Module 13.3

Dynamic Risk Measures
Interest Rate Swaps

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Overview

- Assume two rate curves
  - Forward curve
  - Basis curve (to form discount curve)
- IRS model via LSC
- Monte Carlo simulations
- LSC parameter correlations with focus on return VaR

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IRS Valuation Challenges

- Linking forward curve to forward rates
- Basis curve to link forward curve to discount curve
  - Generally various interest rates move together
  - Seek mechanism to capture this correlation

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Collateral and HPRs

- Because IRS values can range from positive to negative, we need some basis for computing HPRs
- Cash collateral provides the appropriate mechanism
- Holding period returns assume return of cash collateral

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DRM Assumed Parameters

- Evaluation Date (EM, ED, EY): 12/01/2023 (EM = 12, ED = 1, EY = 2023, three separate variables)
- Maturity Date (MM, MD, MY): 12/01/2028 (MM = 12, MD = 1, MY = 2023, three separate variables, say +5 years)
- Payment Frequency (FixPF, FltPF): FixPF = 2, FltPF = 4
- Number of Accrued Days Indicator Function (FixNAD, FltNAD): FixNAD = 1 (30 days/month), FltNAD = 0 (Act days per month)
- Number of Total Days in Year (FixNTD, FltNTD): FixNAD = 360 (360 days/year), FltNTD = 365 (365 days/year)
- Conversion for Holidays and Week-ends (FixConv, FltConv): FixConv = MBF, FltConv = MBF (Modified Business Following (MBF) or Modified Business Preceding, MBP)
- Notional Amount (NAmt): NAmt = 1,000,000
- Discount Type (DiscountType) = BC (BC denotes two curves, forward curve and basis curve, otherwise, swap based on forward curve = discount curve)

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DRM Assumed Parameters

- Fixed Swap Rate (FixedRate) = 5.015139 (or set such that initial swap value is zero)
- Equivalent Cash Collateral (CashCollateral) = 10 (percent of notional amount)
- Number of LSC parameters (NLSCFR, NLSCGR): NLSCFR = 3 (level, slope, and curvature1 for forward rate), NLSCGR = 3 (level, slope, and curvature1 for generic rate, basis curve if DiscountType = BC)
- LSC Parameters (FRParmX, GRParmX, X=1 to 6): FRParm1 = 6 (forward rate level = 6%), FRParm2 = −2 (forward rate slope = −2%, upward sloping), FRParm3 = −1 (forward rate curvature1 = −1%), and the remaining forward rate parameters are 0. Further, all the basis curve parameters, GRParmX, are assume to be zero.
- Scalars (FRScalarX, GRScalarX, X = 1 to 6): LSC scalars corresponding to LSC parameters
- Output: Type of output (Value1 denotes computation of value using two curves)

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Base Simulation Parameters

- VaR horizon = 30 days
- Confidence level = 95%
- Number of simulations = 10,000 (runs slow)
- Means, standard deviations, and correlations provided in text

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IRS Related Strategies

1. Underlying swap (initial case) (SIC)
2. Underlying swap with maturity date = 1 year (SMD1Y)
3. Underlying swap with maturity date = 2 year (SMD1Y)
4. Underlying swap with maturity date = 3 year (SMD1Y)
5. Underlying swap with maturity date = 4 year (SMD1Y)
6. Underlying swap with maturity date = 5 year (SMD1Y)
7. Underlying swap with maturity date = 10 year (SMD1Y)
8. Underlying swap with maturity date = 20 year (SMD1Y)
9. Underlying swap with maturity date = 30 year (SMD1Y)
10. Underlying swap with fixed 30/360, Semi and floating ACT/360, Quart: (SDCPV)

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IRS Related Strategies

11. Underlying swap with both legs 30/360 Semi : (SDC30S)
12. Underlying swap with both legs 30/360 Quart: (SDC30Q)
13. Underlying swap with both legs ACT/360 (SDCACTQ)
14. Underlying swap with 2 percent decrease in fixed rate = –2.0 (SFRM2)
15. Underlying swap with 1 percent decrease in fixed rate = –1.0 (SFRM1)
16. Underlying swap with 1 percent increase in fixed rate = 1.0 (SFRMP1)
17. Underlying swap with 2 percent increase in fixed rate = 2.0 (SFRMP2)
18. Underlying swap with +10 percent increase in cash collateral = 10.0 (SCC10)
19. Underlying swap with +20 percent increase in cash collateral = 20.0 (SCC20)

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Table 13.3.1 Return VaR Based on Forward Curve Level and Basis Curve Level Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 95%

Strategy\Correlation	-0.75	0.00	0.75
SIC	33.69	35.20	38.63
SMD1Y	5.43	5.87	6.49
SMD2Y	12.49	13.52	15.11
SMD3Y	20.35	21.22	23.62
SMD4Y	27.52	28.76	31.65
SMD5Y	33.69	35.20	38.63
SMD10Y	55.78	58.02	64.75
SMD20Y	80.33	81.82	92.36
SMD30Y	91.44	93.15	104.56
SDCPV	33.69	35.20	38.63
SDC30S	33.61	35.12	38.53
SDC30Q	33.42	34.93	38.33
SDCACTQ	37.64	39.15	42.56
SFRM2	31.47	32.96	36.22
SFRM1	32.54	34.08	37.38
SFRP1	34.84	36.42	39.84
SFRP2	36.00	37.42	41.14
SCC10	33.69	35.20	38.63
SCC20	16.85	17.60	19.32

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Table 13.3.2 Return VaR Based on Forward Curve Slope and Basis Curve Slope Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 95%

Strategy\Correlation	-0.75	0.00	0.75
SIC	38.12	37.57	39.08
SMD1Y	6.51	6.50	7.11
SMD2Y	14.93	14.95	15.64
SMD3Y	23.21	23.29	23.94
SMD4Y	31.13	30.63	31.71
SMD5Y	38.12	37.57	39.08
SMD10Y	61.72	62.51	64.06
SMD20Y	87.97	89.08	91.97
SMD30Y	99.34	100.76	104.12
SDCPV	38.12	37.57	39.08
SDC30S	38.02	37.50	39.00
SDC30Q	37.81	37.29	38.79
SDCACTQ	42.04	41.51	43.02
SFRM2	35.92	35.47	36.48
SFRM1	37.00	36.58	37.82
SFRP1	39.20	38.63	40.22
SFRP2	40.28	39.72	41.36
SCC10	38.12	37.57	39.08
SCC20	19.06	18.79	19.54

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Table 13.3.3 Return VaR Based Forward Curve Curvature and Basis Curve Curvature Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	39.04	38.91	38.34
SMD1Y	6.77	6.56	6.80
SMD2Y	15.37	15.22	15.39
SMD3Y	24.15	23.68	23.49
SMD4Y	32.08	31.90	31.13
SMD5Y	39.04	38.91	38.34
SMD10Y	64.07	63.39	63.47
SMD20Y	92.05	91.45	92.47
SMD30Y	104.59	103.53	105.40
SDCPV	39.04	38.91	38.34
SDC30S	38.95	38.92	38.25
SDC30Q	38.75	38.68	38.03
SDCACTQ	42.99	42.89	42.27
SFRM2	36.30	36.54	35.92
SFRM1	37.73	37.74	36.97
SFRP1	40.41	40.31	39.52
SFRP2	41.65	41.45	40.54
SCC10	39.04	38.91	38.34
SCC20	19.52	19.45	19.17

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Table 13.3.4 Return VaR Based Forward Curve Level and Basis Curve Slope Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	34.48	36.38	39.37
SMD1Y	5.51	6.32	6.95
SMD2Y	12.86	14.26	15.74
SMD3Y	20.56	22.11	24.46
SMD4Y	28.04	29.64	32.73
SMD5Y	34.48	36.38	39.37
SMD10Y	56.99	59.28	64.33
SMD20Y	82.11	85.29	91.29
SMD30Y	93.56	97.37	103.78
SDCPV	34.48	36.38	39.37
SDC30S	34.38	36.29	39.24
SDC30Q	34.20	36.08	39.04
SDCACTQ	38.42	40.30	43.30
SFRM2	32.15	33.87	36.63
SFRM1	33.40	35.13	38.02
SFRP1	35.61	37.59	40.69
SFRP2	36.85	38.84	41.97
SCC10	34.48	36.38	39.37
SCC20	17.24	18.19	19.69

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Table 13.3.5 Return VaR Based Forward Curve Slope and Basis Curve Level Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	37.21	37.39	38.68
SMD1Y	6.69	6.66	6.82
SMD2Y	14.66	14.96	15.34
SMD3Y	22.92	23.29	23.91
SMD4Y	30.50	30.94	31.59
SMD5Y	37.21	37.39	38.68
SMD10Y	61.24	62.84	63.97
SMD20Y	88.00	90.23	91.87
SMD30Y	99.85	102.44	104.21
SDCPV	37.21	37.39	38.68
SDC30S	37.14	37.31	38.56
SDC30Q	36.92	37.11	38.36
SDCACTQ	41.13	41.35	42.60
SFRM2	35.01	35.09	36.19
SFRM1	36.14	36.23	37.42
SFRP1	38.35	38.55	39.85
SFRP2	39.48	39.83	40.94
SCC10	37.21	37.39	38.68
SCC20	18.61	18.70	19.34

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Table 13.3.6 Return VaR Based Forward Curve Slope and Basis Curve Curvature Correlation
Margin = 10%, Number of Simulations = 10,000, Confidence Level = 99%

Strategy\Correlation	-0.75	0.00	0.75
SIC	38.87	38.28	41.97
SMD1Y	6.96	6.59	7.55
SMD2Y	15.65	14.87	17.08
SMD3Y	24.17	23.26	26.39
SMD4Y	31.93	31.03	34.78
SMD5Y	38.87	38.28	41.97
SMD10Y	64.23	64.20	66.75
SMD20Y	90.86	92.05	93.41
SMD30Y	103.69	103.76	104.81
SDCPV	38.87	38.28	41.97
SDC30S	38.79	38.27	41.82
SDC30Q	38.57	38.03	41.62
SDCACTQ	42.80	42.23	45.88
SFRM2	36.64	35.92	39.14
SFRM1	37.72	37.14	40.50
SFRP1	40.11	39.62	43.33
SFRP2	41.42	40.72	44.70
SCC10	38.87	38.28	41.97
SCC20	19.43	19.14	20.99

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IRS Valuation

$$V_{Swap} = \sum_{i'=1}^{N_{Flt}} DF_{Flt,i'} NA_{Flt,i'} \left(\frac{NAD_{Flt,i'}}{NTD_{Flt,i'}} \right) r_{FR,i'-1} - \sum_{i=1}^{N_{Fix}} DF_{Fix,i} NA_{Fix,i} \left(\frac{NAD_{Fix,i}}{NTD_{Fix,i}} \right) r_{Fix}. \quad (13.3.1)$$

where

i, i' counters for fixed cash flows (i) and floating cash flows (j),

$NAD_{Fix,i}$, $NAD_{Flt,i'}$ number of accrued days for each cash flow (fixed or floating),

$NTD_{Fix,i}$, $NTD_{Flt,i'}$ number of total days per year for each cash flow (fixed or floating),

$NA_{Fix,i}$, $NA_{Flt,i'}$ notional amount outstanding for each cash flow (fixed or floating),

$DF_{Fix,i}$, $DF_{Flt,i'}$ discount factor for each cash flow (fixed or floating, discounting only a function of time),

r_{Fix} annualized fixed rate associated with a particular interest rate swap, and

$r_{FR,i'-1}$ annualized forward rate associated with the jth cash flow (advanced set, settled in arrears).

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IRS Valuation (LSC Model)

$$V_{Swap,t}^{LSC,J} = \sum_{i'=1}^{N_{Flt}} TFlt_{1,i',t}^{LSC,J} - \sum_{i'=1}^{N_{Flt}} TFlt_{2,i',t}^{LSC,J} - r_{Fix} \sum_{i=1}^{N_{Fix}} TFix_{1,i,t}^{LSC,J}, \quad (13.3.2)$$

where we have three terms, two related to floating rates and one related to fixed rates,

$$TFlt_{1,i',t}^{LSC,J} = NA_{Flt,i'} e^{-\left(L_{BC,i'} + S_{BC,i'} \beta_{BC,J,S,t} + C_{BC,i'} \beta_{BC,J,C,t}\right) \tau_{i',t} - \left(L_{FC,i'} + S_{FC,i'} \beta_{FC,J,S,t} + C_{FC,i'} \beta_{FC,J,C,t}\right) \tau_{i'-1,t}}$$

$$TFlt_{2,i',t}^{LSC,J} = NA_{Flt,i'} e^{-\left[(L_{FC,i'} + L_{BC,i'}) + (S_{FC,i'} \beta_{FC,J,S,t} + S_{BC,i'} \beta_{BC,J,S,t}) + (C_{FC,i'} \beta_{FC,J,C,t} + C_{BC,i'} \beta_{BC,J,C,t})\right] \tau_{i',t}}$$

$$TFix_{1,i,t}^{LSC,J} = \hat{A}_{Fix,i} e^{-\left[(L_{FC,i} + L_{BC,i}) + (S_{FC,i} \beta_{FC,J,S,t} + S_{BC,i} \beta_{BC,J,S,t}) + (C_{FC,i} \beta_{FC,J,C,t} + C_{BC,i} \beta_{BC,J,C,t})\right] \tau_{i,t}}$$ (13.3.3)

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IRS Terminal Valuation

$$\tilde{V}_{Swap,t+h}^{LSC,t+h} = \sum_{i'=1}^{N_{Flt}} T\tilde{F}lt_{1,i',t+h}^{LSC,t+h} - \sum_{i'=1}^{N_{Flt}} T\tilde{F}lt_{2,i',t+h}^{LSC,t+h} - r_{Fix} \sum_{i=1}^{N_{Fix}} T\tilde{F}ix_{1,i,t+h}^{LSC,t+h}, \text{ (Swap Value at } t + \Delta) \quad (13.3.4)$$

where we have three terms (note the LSC factor subscripts are t and the maturity coefficients and maturity time subscripts are $t + \Delta$)

$$T\tilde{F}lt_{1,i',t+h}^{LSC,t+h} = NA_{Flt,i'} e^{-\left(\hat{L}_{BC,i'+h} + \hat{S}_{BC,i'+h} \beta_{BC,J,S,t+h} + \hat{C}_{BC,i'+h} \beta_{BC,J,C,t+h}\right) \tau_{i',t+h} - \left(\hat{L}_{FC,i'+h} + \hat{S}_{FC,i'+h} \beta_{FC,J,S,t+h} + \hat{C}_{FC,i'+h} \beta_{FC,J,C,t+h}\right) \tau_{i'-1,t+h}}$$

$$T\tilde{F}lt_{2,i',t+h}^{LSC,t+h} = NA_{Flt,i'} e^{-\left[(\hat{L}_{FC,i'+h} + \hat{L}_{BC,i'+h}) + (\hat{S}_{FC,i'+h} \beta_{FC,J,S,t+h} + \hat{S}_{BC,i'+h} \beta_{BC,J,S,t+h}) + (\hat{C}_{FC,i'+h} \beta_{FC,J,C,t+h} + \hat{C}_{BC,i'+h} \beta_{BC,J,C,t+h})\right] \tau_{i',t+h}}$$

$$T\tilde{F}ix_{1,i,t+h}^{LSC,t+h} = \hat{A}_{Fix,i} e^{-\left[(\hat{L}_{FC,i+h} + \hat{L}_{BC,i+h}) + (\hat{S}_{FC,i+h} \beta_{FC,J,S,t+h} + \hat{S}_{BC,i+h} \beta_{BC,J,S,t+h}) + (\hat{C}_{FC,i+h} \beta_{FC,J,C,t+h} + \hat{C}_{BC,i+h} \beta_{BC,J,C,t+h})\right] \tau_{i,t+h}}$$ (13.3.5)

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DRM and Swaps

- Swap profits and losses

$$\Delta\tilde{\Pi}_{t,t+h} = \tilde{V}_{Swap,t+h}^{LSC,t+h} - V_{Swap,t}^{LSC,t}$$

- Equivalent cash collateral as percentage of notional amount
- Holding period returns computed as basis for return VaR

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Summary

- Assumed two rate curves
  - Forward curve
  - Basis curve (to form discount curve)
- IRS model via LSC
- Monte Carlo simulations
- LSC parameter correlations with focus on return VaR

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