Portfolio Issues

Module 14.2
Return Attribution

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Overview

- Performance attribution – focus on decomposing managerial performance
  - Alpha = Managed return – Benchmark return
  - EV change = Asset change – Liability change
- Return attribution – allocate alpha to various managerial decision
- Risk attribution – allocate excess return variance to various managerial decisions

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Return Attribution

- Return attribution decomposes the total excess return into a variety of statistics including excess return by stock, sector, sector allocation decision, security selection decision, and interaction
  - Numerous applications
  - Aid in investment decision-making

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Basic Identities

- Value of managed and benchmark portfolios

$$\Pi_{M,t} = \sum_{j=1}^{n_M} N_{M,j,t} P_{j,t} \qquad \Pi_{B,t} = \sum_{j=1}^{n_B} N_{B,j,t} P_{j,t}$$

N–number of shares, P–price, n–unique shares

- Return on managed and benchmark portfolios

$$\tilde{R}_{M,t} = \sum_{j=1}^{n_M} w_{M,j,t} \tilde{R}_{j,t} \qquad \tilde{R}_{B,t} = \sum_{j=1}^{n_B} w_{B,j,t} \tilde{R}_{j,t}$$

w–weight, R–discretely compounded, periodic return $w_{M,j,t} = \frac{N_{M,j,t} P_{M,j,t}}{\sum_{j=1}^{n_M} N_{M,j,t} P_{M,j,t}}$ $w_{B,j,t} = \frac{N_{B,j,t} P_{B,j,t}}{\sum_{j=1}^{n_B} N_{B,j,t} P_{B,j,t}}$ $\tilde{R}_{j,t} = \frac{\tilde{P}_{j,t+1}}{P_{j,t}} - 1$

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Return Attribution Example

Figure 14.2.1. Inputs for return attribution

	A	B	C	D	E
1	Performance Attribution				
2	Asset Allocation				
3		Benchmark		Managed	
4		Weights	Return	Weights	Return
5	Stocks	60.00%	6.00%	70.00%	7.00%
6	Bonds	40.00%	3.00%	25.00%	2.50%
7	Cash	0.00%	1.00%	5.00%	1.20%
8	Total Weights	100.00%		100.00%	
9	Return		4.8000%		5.5850%
10	Excess Return				0.7850%

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Return Attribution Example

Figure 14.2.2. Return attribution analysis

	A	B	C	D	E	F	G	H
1	Performance Attribution							
2	Asset Allocation							
3		Benchmark		Managed				
4		Weights	Return	Weights	Return	SAD	SSD	Interaction
5	Stocks	60.00%	6.00%	70.00%	7.00%	0.6000%	0.6000%	0.1000%
6	Bonds	40.00%	3.00%	25.00%	2.50%	-0.4500%	-0.2000%	0.0750%
7	Cash	0.00%	1.00%	5.00%	1.20%	0.0500%	0.0000%	0.0100%
8	Total Weights	100.00%		100.00%				
9	Return		4.8000%		5.5850%	0.2000%	0.4000%	0.1850%
10	Excess Return				0.7850%			0.7850%

SAD – sector allocation decision
SSD – security selection decision

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Excess Return

- Excess return: $ER_t = \tilde{R}_{M,t} - \tilde{R}_{B,t} = \sum_{j=1}^{n_M} w_{M,j,t}\tilde{R}_{j,t} - \sum_{j=1}^{n_B} w_{B,j,t}\tilde{R}_{j,t}$
- Excess return decomposed:

$$ER_t = SAD_t + SSD_t + I_t = \sum_{k=1}^{K}\left(SAW_{M,k,t} - SAW_{B,k,t}\right)\tilde{R}_{B,k,t}$$

$$+\sum_{k=1}^{K} SAW_{B,k,t}\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right) + \sum_{k=1}^{K}\left(SAW_{M,k,t} - SAW_{B,k,t}\right)\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right)$$

$$= \sum_{k=1}^{K} SAW_{M,k,t}\tilde{R}_{M,k,t} - SAW_{B,k,t}\tilde{R}_{B,k,t}$$

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Raw Attribution

- Excess sector allocation weights (EW)

$$EW_{k,t} = \left(SAW_{M,k,t} - SAW_{B,k,t}\right); t, k$$

  - t – period of time; k – sector
  - M – manager; B – benchmark
  - SAW – sector allocation weight

	SAW(B)	SAW(M)	EW
Stocks	60.00%	70.00%	10.00%
Bonds	40.00%	25.00%	-15.00%
Cash	0.00%	5.00%	5.00%
Total Weights	100.00%	100.00%	0.00%

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Sector Allocation Weights

- SAW: $SAW_{M,k,t} = \sum_{j=1}^{n_M} w_{M,j,t} I_{\text{Sector}_k}(j) \quad SAW_{B,k,t} = \sum_{j=1}^{n_B} w_{B,j,t} I_{\text{Sector}_k}(j)$
- Indicator function: $I_{\text{Sector}_k}(j)$= 1 if in sector else 0
- Identities: $1.0 = \sum_{k=1}^{K} SAW_{M,k,t} \quad 1.0 = \sum_{k=1}^{K} SAW_{B,k,t}$
- Weighted average return for each sector

$$\tilde{R}_{M,k,t} = \sum_{j=1}^{n_M}\left(\frac{w_{M,j,t}}{SAW_{M,k,t}}\right)\tilde{R}_{j,t} I_{\text{Sector}_k}(j) \qquad \tilde{R}_{B,k,t} = \sum_{j=1}^{n_B}\left(\frac{w_{B,j,t}}{SAW_{B,k,t}}\right)\tilde{R}_{j,t} I_{\text{Sector}_k}(j)$$

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Excess Return Attribution

- Excess sector return (ER)

$$ER_{k,t} = \left(R_{M,k,t} - R_{B,k,t}\right); t, k$$

- Sector allocation decision (SAD)

$$SAD_{k,t} = \left(SAW_{M,k,t} - SAW_{B,k,t}\right)\tilde{R}_{B,k,t}$$

- Security selection decision (SSD)

$$SSD_{k,t} = SAW_{B,k,t}\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right)$$

- Interaction (I)

$$I_{k,t} = \left(SAW_{M,k,t} - SAW_{B,k,t}\right)\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right)$$

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Time Series Problem

Excess return is only additive across time (t) when forced with a time series adjustment term. The decomposition terms (sector allocation decision SAD, security selection decision SSD, and interaction I) are additive across sectors (k) at a point in time:

$$ER = SAD + SSD + I + I_{TS} = \sum_{t=1}^{T} ER_t = \sum_{t=1}^{T} SAD_t + \sum_{t=1}^{T} SSD_t + \sum_{t=1}^{T} I_t + I_{TS},$$

where

$$SAD_t = \sum_{k=1}^{K} SAD_{k,t},$$

$$SSD_t = \sum_{k=1}^{K} SSD_{k,t}, \text{ and}$$

$$I_t = \sum_{k=1}^{K} I_{k,t}.$$

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Time Series Solution

We define the time series adjustment term as simply a plug figure to account for the non-additivity of the decomposition process or

$$I_{TS} = ER - SAD - SSD - I = \sum_{t=1}^{T} ER_t - \sum_{t=1}^{T} SAD_t - \sum_{t=1}^{T} SSD_t - \sum_{t=1}^{T} I_t.$$

Hence for each measurement period (e.g., a quarter), reported statistics include excess return (by subperiods and by sectors) and allocation decisions (also by subperiods and by sectors).

$$ER_t = SAD_t + SSD_t + I_t = \sum_{k=1}^{K}\left(SAW_{M,k,t} - SAW_{B,k,t}\right)\tilde{R}_{B,k,t}$$

$$+\sum_{k=1}^{K} SAW_{B,k,t}\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right) + \sum_{k=1}^{K}\left(SAW_{M,k,t} - SAW_{B,k,t}\right)\left(\tilde{R}_{M,k,t} - \tilde{R}_{B,k,t}\right).$$

$$= \sum_{k=1}^{K} SAW_{M,k,t}\tilde{R}_{M,k,t} - \sum_{k=1}^{K} SAW_{B,k,t}\tilde{R}_{B,k,t}$$

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Return Identities

$$\tilde{R}_M = \sum_{j=1}^{n_M} w_{M,j}\tilde{R}_j = \sum_{k=1}^{K} SA\tilde{R}_{M,k} = \sum_{k=1}^{K} SAW_{M,k}\tilde{R}_{M,k} \quad \text{return on managed portfolio}$$

$$SA\tilde{R}_{M,k} = SAW_{M,k}\tilde{R}_{M,k} \quad \text{managed, weight-adjusted, sector allocation return}$$

$$\tilde{R}_B = \sum_{j=1}^{n_B} w_{B,j}\tilde{R}_j = \sum_{k=1}^{K} SA\tilde{R}_{B,k} = \sum_{k=1}^{K} SAW_{B,k}\tilde{R}_{B,k} \quad \text{return on benchmark portfolio}$$

$$SA\tilde{R}_{B,k} = SAW_{B,k}\tilde{R}_{B,k} \quad \text{benchmark, weight-adjusted, sector allocation return}$$

Summary

- Return attribution decomposes the reported alpha into various managerial decision categories
- Two common categories
  - Sector allocation decision
  - Security selection decision
- Quantitative details explored

