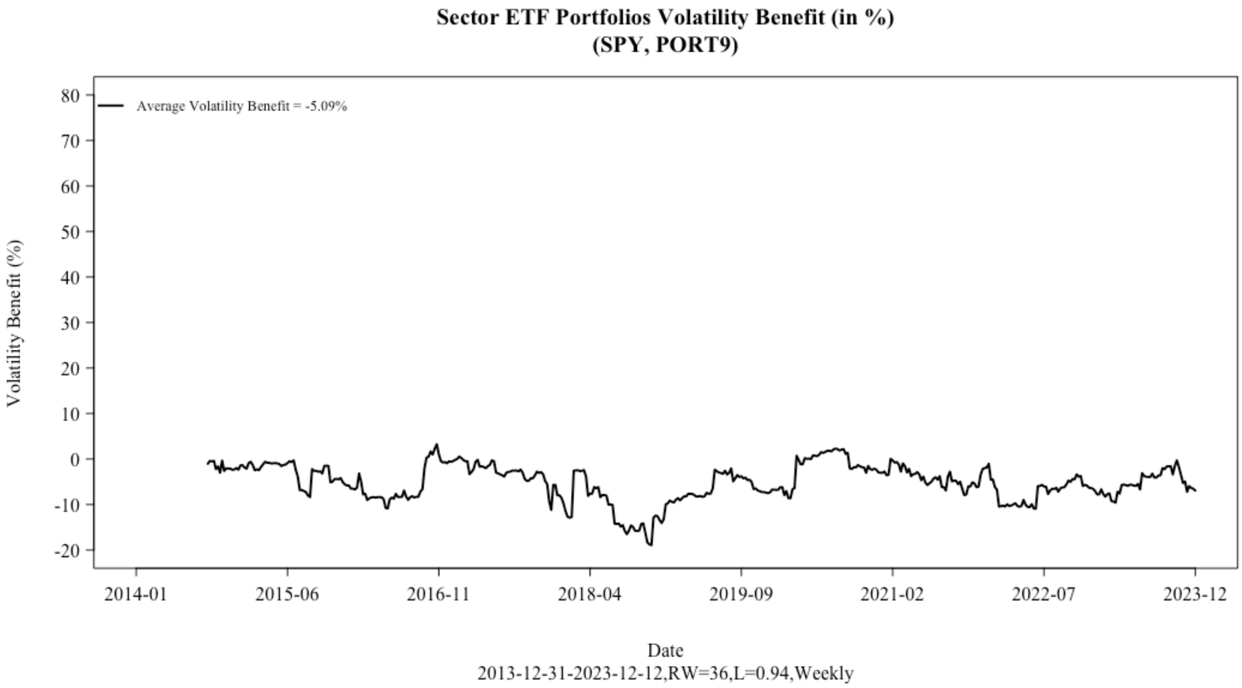
Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), and Utilities (XLU).

Panel H: Eight Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), and Consumer Staples (XLP).

Panel I: Nine Sector ETFs with SPY



Note: Technology (XLK), Finance (XLF), Industrial (XLI), Consumer Discretionary (XLY), Materials (XLB), Healthcare (XLV), Utilities (XLU), Consumer Staples (XLP), and Energy (XLE).

Figure 14.1.21 summarizes the marginal influence on rolling volatility benefit with adding additional stocks in one bar chart. As the number of stocks increase, we see the average of equally weighted portfolio rolling volatility benefit trending toward zero. Note for these nine stocks, the average of the rolling volatility benefit is over 50%.

Figure 14.1.21. Individual Stock Average Rolling Portfolio Volatility Benefit

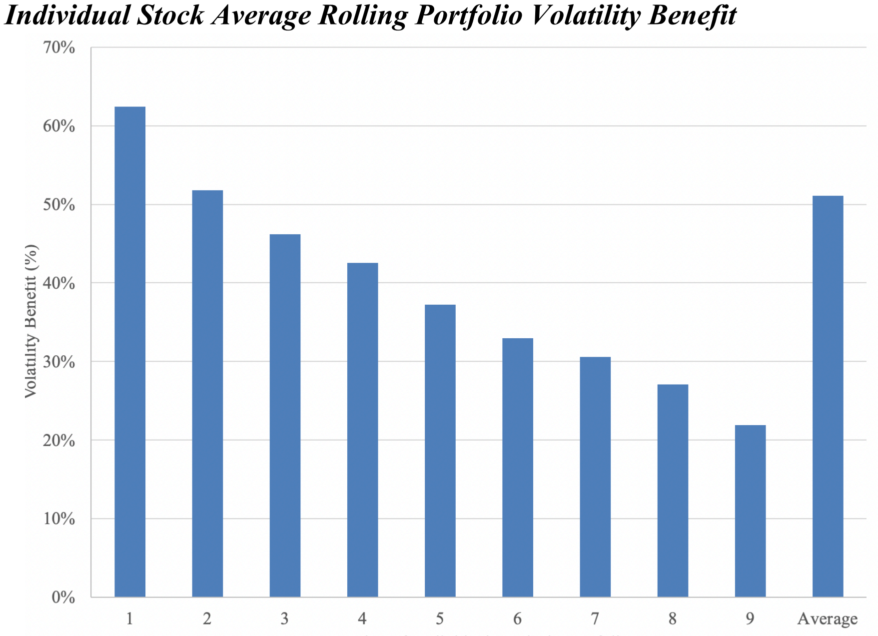


Figure 14.1.22 again summarizes the prior results as a percentage of the average volatility benefit. Again, as the number of stocks increase, we see this percentage drops quickly.

Figure 14.1.22. Individual Stock Percent of SPY Average Rolling Portfolio Volatility Benefit

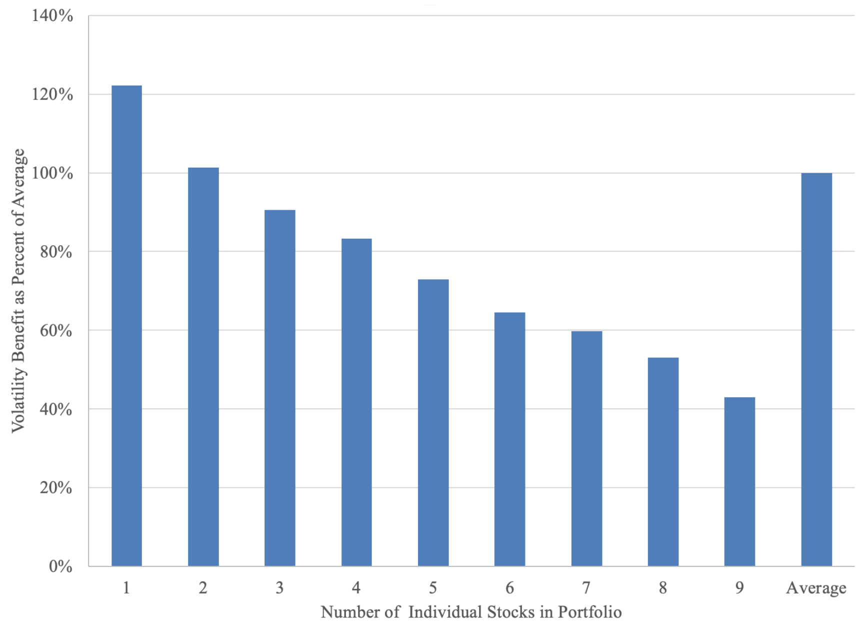


Figure 14.1.23 summarizes the marginal influence on rolling volatility benefit with adding additional sector ETFs in one bar chart. As the number of sector ETFs increase, we see the average of equally weighted portfolio rolling volatility benefit trending toward zero and even going negative.

Figure 14.1.23. Sector ETFs Average Rolling Portfolio Volatility Benefit

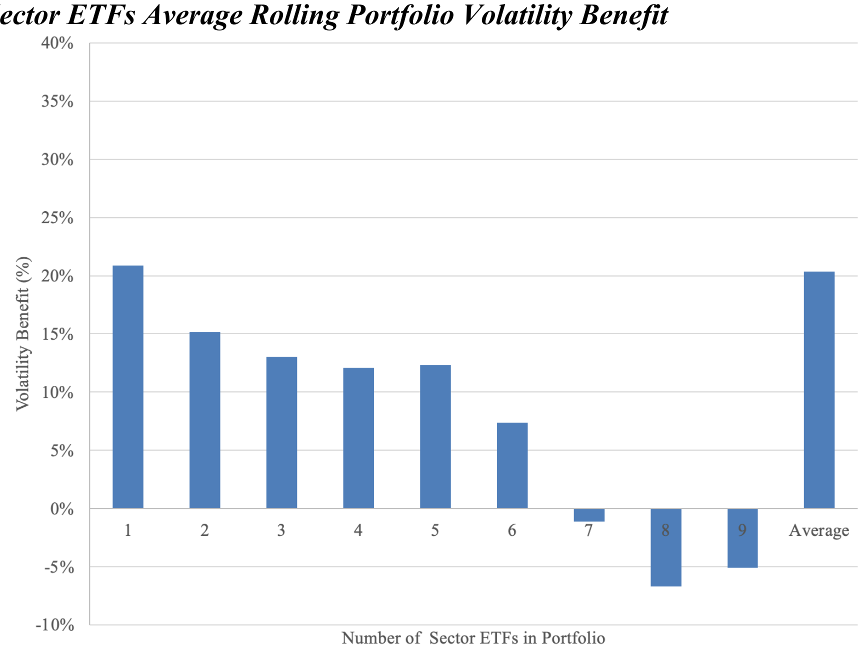
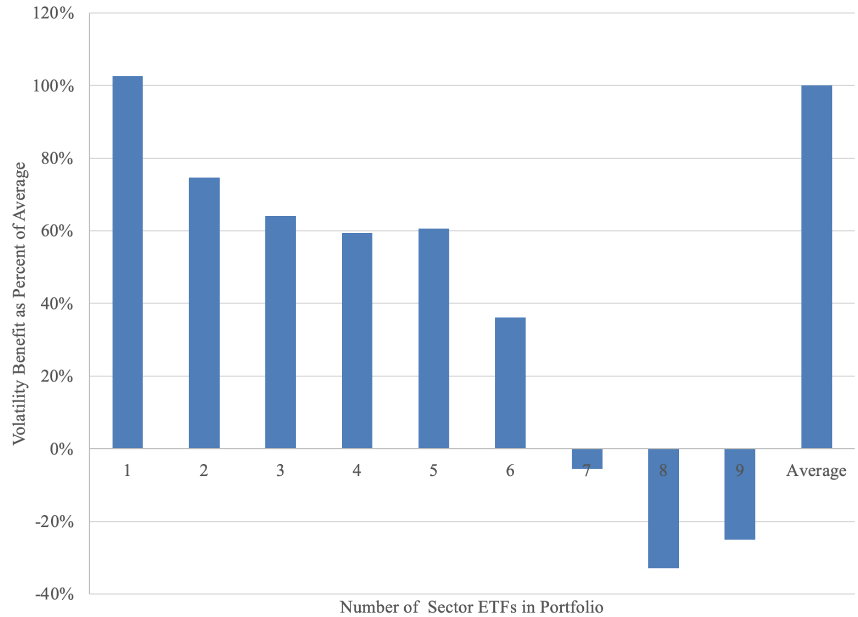


Figure 14.1.24 again summarizes the prior results as a percentage of the average of the marginal influence on rolling volatility benefits with adding additional sector ETFs in one bar chart. Again, as the number of sector ETFs increase, we see this percentage drops significantly and even turns negative.

Figure 14.1.24. Sector ETFs Percent of SPY Average Rolling Portfolio Volatility Benefit



Again, we have four key insights for sector ETFs: First, sector ETF risk or even average of sector ETF risk overstates portfolio uncertainty. Second, risk reduction with ETFs is marginal. Third, marginal diversification benefit declines as number of sector ETFs increases and eventually increases. Finally, even with sector ETF diversification, significant risk remains in portfolios.

We now turn to examine selected quantitative finance materials related to portfolio risk management.

Quantitative finance materials

We now provide more details related to diversification benefits.

Diversification Benefit

Recall the diversification benefits (DB) can be expressed in terms of reduction of risk, the volatility benefit (VB), or in terms of contribution to the expected return, the return due to diversification (RDD). We provide the mathematical framework for both DBs.

Volatility Benefit

Suppose we had just two asset classes (say stocks and bonds) and two annual historic returns are given in Table 14.1.1 along with a portfolio of 37.5% stock (asset class 1) and 62.5% bonds (asset class 2). Notice that these weights result in a portfolio that has no risk historically. How can we measure the benefits achieved by diversification? One way is to document the reduction in risk through the statistical measure of standard deviation. The standard deviation of the portfolio is zero compared with 25% and 15% for asset class 1 and 2, respectively. Table 14.1.1 reports just the discretely compounded returns.

TABLE 14.1.1. Basic Data for Illustration of Concepts

	Asset Class 1	Asset Class 2	Portfolio
Portfolio Weights (%)	37.50%	62.50%	100.00%
Period 1	40%	–10%	8.75%
Period 2	–10%	20%	8.75%
Average Returns (%)	15.00%	5.00%	8.75%
Standard Deviations (%)	25.00%	15.00%	0.00%

One measure of DB is the volatility benefit. Due to the covariance term related to the variance of the portfolio, the standard deviation of a portfolio is less than the weighted average of the standard deviations of each position within the portfolio. Thus, we define the DB in terms of volatility as the volatility benefit (*VB*) as

$$VB_{\pi,t} = 1 - \frac{\sigma_{\pi,t}}{\sum_{i=1}^{N} w_{i,t}\sigma_{i,t}}. \tag{14.1.1}$$

Thus, *VB* is the percentage gain over the weighted average standard deviation of instruments within the portfolio. In the illustration given in Table 1a, the *VB* is equal to 100% as the standard deviation of the portfolio is zero by design or

$$VB_{\pi,t} = 1 - \frac{\sigma_{\pi,t}}{\sum_{i=1}^{N} w_{i,t}\sigma_{i,t}} = 1 - \frac{0}{0.375(0.25) + 0.6250(0.15)} = 1.0.$$

Return due to diversification
Table 14.1.2 provides additional calculations highlighting the distinction between discrete compounding and continuous compounding.

TABLE 14.1.2. Expanded Data for Illustration of Concepts

Asset Class	**1**			**2**			**Portfolio**		
	Value	**r_d**	**r_c**	**Value**	**r_d**	**r_c**	**Value**	**r_d**	**r_c**
Weights (%)	37.50			62.50			100		
Period 0	\$100			\$100			\$100		
Period 1	\$140	40%	33.6472%	\$90	–10%	–10.5361%	\$108.75	8.75%	8.388%
Period 2	\$126	–10%	–10.5361%	108	20%	18.2322%	\$118.2656	8.75%	8.388%
Average (%)		15%	11.5555%		5%	3.84805%		8.75%	8.388%
St. Dev. (%)		25%			15%			0%	0%

One well-known phenomena in measuring rates of return is that the arithmetic average rate of return (labeled Average Returns (%) in Table 14.1.1 or r_d in Table 14.1.2 above) is biased high compared to how much an investor actually earns from the investment. The appropriate measure of true earning is the geometric average of discretely compounded returns. These two measures of return are illustrated for asset class 1:

$$\text{Arithmetic Average: } AR_1 = \frac{1}{n}\sum_{t=1}^{n} R_{1,t} = \frac{1}{2}\left[0.4 + (-0.1)\right] = 0.15 \text{ and} \tag{14.1.2}$$

$$\text{Geometric Average: } GR_1 = \left[\prod_{t=1}^{n}\left(1 + R_{1,t}\right)\right]^{1/n} - 1 = \left\{(1+0.4)\left[1+(-0.1)\right]\right\}^{1/2} - 1 = 0.12249722. \tag{14.1.3}$$

where $R_{i,t}$ denotes the rate of return on asset class 1 over time period 1, *n* denotes the number of historical periods, Σ denotes the summation operator, and $\prod$ denotes the multiplication operator. The geometric average measures the true growth rate of a \$1 investment in asset class 1. One dollar invested in the first period would accumulate to \$1.4 [\$1(1+0.4)] and the \$1.4 invested in the second period would accumulate to \$1.26 {\$1.4[1+(–0.1)]}. This is precisely the cumulative value of \$1 if it earned 12.249722% each year [\$1(1+0.12249722)(1+0.12249722)=\$1.26]. The \$1 invested at the arithmetic average, however, would accumulate to \$1.3225 [\$1(1+0.15)(1+0.15)].

Note that the geometric average can be computed with average continuously compounded returns. From Table 14.1.2, we note the average continuously compounded for asset class 1 is 11.5555% [=(0.336472) +(–0.105361)]. The geometric average can be expressed as

$$GR_1 = e^{\left(\sum_{t=1}^{n} r_{c,1,t}\right)/n} - 1 = e^{(0.336472 - 0.105361)/2} - 1 = 0.12249625. \quad (14.1.4)$$

The arithmetic average is biased high when measuring the growth of an investment over time. The arithmetic average is an unbiased estimate of the single period rate of return. For asset class 1, the actual bias ($ActBias_1$) is:

$$ActBias_1 = AR_1 - GR_1 = 0.15 - 0.12249722 = 0.02750278. \quad (14.1.5)$$

The magnitude of this bias can be estimated by the following equation:[1]

$$EstBias_1 = AR_1 - \ln\left(1 + AR_1\right) + \frac{\sigma_1^2}{2\left(1 + AR_1\right)^2}, \quad (14.1.6)$$

which in our case is

$$EstBias_1 = 0.15 - \ln(1 + 0.15) + 0.25^2/[2(1+0.15)^2]$$
$$= 0.15 - 0.13976194 + 0.02362949 = 0.03386755.$$

The difference between the actual bias and the estimated bias is 0.00636477 (0.03386755 – 0.02750278) or 0.636477%. Although this error may seem very high, when working with actual monthly data this estimation error is dramatically less (around 0.4 basis points or 0.004%).

An alternative way to represent this estimation of the bias is estimating the geometric rate of return based on the arithmetic rate of return and the standard deviation. That is, the estimated geometric return is:

$$EstGR_1 = \ln\left(1 + AR_1\right) - \frac{\sigma_1^2}{2\left(1 + AR_1\right)^2} = \ln\left(1 + 0.15\right) - \frac{0.25^2}{2\left(1 + 0.15\right)^2} = 0.11613245. \quad (14.1.7)$$

From this expression of the estimated geometric rate of return we observe that the magnitude of the difference between the arithmetic return and geometric return is due, in part, to the standard deviation. Hence, we can estimate the geometric return of a portfolio and then identify how much contribution each security is making to the geometric return of the portfolio. If we define the return contribution of asset class j as

$$RC_j = \frac{AR_j}{AR_p} \ln\left(1 + AR_p\right) - \frac{\operatorname{cov}\left(R_j, R_p\right)}{2\left(1 + AR_p\right)^2}, \quad (14.1.8)$$

then it can be shown that the estimated geometric return is (or the return contribution of the portfolio):

$$RC_p = \sum_{j=1}^{n} w_j RC_j = \sum_{j=1}^{n} w_j \left[\frac{AR_j}{AR_p} \ln\left(1 + AR_p\right) - \frac{\operatorname{cov}\left(R_j, R_p\right)}{2\left(1 + AR_p\right)^2}\right] = \ln\left(1 + AR_p\right) - \frac{\sigma_p^2}{2\left(1 + AR_p\right)^2}. \quad (14.1.9)$$

Table 14.1.3 illustrates several insights from this simple two-period example. First, because the standard deviation is zero for the portfolio, the geometric average is equal to the arithmetic average (8.75%). The weighted average of the geometric returns ($WAGR_p$) can be expressed as

$$WAGR_p = \sum_{j=1}^{n} w_j GR_j. \quad (14.1.10)$$

and in this example is 7.045551% [= 0.375(0.12249722) + 0.625(0.03923048)] that is dramatically different than the actual performance of the portfolio. The difference between 8.75% (actual return) and 7.045551%

[1] See Booth and Fama (1992).

(average of each asset class return) represents one measure of the portfolio return attributable to diversification. In this example, the return due to diversification for the portfolio is 1.704449%.

Table 14.1.3. Decomposing Estimated Geometric Returns with Simple Example

	Asset Class 1	**Asset Class 2**	**Portfolio**
Geometric Return	12.249722%	3.923048%	8.750000%
Weighted Average Geometric Return			7.045551%
Return Due to Diversification	2.129961%	0.870179%	1.704449%
Return Contribution	14.379683%	4.793228%	8.388148%
Weighted Average Return Due to Diversification			1.342598%

Decomposing the return contribution for each asset class (see the Return Contribution Row in Table 2), affords us the ability to compute the additional return above the asset class geometric return that is directly attributable to diversification. The Return Due to Diversification (RDD_j) is simply the difference between the Return Contribution (RC_j) and the Geometric Return (GR_j) for each asset class. For the portfolio, the Return Due to Diversification (RDD_p) is the difference between the Geometric Return (GR_p) on the portfolio and the $WAGR_p$. That is,

$$RDD_j = RC_j - GR_j \text{ and} \tag{14.1.11}$$

$$RDD_p = GR_p - WAGR_p. \tag{14.1.12}$$

In practice, the RDD_p (1.704449%) and the weighted average return due to diversification (1.342598%) are approximately equal. The dramatic difference here is due to using annual data and extreme diversification levels.

We now turn to a typical asset allocation across five asset classes using monthly historical data from an approximately 25 year period. Table 14.1.4 provides the salient information. We assume an equally-weighted portfolio. There are several interesting observations. First, notice that the approximation of the geometric returns (Return Contribution–Portfolio 11.0944%) is very close to the actual geometric returns (11.0907%). Hence our approximation method is fairly accurate in practice with monthly returns. Second, notice that the geometric returns are all below the arithmetic annual returns and the magnitude of this difference is related to the standard deviation. The higher standard deviation asset classes have higher differences.

Table 14.1.4. Return to Diversification with Five Asset Classes

	Equity Fund	Corp Bond Fund	Interm. Bond	Internat. Fund	Small Cap Fund	Portfolio
Portfolio Weights (%)	20.00%	20.00%	20.00%	20.00%	20.00%	100.00%
Monthly Average Returns (%)	1.1240%	0.7426%	0.7230%	1.0669%	1.2043%	0.9722%
Monthly Standard Deviations (%)	4.4281%	1.8602%	1.7638%	5.0164%	6.4473%	2.9647%
Annualized Arithmetic Return	13.4882%	8.9113%	8.6765%	12.8031%	14.4522%	11.6662%
Annualized Standard Deviation	15.3392%	6.4438%	6.1101%	17.3772%	22.3340%	10.2700%
Annualized Geometric Return	12.2498%	8.6755%	8.4633%	11.2526%	11.8543%	11.0907%
Weighted Average Geometric Return						10.4991%
Return Due to Diversification	0.4820%	0.0229%	0.0380%	0.8693%	1.5641%	0.5916%
Return Contribution	12.7318%	8.6984%	8.5014%	12.1220%	13.4183%	11.0944%
Covar(j, Portfolio)	0.001175	0.000289	0.000226	0.001052	0.001638	0.000876
Weighted Average Return Due to Diversification						0.59526%

With this monthly data, RDD_p (0.5916%) and the weighted average return due to diversification (0.59526%) are approximately equal. This small difference is typical with real world data.

Summary

In this module, we introduced the notion of the measurable benefits from diversification. Specifically, we quantified in terms in a few ways the benefits of lower risk. We also provided extensive graphical analysis of selected individual stocks and ETFs.

References

Booth, David G. and Eugene F. Fama, "Diversification Returns and Asset Contributions," *Financial Analysts Journal* (May-June 1992), p. 26-32.