Module 13.2: DRM Selected Other Futures

Learning objectives

- Apply Monte Carlo simulation to explore risks that arise from inter-market exposure and this risk's influence on return value-at-risk.
- Illustrate the insights gained from Monte Carlo simulation with a focus on various correlations measured by return value-at-risk.
- Demonstrate the important role of margin and its influence on return value-at-risk with inter-market risk.

Executive summary

We focus here on the influence of inter-market risk on the analysis conducted in Module 13.1. We again focus on the correlation between the underlying growth rates and parameters related to the all-in carry costs (level and slope of the fitted carry cost curve). In this module we introduce inter-market risk between two different underlying instruments and their respective futures contracts. We assume both instrument's futures contracts are consistent with a fully-arbitraged market. We again illustrate with Monte Carlo simulation the influence of different correlations. Finally, we address the role of margin in the form of cash collateral on the return value-at-risk.

Within the quantitative finance materials, we work out the appropriate mathematical relationships required for the simulations with basis risk.

Central finance concepts

After again briefly reviewing the carry arbitrate model, we explain one way to develop dynamic risk measures related to selected other futures. Specifically, we generalize the framework to incorporate other futures instruments that contain inter-market risk.

Carry arbitrage model (CAM) assumptions

Recall from Module 6.1, there are five key assumptions underlying the carry arbitrage model. In its purest form, (1) at least some market participants pursue arbitrage opportunities; (2) there are no market frictions such as transaction costs and differential taxes; (3) borrowing and lending at the risk-free interest rate is feasible; (4) short selling is allowed with full use of proceeds; and (5) futures margin requirements are very small and can be ignored.

To develop dynamic risk measures, we further explore futures margin requirements as they are not large enough to rationally management futures contract portfolios over long periods of time. The initial and maintenance margins simply seek to assure solvency over the next day or until a variation margin can be demanded. The likelihood of a 100% loss is almost 100% if the allocated capital is merely the initial margin requirement. To address this challenge with a reasonable approach, we focus on additional cash collateral set aside to support the futures contract portfolio.

Futures margin and cash collateral

Futures positions can result in zero or negative accumulated values making futures trading extremely risky. Further, considering minimal margin requirements due to daily marking-to-market, holding period return calculations are not only difficult but not appropriate. Also, rehypothecatable collateral is preferred by brokerage firm because the process of rehypothecation allows the brokerage firm the ability to use the trader's collateral for their own purposes. Given the typically short horizon for DRMs analysis, we adopt an equivalent cash collateral amount denoted ECC. Thus, if interest bearing collateral is posted, we assume a cash equivalent amount to simplify the DRMs.

One reasonable way to handle this problem is to assume futures contract portfolios have this ECC allocated to it far beyond the required initial margin. It is important to understand that this measure is different from the required margin accounts that support futures contracts. Here, the goal is to establish an internal ECC account that is expected to support the futures contract portfolio over the entire life of the futures contract portfolio, not just the daily initial or maintenance margin. Recall, a margin call will require the trader to come up with additional collateral. Thus, with respect to dynamic risk measures, we are evaluating a futures contract portfolio. Further, a positive ECC amount will result in rational holding period return calculations. Note that the subsequent futures contract portfolio cash flows will include the return of the ECC amount.

Dynamic risk measures and selected other futures contracts

Like our options DRMs, we report the influence of various correlations on the return value-at-risk (RVaR). Unless otherwise indicated, we assume the following futures contract parameters:

- Underlying instrument 1 value (UI1) = 100
- Underlying instrument 2 value (UI2) = 100
- Level all-in carry costs for UI1 (LCC1) = 4%
- Level all-in carry costs for UI2 (LCC2) = –2%
- Slope all-in carry costs for UI1 (SCC1) = 0%
- Slope all-in carry costs for UI2 (SCC2) = 0%
- Nearby time to maturity for UI1 (TimeToMaturity1) = 0.25 years
- Nearby time to maturity for UI2 (TimeToMaturity2) = 0.25 years
- LSC model scalar (sc) = 0.5 years (short due to short dates futures contracts)
- Equivalent cash collateral (ECC) = 10%

Unless otherwise indicated, we assume the following simulation parameters:

- VaR horizon = 1 month
- Number of simulations = 100,000
- Means
    - UI1 = 0% (annualized, continuously compounded)
    - UI1 = 0% (annualized, continuously compounded)
    - LCC1 = 0.0 (annualized, discretely compounded, unit change)
    - LCC2 = 0.0 (annualized, discretely compounded, unit change)
    - SCC1 = 0.0 (annualized, discretely compounded, unit change)
    - SCC2 = 0.0 (annualized, discretely compounded, unit change)
- Standard deviations
    - UI1 = 20% (annualized, continuously compounded)
    - UI1 = 20% (annualized, continuously compounded)
    - LCC1 = 10.0 (annualized, discretely compounded, unit change)
    - LCC2 = 10.0 (annualized, discretely compounded, unit change)
    - SCC1 = 20.0 (annualized, discretely compounded, unit change)
    - SCC2 = 20.0 (annualized, discretely compounded, unit change)
- Correlations
    - UI1, UI2 = 0.8
    - UI1, LCC1 = –0.3
    - UI1, LCC2 = –0.3
    - UI1, SCC1 = 0.5
    - UI1, SCC2 = 0.5
    - UI2, LCC1 = –0.3
    - UI2, LCC2 = –0.3
    - UI2, SCC1 = 0.5
    - UI2, SCC2 = 0.5
    - LCC1, LCC2 = 0.8
    - LCC1, SCC1 = 0.7
    - LCC1, SCC2 = 0.6
    - LCC2, SCC1 = 0.7

- LCC2, SCC2 = 0.6
- SCC1, SCC2 = 0.6

Again, these parameters were not selected based on an in depth study of a particular Selected Other Futures complex, rather they were chosen simply for illustration purposes. The LCC1 could be thought of as a 6% interest rate less a 2% dividend yield and LCC2 could be thought of as a commodity with a 6% interest rate and a 2% storage fee. The specific parameterization is not the main issue, rather we focus on the influence of the interaction between the carry costs and various Selected Other Futures strategies.

We explore 19 strategies based on interactions between two futures market. Further, we adopt a simplified notation scheme. The spread trades explored here are inter-market, that is, with futures contracts having different underlying instruments.

1. Long underlying instrument 1 (LUI1)
2. Long underlying instrument 1, short nearby of underlying instrument 2 (LUI1SF21)
3. Long underlying instrument 1, short second nearby of underlying instrument 2 (LUI1SF22)
4. Long underlying instrument 1, short third nearby of underlying instrument 2 (LUI1SF23)
5. Short underlying instrument 1, long nearby of underlying instrument 2 (SUI1LF21)
6. Short underlying instrument 2, long second nearby of underlying instrument 2 (SUI1LF22)
7. Short underlying instrument 3, long third nearby of underlying instrument 2 (SUI1LF23)
8. Long nearby of underlying instrument 1, short nearby of underlying instrument 2 (LF11SF21)
9. Long second nearby of underlying instrument 1, short second nearby of underlying instrument 2 (LF12SF22)
10. Long third nearby of underlying instrument 1, short third nearby of underlying instrument 2 (LF13SF23)
11. Short nearby of underlying instrument 1, long nearby of underlying instrument 2 (SF11LF21)
12. Short second nearby of underlying instrument 1, long second nearby of underlying instrument 2 (SF12LF22)
13. Short third nearby of underlying instrument 1, long third nearby of underlying instrument 2 (SF13LF23)
14. Long nearby of underlying instrument 1, short second nearby of underlying instrument 2 (LF11SF22)
15. Long nearby of underlying instrument 1, short third nearby of underlying instrument 2 (LF11SF23)
16. Long nearby of underlying instrument 1, short fourth nearby of underlying instrument 2 (LF11SF24)
17. Short nearby of underlying instrument 1, long second nearby of underlying instrument 2 (SF11LF22)
18. Short nearby of underlying instrument 1, long third nearby of underlying instrument 2 (SF11LF23)
19. Short nearby of underlying instrument 1, long fourth nearby of underlying instrument 2 (SF11LF24)

There are 15 different correlations that could be produced and reported. The R code provided generates all 15 for the curious amongst us. We highlight only four casesd here.

Table 13.2.1 illustrates the influence of the correlation between underlying instrument 1 and underlying instrument 2. As expected with futures-based strategies, the higher the correlation, the lower the return VaR. Further, compared with results in Module 13.1, the return VaRs are dramatically high with the existence of inter-market risk.

Table 13.2.1 Return VaR Based CAM Selected Other Futures UI1 and UI2 Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	33.04	31.86	31.27	30.95	30.70	30.49	30.08
LUI1SF21	75.01	68.97	63.81	56.34	49.25	40.68	29.26
LUI1SF22	74.80	68.75	63.61	56.12	49.12	40.54	29.15
LUI1SF23	74.56	68.51	63.40	55.93	48.99	40.40	29.02
SUI1LF21	71.77	65.59	59.86	53.24	46.60	37.59	25.90
SUI1LF22	71.63	65.48	59.80	53.22	46.52	37.53	25.88
SUI1LF23	71.51	65.38	59.70	53.10	46.48	37.49	25.89
LF11SF21	78.54	72.49	67.32	59.82	52.74	44.14	32.62
LF12SF22	78.62	72.56	67.36	59.84	52.79	44.14	32.65
LF13SF23	78.75	72.61	67.43	59.81	52.77	44.18	32.61
SF11LF21	68.73	62.52	56.82	50.13	43.45	34.43	22.70
SF12LF22	69.09	62.85	57.07	50.44	43.73	34.63	22.93
SF13LF23	69.38	63.13	57.29	50.71	44.00	34.85	23.18
LF11SF22	78.30	72.27	67.11	59.64	52.57	43.98	32.53
LF11SF23	78.08	72.03	66.90	59.43	52.43	43.86	32.42
LF11SF24	77.82	71.81	66.70	59.23	52.27	43.70	32.32
SF11LF22	68.59	62.41	56.74	50.06	43.36	34.38	22.70
SF11LF23	68.50	62.30	56.62	49.98	43.32	34.31	22.68
SF11LF24	68.39	62.21	56.48	49.91	43.29	34.26	22.70

Table 13.2.2 illustrates the influence of the correlation between the underlying instrument 1 and instrument 2's level all-in carry costs. Although not as dramatic in magnitude as Table 13.2.1, the higher the correlation, the higher the return VaRs.

Table 13.2.2 Return VaR Based CAM Selected Other Futures UI1 and LCC2 Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	30.64	30.58	30.07	29.94	30.12	30.44	30.77
LUI1SF21	27.29	26.62	26.42	26.58	27.68	29.36	31.26
LUI1SF22	27.22	26.53	26.29	26.45	27.53	29.24	31.08
LUI1SF23	27.13	26.44	26.17	26.33	27.36	29.09	30.93
SUI1LF21	24.05	23.18	23.23	23.34	24.51	25.98	28.23
SUI1LF22	24.09	23.22	23.24	23.32	24.48	25.93	28.18
SUI1LF23	24.09	23.26	23.24	23.31	24.41	25.88	28.10
LF11SF21	30.69	30.05	29.79	29.95	31.05	32.79	34.68
LF12SF22	30.65	30.03	29.74	29.89	30.95	32.70	34.57
LF13SF23	30.65	30.05	29.68	29.87	30.88	32.66	34.44
SF11LF21	20.84	19.98	20.03	20.17	21.28	22.78	25.05
SF12LF22	21.10	20.24	20.19	20.34	21.47	22.94	25.21
SF13LF23	21.35	20.50	20.35	20.53	21.63	23.17	25.34
LF11SF22	30.59	29.94	29.69	29.84	30.90	32.62	34.51
LF11SF23	30.51	29.85	29.58	29.69	30.75	32.46	34.31
LF11SF24	30.40	29.76	29.43	29.57	30.61	32.29	34.12
SF11LF22	20.88	20.04	20.02	20.15	21.24	22.73	24.98
SF11LF23	20.93	20.08	20.02	20.12	21.22	22.70	24.90
SF11LF24	20.99	20.11	20.01	20.09	21.18	22.63	24.85

Table 13.2.3 illustrates the influence of the correlation between the underlying instrument 1 and instrument 2's slope all-in carry costs. Here, the higher the correlation, the lower the return VaRs.

Table 13.2.3 Return VaR Based CAM Selected Other Futures UI1 and SCC2 Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	33.06	31.94	30.73	29.93	29.81	30.59	30.69
LUI1SF21	39.60	36.45	31.94	27.24	26.39	26.24	26.74
LUI1SF22	39.48	36.36	31.84	27.17	26.30	26.10	26.60
LUI1SF23	39.38	36.29	31.73	27.07	26.19	25.98	26.52
SUI1LF21	36.97	32.91	28.75	24.10	23.23	23.00	23.49
SUI1LF22	37.01	32.98	28.78	24.14	23.25	22.99	23.47
SUI1LF23	37.00	32.99	28.84	24.16	23.25	22.99	23.47
LF11SF21	42.99	39.89	35.33	30.63	29.77	29.59	30.16
LF12SF22	43.03	39.92	35.30	30.61	29.75	29.58	30.14
LF13SF23	43.08	39.94	35.28	30.57	29.79	29.57	30.13
SF11LF21	33.84	29.74	25.57	20.89	20.02	19.78	20.28
SF12LF22	34.12	30.01	25.86	21.15	20.20	19.98	20.51
SF13LF23	34.42	30.31	26.16	21.38	20.37	20.23	20.73
LF11SF22	42.93	39.80	35.23	30.54	29.67	29.47	30.03
LF11SF23	42.80	39.71	35.14	30.45	29.56	29.38	29.93
LF11SF24	42.67	39.61	35.01	30.33	29.48	29.29	29.81
SF11LF22	33.86	29.77	25.62	20.94	20.05	19.79	20.27
SF11LF23	33.88	29.76	25.65	20.96	20.05	19.78	20.26
SF11LF24	33.86	29.77	25.66	20.97	20.02	19.79	20.28

Table 13.2.4 illustrates the influence of margin on the return VaR. Similar to the results in Module 13.1, return VaR is very sensitive to margin. The higher the margin, the lower the return VaR. Again, an interesting insight is the ability to dial up or dial down the risk level of various futures portfolios simply by manipulating cash collateral held. Thus, the riskiness of futures portfolios are directly linked to cash collateral held.

Table 13.2.4 Return VaR Based CAM Selected Other Futures Margin Requirements Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Margin (%)	10	20	30	40	50	60	70	80	90	100
LUI1	30.37	15.01	10.06	7.61	6.03	4.97	4.32	3.74	3.38	3.02
LUI1SF21	26.24	13.20	8.75	6.57	5.26	4.37	3.79	3.30	2.92	2.63
LUI1SF22	26.13	13.14	8.72	6.54	5.24	4.35	3.77	3.29	2.91	2.62
LUI1SF23	26.03	13.09	8.68	6.50	5.22	4.33	3.76	3.27	2.89	2.60
SUI1LF21	23.05	11.54	7.63	5.72	4.60	3.85	3.30	2.88	2.59	2.30
SUI1LF22	23.05	11.53	7.64	5.72	4.60	3.84	3.30	2.89	2.59	2.30
SUI1LF23	23.02	11.53	7.63	5.71	4.60	3.84	3.30	2.88	2.59	2.30
LF11SF21	29.64	14.90	9.88	7.41	5.94	4.94	4.27	3.73	3.30	2.97
LF12SF22	29.59	14.87	9.86	7.39	5.93	4.93	4.27	3.72	3.29	2.96
LF13SF23	29.53	14.84	9.84	7.39	5.92	4.93	4.26	3.71	3.29	2.95
SF11LF21	19.81	9.93	6.57	4.91	3.96	3.30	2.83	2.48	2.24	1.98
SF12LF22	19.97	10.04	6.66	4.96	4.00	3.34	2.86	2.51	2.26	2.00
SF13LF23	20.18	10.17	6.72	5.01	4.04	3.38	2.89	2.53	2.28	2.02
LF11SF22	29.53	14.83	9.84	7.38	5.92	4.91	4.26	3.71	3.28	2.95
LF11SF23	29.42	14.78	9.80	7.35	5.90	4.90	4.24	3.70	3.27	2.94
LF11SF24	29.28	14.72	9.76	7.32	5.87	4.88	4.22	3.69	3.26	2.93
SF11LF22	19.81	9.93	6.57	4.91	3.96	3.31	2.83	2.48	2.23	1.98
SF11LF23	19.80	9.94	6.58	4.91	3.96	3.30	2.83	2.49	2.23	1.98
SF11LF24	19.81	9.93	6.57	4.91	3.96	3.31	2.84	2.49	2.23	1.98

Quantitative finance materials

We first briefly review the carry arbitrage model from Modules 6.1 and 9.1 for the purpose of developing potential simulations. We then develop a dynamic risk measures (DRMs) framework for appraising futures portfolio risk. The focus here is with two underlying instruments.

Carry arbitrage model review

Recall one simple way to express these two expressions of the CAM is

$$F_{1,n,t} = FV_{cc1,n}\left(UI_{1,t}\right) \text{ for all } n \tag{13.2.1}$$

$$F_{2,n,t} = FV_{cc2,n}\left(UI_{2,t}\right) \text{ for all } n \tag{13.2.2}$$

where $FV_{cci,n}$ denotes the future value operator based on specified carry costs for instrument i (i = 1,2), the n^{th} contract observed at time t. Carry costs include the financing costs and any other associated revenues and costs related to the underlying instrument, such as dividend yield, foreign interest rates, storage costs, insurance, and so on. Thus, in a futures market that reflects full arbitrage as expressed in the equation above should approximate the futures price. Further, changes in the futures price should depend solely on changes in the underlying instrument and changes in the various carrying costs.

As previously covered, we allow the carry costs to be maturity varying based on the LSC model. Recall from Module 3.4 we introduced a generalized LSC model expressed as

$$y_i = \sum_{j=0}^{N} x_{i,j} f_j, \tag{13.2.3}$$

where y_i denotes in this context is the carry costs for the i^{th} futures contract that varies in maturity time, $x_{i,j}$ denotes input LSC coefficients based on some maturity and some factor, N denotes the number of LSC factors (plus level), and f_j denotes the output factors. The general LSC model assumes

$$x_{i,0} = 1,\ x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i/s_1}\right), \text{ and } x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i/s_j}\right) - e^{-\tau_i/s_j}; j > 1. \tag{13.2.4}$$

In this module, we apply only a two-factor model. Recall we assume the input scalars, s_j, are provided by the user, where $s_1 = s_2$. Again s_j denotes scalars that applies various weights to different locations on the carry cost maturity structure, $x_{i,j}$ denotes LSC maturity coefficients, a parameter that depends solely on maturity time and selected scalars, and f_j denotes the output LSC factor, a parameter that is typically found using ordinary least squares regression applied to estimated maturity time carry costs. In this module, the LSC factor values as well as their stochastic behaviors are inputs into the DRMs analysis.

For this module, we assume a simple two-factor model. With two instruments and continuous compounding, we have

$$F_{1,n,t} = UI_{1,t} e^{cc_{1,n}\tau_{1,n}} \text{ for all } n \tag{13.2.5}$$

$$F_{2,n,t} = UI_{2,t} e^{cc_{2,n}\tau_{2,n}} \text{ for all } n \tag{13.2.6}$$

where $\tau_{i,n}$ (= $T_{i,n} - t$) denotes the time to maturity in years for the nth nearby futures contract. Thus, based on a two-factor LSC model, we have

$$F_{1,n,t} = UI_{1,t} e^{\left(L_{1,cc} + x_{1,n,1} S_{1,cc}\right)\tau_{1,n}} = UI_{1,t} e^{\left[L_{1,cc} + \frac{s_1}{\tau_{1,n}}\left(1 - e^{-\tau_{1,n}/s_1}\right) S_{1,cc}\right]\tau_{1,n}} \text{ for all } n, \tag{13.2.7}$$

$$F_{2,n,t} = UI_{2,t} e^{\left(L_{2,cc} + x_{2,n,1} S_{2,cc}\right)\tau_{2,n}} = UI_{2,t} e^{\left[L_{2,cc} + \frac{s_1}{\tau_{2,n}}\left(1 - e^{-\tau_{2,n}/s_1}\right) S_{2,cc}\right]\tau_{2,n}} \text{ for all } n, \tag{13.2.8}$$

where $L_{i,cc}$ denotes all-in carry cost level factor and $S_{i,cc}$ denotes all-in carry cost slope factor. Thus, we assume that given these two factors combined with the underlying value, then the futures price is known.

Dynamic risk measures applied to selected other futures portfolio

Assuming a risk management horizon, h, the horizon futures price can be modeled as

$$\tilde{F}_{1,n,t+h} = U\tilde{I}_{1,t+h}\left(\tilde{L}_{1,cc,t+h}, \tilde{S}_{1,cc,t+h}, \varepsilon_{t+h}\right) e^{\left[\tilde{L}_{1,cc,t+h} + \frac{s_1}{\tau_{1,n} - h}\left(1 - e^{-(\tau_{1,n} - h)/s_1}\right)\tilde{S}_{1,cc,t+h}\right](\tau_{1,n} - h)} \text{ for all } n, \tag{13.2.9}$$

$$\tilde{F}_{2,n,t+h} = U\tilde{I}_{2,t+h}\left(\tilde{L}_{2,cc,t+h}, \tilde{S}_{2,cc,t+h}, \varepsilon_{t+h}\right) e^{\left[\tilde{L}_{2,cc,t+h} + \frac{s_1}{\tau_{2,n} - h}\left(1 - e^{-(\tau_{1,n} - h)/s_1}\right)\tilde{S}_{2,cc,t+h}\right](\tau_{2,n} - h)} \text{ for all } n, \tag{13.2.10}$$

We assume the correlated risk factors driving the underlying value at time $t + h$, the level and slope of the all-in carry costs as well as a residual value accounting for all other underlying value variation. As written, we explicitly recognize the possibility that the underlyings are correlated with parameters of the LSC models.

We model changes in the portfolio as

$$\Delta\tilde{\Pi}_{t,t+h} = \sum_{j=1}^{N_m} N_j \Delta\tilde{F}_{n,t+h} = \sum_{j=1}^{N_m} N_j \left(\tilde{F}_{n,t+h} - F_{n,t}\right) \text{ for all } n, \tag{13.2.11}$$

where N_j denotes the number of nth nearby futures contracts (positive denotes long and negative denotes short). For positions involving the underlying, the portfolio change is similarly computed.

The challenge is developing a rational basis for the cash collateral amount. As with interest rate swaps, we seek a proportion of some notional amount but with the capacity to be long, short, or some combination of both, it is fraught with the potential for unusual cases. We assume only underlying instrument 1 is used in the portfolio. We propose the following ECC representation:

$$ECC_t = \omega \max\left(\left|\sum_{j=1}^{N_m} N_j F_{n,t}\right|, UI_{1,t}\right). \tag{13.2.12}$$

First, we aggregate the futures portfolio notional value. As this amount may be negative, we compute the absolute value of the net portfolio. As this amount may be zero, we also take the maximum with respect to the underlying instrument value. Finally, we allow the analyst to choose a proportion of this amount for the final ECC value, but we require ω to be positive.

Thus, the holding period returns from which RVaR will be extracted is

$$R_t = \frac{\Delta\tilde{\Pi}_{t,t+h}}{\omega \max\left(\left|\sum_{j=1}^{N_m} N_j F_{n,t}\right|, S_t\right)}. \tag{13.2.13}$$

Summary

We focused on the influence of various correlations. We illustrated our results with Monte Carlo simulation. Finally, we addressed the role of margin in the form of cash collateral on the return value-at-risk.

Within the quantitative finance materials, we worked out the appropriate mathematical relationships required for the simulations. We specifically address the role of cash collateral on holding period returns for futures-related positions.

References

See Module 6.1.