

## Module 13.1

### Dynamic Risk Measures Stock Index Futures

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## Overview

- Carry arbitrage model
  - Multivariate simulations
  - Multiple strategies
  - Focus on correlations
- Role of collateral
- Interpreting role of correlation
- Holding period return-based VaR



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## Carry Arbitrage Model

- Carry arbitrage: Underlying instrument is purchased and thus 'carried' through time
- Reverse carry arbitrage: Underlying instrument is sold short
- CAM: Futures price is the future value of the current spot price where the implied rate is based on the all-in carrying cost of the underlying instrument, including financing



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## Margin and Collateral

- Futures margins are minimal
  - Initial and maintenance margins poor measure of required capital
  - Additional cash required to support position
- Cash collateral forms basis for holding period returns
  - Higher cash collateral, lower HPRs
  - Explore various implied risk measures



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## Stock Index Parameters

- Stock index value (S) = 100
- Level all-in carry cost (LCC) = 4%
- Slope all-in carry cost (SCC) = 0%
- Nearby maturity = 0.25 years
- Quarterly cycle
- LSC model scalar = 0.5
- Equivalent cash collateral (ECC) = 10%



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## Simulation Parameters

- VaR horizon = 1 month
- Number of simulations = 100,000
- Means: Stock index growth rate (g) = 10%, LCC = 0, SCC = 0
- Standard deviations: g = 20%, LCC = 10, SCC = 20
- Correlations: g, LCC = -0.3; g, SCC = 0.5; LCC, SCC = -0.5



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## Strategies Considered

1. Long stock index (LS)
2. Long stock index futures nearby contract (LSFN1)
3. Long stock index futures second nearby contract (LSFN2)
4. Long stock index futures third nearby contract (LSFN3)
5. Short stock index futures nearby contract (SSFN1)
6. Short stock index futures second nearby contract (SSFN2)
7. Short stock index futures third nearby contract (SSFN3)
8. Long stock index futures spread trade (version 1), long nearby contract and short second nearby contract (only one maturity apart, LST1N1)
9. Long stock index futures spread trade (version 1), long second nearby contract and short third nearby contract (only one maturity apart, LST1N2)
10. Long stock index futures spread trade (version 1), long third nearby contract and short fourth nearby contract (only one maturity apart, LST1N3)



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## Strategies Considered

11. Short stock index futures spread trade (version 1), short nearby contract and long second nearby contract (only one maturity apart, SST1N1)
12. Short stock index futures spread trade (version 1), short second nearby contract and long third nearby contract (only one maturity apart, SST1N2)
13. Short stock index futures spread trade (version 1), short third nearby contract and long fourth nearby contract (only one maturity apart, SST1N3)
14. Long stock index futures spread trade (version 2), long first nearby contract and short third nearby contract (two maturities apart, LST2N1)
15. Long stock index futures spread trade (version 2), long first nearby contract and short fourth nearby contract (three maturities apart, LST2N2)
16. Long stock index futures spread trade (version 2), long first nearby contract and short fifth nearby contract (four maturities apart, LST2N3)
17. Short stock index futures spread trade (version 2), short first contract and long third nearby contract (two maturities apart, SST2N1)
18. Short stock index futures spread trade (version 2), short first nearby contract and long fourth nearby contract (three maturities apart, SST2N2)
19. Short stock index futures spread trade (version 2), short first nearby contract and long fifth nearby contract (four maturities apart, SST2N3)

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**Table 13.1.1 Return VaR Based CAM Stock Index Futures Stock and Level Factor Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.73	29.58	30.02	30.06	29.83	29.68	30.14
LSFN1	32.97	32.81	33.28	33.31	33.09	32.96	33.41
LSFN2	32.97	32.83	33.29	33.33	33.13	33.01	33.47
LSFN3	32.95	32.83	33.30	33.35	33.16	33.04	33.52
SSFN1	45.17	45.37	45.04	45.33	45.39	45.29	45.47
SSFN2	45.58	45.83	45.50	45.82	45.89	45.80	45.99
SSFN3	46.02	46.28	45.95	46.31	46.39	46.30	46.50
LST1N1	0.45	0.46	0.47	0.49	0.50	0.51	0.52
LST1N2	0.44	0.45	0.47	0.48	0.49	0.50	0.51
LST1N3	0.44	0.45	0.46	0.48	0.49	0.50	0.51
SST1N1	0.33	0.34	0.35	0.37	0.37	0.38	0.40
SST1N2	0.32	0.33	0.35	0.36	0.37	0.38	0.39
SST1N3	0.32	0.33	0.34	0.36	0.37	0.38	0.39
LST2N1	0.89	0.92	0.94	0.97	0.99	1.02	1.03
LST2N2	1.32	1.36	1.40	1.45	1.49	1.52	1.55
LST2N3	1.76	1.81	1.86	1.93	1.98	2.02	2.06
SST2N1	0.65	0.67	0.70	0.73	0.74	0.76	0.79
SST2N2	0.96	1.00	1.05	1.08	1.11	1.14	1.18
SST2N3	1.28	1.33	1.39	1.44	1.48	1.51	1.57



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**Table 13.1.2 Return VaR Based CAM Stock Index Futures Stock and Slope Factor Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.83	30.29	30.05	30.07	30.19	29.81	29.95
LCXL	33.01	33.49	33.25	33.29	33.42	33.06	33.20
LCX	32.96	33.44	33.22	33.28	33.40	33.08	33.22
LCXH	32.92	33.40	33.20	33.27	33.42	33.08	33.24
LPXL	45.27	45.54	45.21	45.40	44.85	45.06	45.39
LPX	45.67	45.95	45.63	45.82	45.29	45.53	45.86
LPXH	46.07	46.37	46.04	46.26	45.74	45.99	46.33
LCCWXL	0.40	0.42	0.43	0.44	0.46	0.47	0.49
LCCWX	0.42	0.43	0.44	0.44	0.45	0.46	0.47
LCCWXH	0.43	0.44	0.44	0.45	0.45	0.46	0.47
LPPBXL	0.28	0.30	0.31	0.33	0.34	0.35	0.37
LPPBX	0.30	0.31	0.32	0.33	0.34	0.34	0.35
LPPBXH	0.32	0.32	0.32	0.33	0.34	0.34	0.35
LLCXL	0.82	0.84	0.87	0.89	0.91	0.93	0.96
LLCX	1.25	1.28	1.31	1.33	1.36	1.39	1.43
LLCXH	1.70	1.73	1.76	1.79	1.82	1.85	1.89
LLPXL	0.58	0.61	0.63	0.65	0.68	0.69	0.72
LLPX	0.90	0.93	0.95	0.99	1.02	1.03	1.07
LLPXH	1.22	1.26	1.28	1.32	1.36	1.37	1.41



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**Table 13.1.3 Return VaR Based CAM Stock Index Futures Level and Slope Factor Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.76	30.61	29.87	30.16	30.12	29.76	30.24
LSFN1	33.00	33.86	33.13	33.40	33.37	32.99	33.49
LSFN2	33.01	33.86	33.14	33.42	33.39	32.98	33.52
LSFN3	33.02	33.86	33.14	33.41	33.41	32.97	33.53
SSFN1	45.10	45.12	45.33	44.94	45.39	45.10	45.07
SSFN2	45.57	45.57	45.78	45.40	45.86	45.54	45.54
SSFN3	46.03	46.04	46.25	45.87	46.32	46.01	46.00
LST1N1	0.47	0.47	0.47	0.47	0.48	0.48	0.48
LST1N2	0.46	0.46	0.47	0.46	0.47	0.47	0.47
LST1N3	0.46	0.46	0.46	0.46	0.47	0.46	0.46
SST1N1	0.35	0.36	0.35	0.36	0.36	0.36	0.36
SST1N2	0.34	0.35	0.34	0.35	0.35	0.35	0.35
SST1N3	0.34	0.35	0.34	0.34	0.34	0.34	0.35
LST2N1	0.93	0.93	0.94	0.93	0.95	0.95	0.94
LST2N2	1.39	1.39	1.40	1.39	1.41	1.41	1.41
LST2N3	1.85	1.86	1.87	1.85	1.88	1.87	1.87
SST2N1	0.69	0.71	0.70	0.71	0.71	0.71	0.71
SST2N2	1.03	1.06	1.04	1.05	1.05	1.05	1.06
SST2N3	1.37	1.40	1.38	1.40	1.40	1.39	1.41



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**Table 13.1.4 Return VaR Based CAM Stock Index Futures Margin Requirements**  
Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Margin (%)	10	20	30	40	50	60	70	80	90	100
LS	29.69	14.89	9.99	7.41	6.02	4.98	4.30	3.77	3.34	2.99
LSFN1	32.94	16.51	11.08	8.22	6.67	5.53	4.77	4.18	3.70	3.31
LSFN2	32.96	16.53	11.08	8.23	6.67	5.53	4.77	4.18	3.70	3.32
LSFN3	32.97	16.53	11.08	8.23	6.67	5.53	4.77	4.18	3.70	3.32
SSFN1	45.16	22.71	15.26	11.35	9.04	7.49	6.42	5.68	5.03	4.50
SSFN2	45.63	22.95	15.42	11.47	9.14	7.57	6.49	5.74	5.09	4.55
SSFN3	46.09	23.17	15.57	11.59	9.23	7.64	6.55	5.80	5.14	4.59
LST1N1	0.47	0.24	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
LST1N2	0.46	0.23	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
LST1N3	0.46	0.23	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
SST1N1	0.35	0.18	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.04
SST1N2	0.34	0.17	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.03
SST1N3	0.34	0.17	0.11	0.08	0.07	0.06	0.05	0.04	0.04	0.03
LST2N1	0.93	0.47	0.32	0.24	0.19	0.16	0.13	0.12	0.10	0.09
LST2N2	1.39	0.70	0.47	0.35	0.28	0.23	0.20	0.17	0.16	0.14
LST2N3	1.85	0.93	0.63	0.47	0.37	0.31	0.26	0.23	0.21	0.19
SST2N1	0.69	0.35	0.23	0.17	0.14	0.12	0.10	0.09	0.08	0.07
SST2N2	1.03	0.52	0.35	0.26	0.21	0.17	0.15	0.13	0.12	0.10
SST2N3	1.37	0.69	0.46	0.34	0.28	0.23	0.20	0.17	0.15	0.14



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## Quantitative Finance Materials

- CAM reviewed
- LSC model reviewed
- DRM and model simulated
- Equivalent cash collateral
- Holding period returns for return VaR



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## CAM Model

- Futures price is future value of spot price

$$F_{n,t} = FV_{cc,n}(S_t) \text{ for all } n$$

- $FV_{cc,n}$  – future value operator based on specified carry costs over n periods
- Carry costs include the financing costs and any other associated revenues and costs related to the underlying instrument



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## Stock Index Futures Contracts

- Fully arbitrated market

$$F_0(T) = S_0 e^{(r-\delta)T}$$

- $\delta$  – dividend yield
  - Dividends are rather sticky
  - Dividend yield would rise (fall) when the S&P 500 index falls (rises)



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## LSC Model

- Linear factors:  $y_i = \sum_{j=0}^N x_{i,j} f_j$

- Level:  $x_{i,0} = 1$       Slope:  $x_{i,1} = \frac{S_i}{\tau_i} (1 - e^{-\tau_i/\delta_i})$

- Curvatures:  $x_{i,j} = \frac{S_i}{\tau_i} \left( 1 - e^{-\tau_i/\delta_i} \right) - e^{-\tau_i/\delta_i}; j > 1$

- LSC model has the lowest “average (across the sample) mean (across the curve) absolute yield error” (Steeley) when compared with splines, polynomials and Vasicek’s model



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## DRM and Index Futures

- Futures price simulation model

$$\tilde{F}_{n,t+h} = \tilde{S}_{t+h} \left( \tilde{L}_{cc,t+h} \tilde{S}_{cc,t+h} \tilde{\epsilon}_{t+h} \right) e^{\left[ \tilde{L}_{cc,t+h} + \frac{\delta_1}{\tau_n - h} (1 - e^{-(\tau_n - h)/\delta_1}) \tilde{S}_{cc,t+h} \right] (\tau_n - h)} \text{ for all } n.$$

- Profit/Loss simulation

$$\Delta \tilde{\Pi}_{t,t+h} = \sum_{j=1}^{N_m} N_j \Delta \tilde{F}_{n,t+h} = \sum_{j=1}^{N_m} N_j (\tilde{F}_{n,t+h} - F_{n,t}) \text{ for all } n,$$



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## DRM and Index Futures

- Equivalent cash collateral amount

$$ECC_t = \omega \max \left( \sum_{j=1}^{N_m} N_j F_{n,t} \mid S_t \right).$$

- Holding period return for return VaR

$$R_t = \frac{\Delta \tilde{\Pi}_{t,t+h}}{\omega \max \left( \sum_{j=1}^{N_m} N_j F_{n,t} \mid S_t \right)}.$$



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## Summary

- Reviewed carry arbitrage model
- Simulations to explore role of correlation
- Quantitative framework examined

