

Module 13.3: DRM Interest Rate Swaps

Learning objectives

- Apply Monte Carlo simulation to explore risks that arise from interest rate swaps and evaluate this risk's influence on return value-at-risk.
- Illustrate the insights gained from Monte Carlo simulation with a focus on various correlations measured by return value-at-risk.
- Demonstrate the important role of the correlation between the forward curve and the basis curve (to arrive at the appropriate discount curve and its influence on return value-at-risk).

Executive summary

We focus here on the influence of correlation between several of the LSC parameters (level, slope, and curvature for both the forward curve and the basis curve). We illustrate with Monte Carlo simulation the influence of different correlations. Finally, we address the role of equivalent cash collateral on the return value-at-risk.

Within the quantitative finance materials, we work out the appropriate mathematical relationships required for the simulations.

Central finance concepts

We now briefly review the key finance concepts for interest rate swaps.

Interest rate swap valuation

Recall from Module 6.3 that the interest rate swap's value today can be represented in several ways. We focus here on the fundamental factors driving swap values—the forward curve and the basis curve (to derive the discount curve).

The key to successful development of DRMs is linking the forward curve, say based on a LSC model, with the appropriate discrete forward rate that incorporates the appropriate day count methodology. The important insight is that the LSC-based forward curve is primary and the applied forward rate in valuation is derived from it. Thus, we estimate only one forward curve and a variety of forward rates unique to each swap contract can be derived.

LSC model applied to interest rate swap valuation

The number of DRM variables depends on the number LSC factors assumed for the forward curve and the basis curve used to derive the discount curve. In the quantitative section below, we review the various ways there are to link these two curves. In general, it is inappropriate to assume they are not related. That is, when interest rates rise significantly, then both the forward curve and the discount curve rise.

Functional relationships between the discount curve and the forward curve

Given the high correlation between most interest rates, it again seems unreasonable to assume that movements in the discount curve are unrelated to movements in the forward curve. Establishing this relationship is essential when estimating dynamic risk measures. We need to make one curve primary and the other derived from the primary with the LSC-based basis curve.

At this point, it does not matter whether the discount curve or forward curve is primary. We assume that the forward curve is primary as it is unrelated to the counterparties involved in the swap. For example, historically the most popular forward curve has been tied to Libor although it has now been abandoned. There are numerous forward curves to base the analysis and we sidestep this issue. The discount curve for a particular swap can be estimated based on the spread over or under some base curve depending on the counterparty involved and the collateralization requirements. Often swaps are over-collateralized leading to the discount curving being lower than the selected forward curve.

Collateral

Recall interest rate swaps can take on zero or negative values making holding period return calculations difficult if not impossible. Again, one efficient way to handle this problem is to assume each swap contract has cash collateral allocated to it. It is important to remember that this measure is different from the

traditional margin accounts that support futures contracts. Here, the goal is to establish a cash collateral account that is expected to support the swap contract over the entire life of the swap, not just until the next collateral reset date. Thus, with respect to dynamic risk measures, we are evaluating each swap position in isolation, not in a portfolio context. Further, a positive cash collateral amount will result in rational holding period return calculations. Note that the future swap cash flows will include the return of the cash collateral amount.

Dynamic risk measures and swap contracts

Like other DRMs, we report the influence of various correlations on the return value-at-risk (RVaR). Unless otherwise indicated, we assume the following swap contract parameters:

- Evaluation Date (EM, ED, EY): 12/01/2023 (EM = 12, ED = 1, EY = 2023, three separate variables)
- Maturity Date (MM, MD, MY): 12/01/2028 (MM = 12, MD = 1, MY = 2023, three separate variables, say +5 years)
- Payment Frequency (FixPF, FltPF): FixPF = 2, FltPF = 4
- Number of Accrued Days Indicator Function (FixNAD, FltNAD): FixNAD = 1 (30 days/month), FltNAD = 0 (Act days per month)
- Number of Total Days in Year (FixNTD, FltNTD): FixNAD = 360 (360 days/year), FltNTD = 365 (365 days/year)
- Conversion for Holidays and Week-ends (FixConv, FltConv): FixConv = MBF, FltConv = MBF (Modified Business Following (MBF) or Modified Business Preceding, MBP)
- Notional Amount (NAmt): NAmt = 1,000,000
- Discount Type (DiscountType) = BC (BC denotes two curves, forward curve and basis curve, otherwise, swap based on forward curve = discount curve)
- Fixed Swap Rate (FixedRate) = 5.015139 (or set such that initial swap value is zero)
- Equivalent Cash Collateral (CashCollateral) = 10 (percent of notional amount)
- Number of LSC parameters (NLSCFR, NLSCGR): NLSCFR = 3 (level, slope, and curvature1 for forward rate), NLSCGR = 3 (level, slope, and curvature1 for generic rate, basis curve if DiscountType = BC)
- LSC Parameters (FRParmX, GRParmX, X=1 to 6): FRParm1 = 6 (forward rate level = 6%), FRParm2 = -2 (forward rate slope = -2%, upward sloping), FRParm3 = -1 (forward rate curvature1 = -1%), and the remaining forward rate parameters are 0. Further, all the basis curve parameters, GRParmX, are assume to be zero.
- Scalars (FRScalarX, GRScalarX, X = 1 to 6): LSC scalars corresponding to LSC parameters
- Output: Type of output (Value1 denotes computation of value using two curves)

In this module, we assume a three factor LSC model for both the forward curve and the basis curve. Thus, we assume the following base simulation parameters:

- VaR horizon = 30 days
- Confidence Level = 95%
- Number of simulations = 10,000
- Means (changes in parameters)
 - FDFCLevel = 0.0 (annualized, linear in time)
 - FDFCSlope = 0.0 (annualized, linear in time)
 - FDFCCurvature1 = 0.0 (annualized, linear in time)
 - FDBCLLevel = 0.0 (annualized, linear in time)
 - FDBCSlope = 0.0 (annualized, linear in time)
 - FDBCCurvature1 = 0.0 (annualized, linear in time)
- Standard deviations
 - FDFCLevel = 2.0 (annualized, linear in square root of time)
 - FDFCSlope = 4.0 (annualized, linear in square root of time)
 - FDFCCurvature1 = 6.0 (annualized, linear in square root of time)

- FDBCLevel = 0.5 (annualized, linear in square root of time)
- FDBCSlope = 1.0 (annualized, linear in square root of time)
- FDBCCurvature1 = 2.0 (annualized, linear in square root of time)
- Correlations
 - FDFCLevel, FDFCSlope = -0.75
 - FDFCLevel, FDFCCurvature1 = 0.00
 - FDFCLevel, FDBCLevel = 0.90
 - FDFCLevel, FDBCSlope = 0.75
 - FDFCLevel, FDBCCurvature1 = 0.00
 - FDFCSlope, FDFCCurvature1 = -0.25
 - FDFCSlope, FDBCLevel = 0.75
 - FDFCSlope, FDBCSlope = 0.90
 - FDFCSlope, FDBCCurvature1 = 0.00
 - FDFCCurvature1, FDBCLevel = 0.00
 - FDFCCurvature1, FDBCSlope = 0.00
 - FDFCCurvature1, FDBCCurvature1 = 0.95
 - FDBCLevel, FDBCSlope = -0.75
 - FDBCLevel, FDBCCurvature1 = 0.00
 - FDBCSlope, FDBCCurvature1 = -0.25

Again, it is important to recall the artist analogy in Chapter 1. These parameters were not selected based on an in depth study of a particular interest rate swap market, rather they were chosen simply for illustration purposes. The specific parameterization is not the main issue, rather we focus on the influence of the interaction between various LSC parameters. We are particularly interested in correlations between the forward curve parameters and the basis curve parameters.

We explore 19 strategies focused on different maturities, different horizons for VaR-related calculations, role of fixed rates, and equivalent cash collateral amounts.

1. Underlying swap (initial case) (SIC)
2. Underlying swap with maturity date = 1 year (SMD1Y)
3. Underlying swap with maturity date = 2 year (SMD1Y)
4. Underlying swap with maturity date = 3 year (SMD1Y)
5. Underlying swap with maturity date = 4 year (SMD1Y)
6. Underlying swap with maturity date = 5 year (SMD1Y)
7. Underlying swap with maturity date = 10 year (SMD1Y)
8. Underlying swap with maturity date = 20 year (SMD1Y)
9. Underlying swap with maturity date = 30 year (SMD1Y)
10. Underlying swap with fixed 30/360, Semi and floating ACT/360, Quart: (SDCPV)
11. Underlying swap with both legs 30/360 Semi : (SDC30S)
12. Underlying swap with both legs 30/360 Quart: (SDC30Q)
13. Underlying swap with both legs ACT/360 (SDCACTQ)
14. Underlying swap with 2 percent decrease in fixed rate = -2.0 (SFRM2)
15. Underlying swap with 1 percent decrease in fixed rate = -1.0 (SFRM1)
16. Underlying swap with 1 percent increase in fixed rate = 1.0 (SFRMP1)
17. Underlying swap with 2 percent increase in fixed rate = 2.0 (SFRMP2)
18. Underlying swap with +10 percent increase in cash collateral = 10.0 (SCC10)
19. Underlying swap with +20 percent increase in cash collateral = 20.0 (SCC20)

Table 13.3.1 illustrates the influence of the correlation between forward curve level and basis curve level. As expected, as this correlation increases so does the return VaR. When both the forward curve level and basis curve level move lower, the receive fixed swap loses more value. The longer maturity swaps also have higher return VaRs. For example, with 0.75 correlation the one year swap has a return VaR of 6.49%

whereas the 30 year swap has a return VaR of 104.56%. Finally, when the collateral doubles, the return VaR is cut in half.

Table 13.3.1 Return VaR Based on Forward Curve Level and Basis Curve Level Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 95%

Strategy\Correlation	-0.75	0.00	0.75
SIC	33.69	35.20	38.63
SMD1Y	5.43	5.87	6.49
SMD2Y	12.49	13.52	15.11
SMD3Y	20.35	21.22	23.62
SMD4Y	27.52	28.76	31.65
SMD5Y	33.69	35.20	38.63
SMD10Y	55.78	58.02	64.75
SMD20Y	80.33	81.82	92.36
SMD30Y	91.44	93.15	104.56
SDCPV	33.69	35.20	38.63
SDC30S	33.61	35.12	38.53
SDC30Q	33.42	34.93	38.33
SDCACTQ	37.64	39.15	42.56
SFRM2	31.47	32.96	36.22
SFRM1	32.54	34.08	37.38
SFRP1	34.84	36.42	39.84
SFRP2	36.00	37.42	41.14
SCC10	33.69	35.20	38.63
SCC20	16.85	17.60	19.32

Table 13.3.2 illustrates the influence of the correlation between forward curve slope and basis curve slope. When comparing Table 13.3.1 with 13.3.2, we note that the correlation between slopes has nearly the same impact on return VaR when compared to the correlations between levels. For shorter term swaps, the slopes have a higher return VaR.

Table 13.3.2 Return VaR Based on Forward Curve Slope and Basis Curve Slope Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 95%

Strategy\Correlation	-0.75	0.00	0.75
SIC	38.12	37.57	39.08
SMD1Y	6.51	6.50	7.11
SMD2Y	14.93	14.95	15.64
SMD3Y	23.21	23.29	23.94
SMD4Y	31.13	30.63	31.71
SMD5Y	38.12	37.57	39.08
SMD10Y	61.72	62.51	64.06
SMD20Y	87.97	89.08	91.97
SMD30Y	99.34	100.76	104.12
SDCPV	38.12	37.57	39.08
SDC30S	38.02	37.50	39.00
SDC30Q	37.81	37.29	38.79
SDCACTQ	42.04	41.51	43.02
SFRM2	35.92	35.47	36.48
SFRM1	37.00	36.58	37.82
SFRP1	39.20	38.63	40.22
SFRP2	40.28	39.72	41.36
SCC10	38.12	37.57	39.08
SCC20	19.06	18.79	19.54

Table 13.3.3 illustrates the influence of the correlation between the forward curve curvature and basis curve curvature. Clearly, the curvature correlations do not influence return VaR much, perhaps slightly lower with higher correlations.

Table 13.3.3 Return VaR Based Forward Curve Curvature and Basis Curve Curvature Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	39.04	38.91	38.34
SMD1Y	6.77	6.56	6.80
SMD2Y	15.37	15.22	15.39
SMD3Y	24.15	23.68	23.49
SMD4Y	32.08	31.90	31.13
SMD5Y	39.04	38.91	38.34
SMD10Y	64.07	63.39	63.47
SMD20Y	92.05	91.45	92.47
SMD30Y	104.59	103.53	105.40
SDCPV	39.04	38.91	38.34
SDC30S	38.95	38.92	38.25
SDC30Q	38.75	38.68	38.03
SDCACTQ	42.99	42.89	42.27
SFRM2	36.30	36.54	35.92
SFRM1	37.73	37.74	36.97
SFRP1	40.41	40.31	39.52
SFRP2	41.65	41.45	40.54
SCC10	39.04	38.91	38.34
SCC20	19.52	19.45	19.17

Table 13.3.4 illustrates the influence of the correlation between forward curve level and basis curve slope. The results here are like Table 13.3.1. Again, as expected, as this correlation increases so does the return VaR. When both the forward curve level and basis curve slope move lower, the receive fixed swap loses more value. The longer maturity swaps also have higher return VaRs. For example, with 0.75 correlation the one year swap has a return VaR of 6.95% whereas the 30 year swap has a return VaR of 103.78%.

Table 13.3.4 Return VaR Based Forward Curve Level and Basis Curve Slope Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	34.48	36.38	39.37
SMD1Y	5.51	6.32	6.95
SMD2Y	12.86	14.26	15.74
SMD3Y	20.56	22.11	24.46
SMD4Y	28.04	29.64	32.73
SMD5Y	34.48	36.38	39.37
SMD10Y	56.99	59.28	64.33
SMD20Y	82.11	85.29	91.29
SMD30Y	93.56	97.37	103.78
SDCPV	34.48	36.38	39.37
SDC30S	34.38	36.29	39.24
SDC30Q	34.20	36.08	39.04
SDCACTQ	38.42	40.30	43.30
SFRM2	32.15	33.87	36.63
SFRM1	33.40	35.13	38.02
SFRP1	35.61	37.59	40.69
SFRP2	36.85	38.84	41.97
SCC10	34.48	36.38	39.37
SCC20	17.24	18.19	19.69

Table 13.3.5 illustrates the influence of the correlation between forward curve slope and basis curve level. The results here are like Table 13.3.4. Again, as expected, as this correlation increases so does the return VaR. When both the forward curve slope and basis curve level move lower, the receive fixed swap loses

more value. The longer maturity swaps also have higher return VaRs. For example, with 0.75 correlation the one year swap has a return VaR of 6.82% whereas the 30 year swap has a return VaR of 104.21%.

Table 13.3.5 Return VaR Based Forward Curve Slope and Basis Curve Level Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	37.21	37.39	38.68
SMD1Y	6.69	6.66	6.82
SMD2Y	14.66	14.96	15.34
SMD3Y	22.92	23.29	23.91
SMD4Y	30.50	30.94	31.59
SMD5Y	37.21	37.39	38.68
SMD10Y	61.24	62.84	63.97
SMD20Y	88.00	90.23	91.87
SMD30Y	99.85	102.44	104.21
SDCPV	37.21	37.39	38.68
SDC30S	37.14	37.31	38.56
SDC30Q	36.92	37.11	38.36
SDCACTQ	41.13	41.35	42.60
SFRM2	35.01	35.09	36.19
SFRM1	36.14	36.23	37.42
SFRP1	38.35	38.55	39.85
SFRP2	39.48	39.83	40.94
SCC10	37.21	37.39	38.68
SCC20	18.61	18.70	19.34

Table 13.3.6 illustrates the influence of the correlation between forward curve slope and basis curve curvature. As prior tables, the higher the correlation, the higher the return VaR.

Table 13.3.6 Return VaR Based Forward Curve Slope and Basis Curve Curvature Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	38.87	38.28	41.97
SMD1Y	6.96	6.59	7.55
SMD2Y	15.65	14.87	17.08
SMD3Y	24.17	23.26	26.39
SMD4Y	31.93	31.03	34.78
SMD5Y	38.87	38.28	41.97
SMD10Y	64.22	64.20	66.75
SMD20Y	90.86	92.05	93.41
SMD30Y	103.69	103.76	104.81
SDCPV	38.87	38.28	41.97
SDC30S	38.79	38.27	41.82
SDC30Q	38.57	38.03	41.62
SDCACTQ	42.80	42.23	45.88
SFRM2	36.64	35.92	39.14
SFRM1	37.72	37.14	40.50
SFRP1	40.11	39.62	43.33
SFRP2	41.42	40.72	44.70
SCC10	38.87	38.28	41.97
SCC20	19.43	19.14	20.99

Quantitative finance materials

We first briefly review the interest rate swap valuation model from Modules 6.3 and 9.3 for the purpose of developing potential simulations. We then develop a dynamic risk measures (DRMs) framework for appraising swap portfolio risk.

Interest rate swap valuation

Recall from Module 6.3 that the interest rate swap's value today, V_{Swap} , can be expressed generically as

$$V_{Swap} = \sum_{i'=1}^{N_{Flt}} DF_{Flt,i'} NA_{Flt,i'} \left(\frac{NAD_{Flt,i'}}{NTD_{Flt,i'}} \right) r_{FR,i'-1} - \sum_{i=1}^{N_{Fix}} DF_{Fix,i} NA_{Fix,i} \left(\frac{NAD_{Fix,i}}{NTD_{Fix,i}} \right) r_{Fix}. \quad (13.3.1)$$

where

i, i' counters for fixed cash flows (i) and floating cash flows (j),

$NAD_{Fix,i}, NAD_{Flt,i'}$ number of accrued days for each cash flow (fixed or floating),

$NTD_{Fix,i}, NTD_{Flt,i'}$ number of total days per year for each cash flow (fixed or floating),

$NA_{Fix,i}, NA_{Flt,i'}$ notional amount outstanding for each cash flow (fixed or floating),

$DF_{Fix,i}, DF_{Flt,i'}$ discount factor for each cash flow (fixed or floating, discounting only a function of time),

r_{Fix} annualized fixed rate associated with a particular interest rate swap, and

$r_{FR,i'-1}$ annualized forward rate associated with the j th cash flow (advanced set, settled in arrears).

For holding period returns, we need two points in calendar time. Thus, even though cumbersome, we add an additional superscript t to denote when the LSC model is fit. Thus, the approximate value of the swap at time t can be estimated based on both LSC models (forward curve and basis curve). Module 9.3 develops the following

$$V_{Swap,t}^{LSC,t} = \sum_{i'=1}^{N_{Flt}} TFlt_{1,i',t}^{LSC,t} - \sum_{i'=1}^{N_{Flt}} TFlt_{2,i',t}^{LSC,t} - r_{Fix} \sum_{i=1}^{N_{Fix}} TFix_{1,i,t}^{LSC,t}, \quad (13.3.2)$$

where we have three terms, two related to floating rates and one related to fixed rates,

$$\begin{aligned} TFlt_{1,i',t}^{LSC,t} &= NA_{Flt,i'} e^{-(L_{BC,t} + S_{BC,t} x_{BC,i',S,t} + C_{BC,t} x_{BC,i',C,t})} r_{t,t} e^{-(L_{FC,t} + S_{FC,t} x_{FC,i',S,t} + C_{FC,t} x_{FC,i',C,t})} r_{t-1,t} \\ TFlt_{2,i',t}^{LSC,t} &= NA_{Flt,i'} e^{-[(L_{FC,t} + L_{BC,t}) + (S_{FC,t} x_{FC,i',S,t} + S_{BC,t} x_{BC,i',S,t}) + (C_{FC,t} x_{FC,i',C,t} + C_{BC,t} x_{BC,i',C,t})]} r_{t,t} \\ TFix_{1,i,t}^{LSC,t} &= \hat{A}_{Fix,i} e^{-[(L_{FC,t} + L_{BC,t}) + (S_{FC,t} x_{FC,i,S,t} + S_{BC,t} x_{BC,i,S,t}) + (C_{FC,t} x_{FC,i,C,t} + C_{BC,t} x_{BC,i,C,t})]} r_{t,t} \end{aligned} \quad (13.3.3)$$

Assuming we need to estimate the swap's value after some period of time, h , we thus have

$$\tilde{V}_{Swap,t+h}^{LSC,t+h} = \sum_{i'=1}^{N_{Flt}} \tilde{TFlt}_{1,i',t+h}^{LSC,t+h} - \sum_{i'=1}^{N_{Flt}} \tilde{TFlt}_{2,i',t+h}^{LSC,t+h} - r_{Fix} \sum_{i=1}^{N_{Fix}} \tilde{TFix}_{1,i,t+h}^{LSC,t+h}, \text{ (Swap Value at } t+h \text{)} \quad (13.3.4)$$

where we have three terms (note the LSC factor subscripts are t and the maturity coefficients and maturity time subscripts are $t+h$)

$$\begin{aligned} \tilde{TFlt}_{1,i',t+h}^{LSC,t+h} &= NA_{Flt,i'} e^{-(\tilde{L}_{BC,t+h} + \tilde{S}_{BC,t+h} x_{BC,i',S,t+h} + \tilde{C}_{BC,t+h} x_{BC,i',C,t+h})} r_{t,t+h} e^{-(\tilde{L}_{FC,t+h} + \tilde{S}_{FC,t+h} x_{FC,i',S,t+h} + \tilde{C}_{FC,t+h} x_{FC,i',C,t+h})} r_{t-1,t+h} \\ \tilde{TFlt}_{2,i',t+h}^{LSC,t+h} &= NA_{Flt,i'} e^{-[(\tilde{L}_{FC,t+h} + \tilde{L}_{BC,t+h}) + (\tilde{S}_{FC,t+h} x_{FC,i',S,t+h} + \tilde{S}_{BC,t+h} x_{BC,i',S,t+h}) + (\tilde{C}_{FC,t+h} x_{FC,i',C,t+h} + \tilde{C}_{BC,t+h} x_{BC,i',C,t+h})]} r_{t,t+h} \\ \tilde{TFix}_{1,i,t+h}^{LSC,t+h} &= \hat{A}_{Fix,i} e^{-[(\tilde{L}_{FC,t+h} + \tilde{L}_{BC,t+h}) + (\tilde{S}_{FC,t+h} x_{FC,i,S,t+h} + \tilde{S}_{BC,t+h} x_{BC,i,S,t+h}) + (\tilde{C}_{FC,t+h} x_{FC,i,C,t+h} + \tilde{C}_{BC,t+h} x_{BC,i,C,t+h})]} r_{t,t+h} \end{aligned} \quad (13.3.5)$$

Dynamic risk measures applied to a single swap

Assuming a risk management horizon, h , the horizon swap value can be modeled based on Equation (13.3.4). We assume six correlated risk factors driving the swap value at time $t+h$, the level, slope, and curvature of both the forward curve and the basis curve.

We model changes in the swap contract value as

$$\Delta \tilde{\Pi}_{t,t+h} = \tilde{V}_{Swap,t+h}^{LSC,t+h} - V_{Swap,t}^{LSC,t} \quad (13.3.6)$$

where the level, slope, and curvature of both the forward curve and the basis curve at time h are derived from their values at time t and the simulated changes in these values.

The challenge is developing a rational basis for the cash collateral amount. We assume some equivalent cash collateral amount is provided. Obviously, the holding period returns and thus the return VaR will be highly sensitive to the assumed equivalent cash collateral amount.

Summary

We focused here on the influence of correlation between several of the LSC parameters (level, slope, and curvature for both the forward curve and the basis curve). We illustrated with Monte Carlo simulation the influence of different correlations. Finally, we addressed the role of equivalent cash collateral on the return value-at-risk.

References

See Module 6.3.