Portfolio Issues

Module 14.3

Risk Attribution

1

Risk Attribution

- Risk attribution decomposes the total variance of excess return into a variety of statistics including percentage marginal contribution to risk by stock, sector, sector allocation decision, security selection decision, and interaction
  - Numerous applications
  - Aid in investment decision-making

2

Some Basics

- Variance of portfolio representations

$$\sigma_{\Pi}^2 \equiv E\left\{\left[\tilde{R}_{\Pi} - E\left(\tilde{R}_{\Pi}\right)\right]^2\right\} = Cov\left(\tilde{R}_{\Pi}, \tilde{R}_{\Pi}\right) = Cov\left(\sum_{j=1}^{n} w_j \tilde{R}_j, \tilde{R}_{\Pi}\right) = \sum_{j=1}^{n} w_j Cov\left(\tilde{R}_j, \tilde{R}_{\Pi}\right)$$

- Marginal contribution to risk (MCTR)

$$MCTR_j = w_j Cov\left(\tilde{R}_j, \tilde{R}_{\Pi}\right)$$

- Percentage MCTR (%MCTR): $\%MCTR_j = w_j \beta_{j,\Pi}$

$$1 = \frac{\sum_{j=1}^{n} MCTR_j}{\sigma_{\Pi}^2} = \frac{\sum_{j=1}^{n} w_j Cov\left(\tilde{R}_j, \tilde{R}_{\Pi}\right)}{\sigma_{\Pi}^2} = \sum_{j=1}^{n} w_j \beta_{j,\Pi} = \sum_{j=1}^{n} \%MCTR_j$$

$$\beta_{j,\Pi} = \frac{Cov\left(\tilde{R}_j, \tilde{R}_{\Pi}\right)}{\sigma_{\Pi}^2}$$

3

Selected Variance Properties

$$\sigma^2\left(\tilde{R}_M\right) = \sum_{j=1}^{n_M} w_{M,j} \operatorname{cov}\left(\tilde{R}_j, \tilde{R}_M\right) = \sum_{k=1}^{K} \operatorname{cov}\left(SA\tilde{R}_{M,k}, \tilde{R}_M\right) = \sum_{k=1}^{K} SAW_{M,k} \operatorname{cov}\left(\tilde{R}_{M,k}, \tilde{R}_M\right)$$

$$\sigma^2\left(\tilde{R}_B\right) = \sum_{j=1}^{n_B} w_{B,j} \operatorname{cov}\left(\tilde{R}_j, \tilde{R}_B\right) = \sum_{k=1}^{K} \operatorname{cov}\left(SA\tilde{R}_{B,k}, \tilde{R}_B\right) = \sum_{k=1}^{K} SAW_{B,k} \operatorname{cov}\left(\tilde{R}_{B,k}, \tilde{R}_B\right)$$

$$\sigma^2(ER) = \operatorname{cov}\left(\tilde{R}_M - \tilde{R}_B, ER\right) = \sum_{j=1}^{n_M} w_{M,j} \operatorname{cov}\left(\tilde{R}_j, ER\right) - \sum_{j=1}^{n_B} w_{B,j} \operatorname{cov}\left(\tilde{R}_j, ER\right)$$

$$= \operatorname{cov}(SAD + SSD + I, ER) = \operatorname{cov}(SAD, ER) + \operatorname{cov}(SSD, ER) + \operatorname{cov}(I, ER)$$

4

Excess Return Risk $\left(\sigma_{ER}^2 > 0\right)$

- Excess return risk:

$$1 = \frac{Cov\left(\tilde{R}_M - \tilde{R}_B, E\tilde{R}\right)}{\sigma_{ER}^2} = \frac{Cov\left(\tilde{R}_M, E\tilde{R}\right)}{\sigma_{ER}^2} - \frac{Cov\left(\tilde{R}_B, E\tilde{R}\right)}{\sigma_{ER}^2} = \beta_{M,ER} - \beta_{B,ER}$$

- Decomposition:

$$1 = \frac{Cov\left(S\tilde{A}D + S\tilde{S}D + \tilde{I}, E\tilde{R}\right)}{\sigma_{ER}^2} = \frac{Cov\left(S\tilde{A}D, E\tilde{R}\right)}{\sigma_{ER}^2} + \frac{Cov\left(S\tilde{S}D, E\tilde{R}\right)}{\sigma_{ER}^2} + \frac{Cov\left(\tilde{I}, E\tilde{R}\right)}{\sigma_{ER}^2}$$

$$= \beta_{SAD,ER} + \beta_{SSD,ER} + \beta_{I,ER}$$

5

ER Beta Decompositions

- Sector allocation decision

$$\beta_{SAD,ER} = \frac{Cov\left(S\tilde{A}D, E\tilde{R}\right)}{\sigma_{ER}^2} = \sum_{k=1}^{K} \left(SAW_{M,k} - SAW_{B,k}\right) \beta_{B,k,ER}$$

- Security selection decision

$$\beta_{SSD,ER} = \frac{Cov\left(S\tilde{S}D, E\tilde{R}\right)}{\sigma_{ER}^2} = \sum_{k=1}^{K} SAW_{B,k} \left(\beta_{M,k,ER} - \beta_{B,k,ER}\right)$$

- Interaction

$$\beta_{I,ER} = \frac{Cov\left(\tilde{I}, E\tilde{R}\right)}{\sigma_{ER}^2} = \sum_{k=1}^{K} \left(SAW_{M,k} - SAW_{B,k}\right)\left(\beta_{M,k,ER} - \beta_{B,k,ER}\right)$$

$$\beta_{M,k,ER} = \frac{\operatorname{cov}\left(\tilde{R}_{M,k}, E\tilde{R}\right)}{\sigma_{ER}^2} \quad \beta_{B,k,ER} = \frac{\operatorname{cov}\left(\tilde{R}_{B,k}, E\tilde{R}\right)}{\sigma_{ER}^2} \quad \beta_{SAD,ER} + \beta_{SSD,ER} + \beta_{I,ER} = \beta_{M,ER} - \beta_{B,ER} = 1$$

6

Risk Key Results w/in Sector

- Sector allocation decision

$$\beta_{SAD,k,ER} = \left(SAW_{M,k} - SAW_{B,k}\right)\beta_{B,k,ER}$$

- Security selection decision

$$\beta_{SSD,k,ER} = SAW_{B,k}\left(\beta_{M,k,ER} - \beta_{B,k,ER}\right)$$

- Interaction

$$\beta_{I,k,ER} = \left(SAW_{M,k} - SAW_{B,k}\right)\left(\beta_{M,k,ER} - \beta_{B,k,ER}\right)$$

7

Summary

- Performance attribution identifies strengths and weaknesses in the managerial process
- Return attribution decomposes the reported alpha into various managerial decision categories
- Risk attribution decomposes the excess return variance into various managerial decision categories

8

Research Question

- How will performance attribution change once stochastic liabilities are introduced?
  - Example 1: Small bank
  - Example 2: Defined benefit retirement system
  - Example 3: High net worth family
- See Module 14.4 Keel Model

9