

## Module 13.2

Dynamic Risk Measures  
Other Futures

1

## Overview

- Assume two underlying instruments
- Carry arbitrage model
  - Multivariate simulations
  - Multiple strategies
  - Focus on correlations
- Monte Carlo simulations
- Inter-market correlation focused on return VaR



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2

2

## Carry Arbitrage Model

- Carry arbitrage: Underlying instrument is purchased and thus 'carried' through time
- Reverse carry arbitrage: Underlying instrument is sold short
- CAM: Futures price is the future value of the current spot price where the implied rate is based on the all-in carrying cost of the underlying instrument, including financing



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3

3

## Margin and Collateral

- Futures margins are minimal
  - Initial and maintenance margins poor measure of required capital
  - Additional cash required to support position
- Cash collateral forms basis for holding period returns
  - Higher cash collateral, lower HPRs
  - Explore various implied risk measures



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4

4

## Underlying Instrument's Parameters

- Underlying instr. 1 value,  $UI1=100$ ,  $UI2=100$
- Level all-in carry cost,  $LCC1=4\%$ ,  $LCC2=-2\%$
- Slope all-in carry cost,  $SCC1=0\%$ ,  $SCC2=0\%$
- Nearby maturity = 0.25 years
- Quarterly cycle
- LSC model scalar = 0.5
- Equivalent cash collateral (ECC) = 10%



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5

5

## Simulation Parameters

- VaR horizon = 1 month
- Number of simulations = 100,000
- Means: Underlying growth rate,  $g1=g2=0\%$ ,  $LCC1=LCC2=0$ ,  $SCC1=SCC2=0$
- Standard deviations:  $g1=g2=20\%$ ,  $LCC1=LCC2=10$ ,  $SCC1=SCC2=20$
- Correlations: 15 given in chapter



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6

6

## Strategies Considered

1. Long underlying instrument 1 (LUI1)
2. Long underlying instrument 1, short nearby of underlying instrument 2 (LUI1SF21)
3. Long underlying instrument 1, short second nearby of underlying instrument 2 (LUI1SF22)
4. Long underlying instrument 1, short third nearby of underlying instrument 2 (LUI1SF23)
5. Short underlying instrument 1, long nearby of underlying instrument 2 (SUI1LF21)
6. Short underlying instrument 2, long second nearby of underlying instrument 2 (SUI1LF22)
7. Short underlying instrument 3, long third nearby of underlying instrument 2 (SUI1LF23)
8. Long nearby of underlying instrument 1, short nearby of underlying instrument 2 (LF11SF21)
9. Long second nearby of underlying instrument 1, short second nearby of underlying instrument 2 (LF12SF22)
10. Long third nearby of underlying instrument 1, short third nearby of underlying instrument 2 (LF13SF23)



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7

7

## Strategies Considered

11. Short nearby of underlying instrument 1, long nearby of underlying instrument 2 (SF11LF21)
12. Short second nearby of underlying instrument 1, long second nearby of underlying instrument 2 (SF12LF22)
13. Short third nearby of underlying instrument 1, long third nearby of underlying instrument 2 (SF13LF23)
14. Long nearby of underlying instrument 1, short second nearby of underlying instrument 2 (LF11SF22)
15. Long nearby of underlying instrument 1, short third nearby of underlying instrument 2 (LF11SF23)
16. Long nearby of underlying instrument 1, short fourth nearby of underlying instrument 2 (LF11SF24)
17. Short nearby of underlying instrument 1, long second nearby of underlying instrument 2 (SF11LF22)
18. Short nearby of underlying instrument 1, long third nearby of underlying instrument 2 (SF11LF23)
19. Short nearby of underlying instrument 1, long fourth nearby of underlying instrument 2 (SF11LF24)



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8

8

**Table 13.2.1 Return VaR Based CAM Selected Other Futures UI1 and UI2 Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	33.04	31.86	31.27	30.95	30.70	30.49	30.08
LUI1SF21	75.01	68.97	63.81	56.34	49.25	40.68	29.26
LUI1SF22	74.80	68.75	63.61	56.12	49.12	40.54	29.15
LUI1SF23	74.56	68.51	63.40	55.93	48.99	40.40	29.02
SUI1LF21	71.77	65.59	59.86	53.24	46.60	37.59	25.90
SUI1LF22	71.63	65.48	59.80	53.22	46.52	37.53	25.88
SUI1LF23	71.51	65.38	59.70	53.10	46.48	37.49	25.89
LF11SF21	78.54	72.49	67.32	59.82	52.74	44.14	32.62
LF12SF22	78.62	72.56	67.36	59.84	52.79	44.14	32.65
LF13SF23	78.75	72.61	67.43	59.81	52.77	44.18	32.61
SF11LF21	68.73	62.52	56.82	50.13	43.45	34.43	22.70
SF12LF22	69.09	62.85	57.07	50.44	43.73	34.63	22.93
SF13LF23	69.38	63.13	57.29	50.71	44.00	34.85	23.18
LF11SF22	78.30	72.27	67.11	59.64	52.57	43.98	32.53
LF11SF23	78.08	72.03	66.90	59.43	52.43	43.86	32.42
LF11SF24	77.82	71.81	66.70	59.23	52.27	43.70	32.32
SF11LF22	68.59	62.41	56.74	50.06	43.36	34.38	22.70
SF11LF23	68.50	62.30	56.62	49.98	43.32	34.31	22.68
SF11LF24	68.39	62.21	56.48	49.91	43.29	34.26	22.70



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9

9

**Table 13.2.2 Return VaR Based CAM Selected Other Futures UI1 and LCC2 Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	30.64	30.58	30.07	29.94	30.12	30.44	30.77
LUI1SF21	27.29	26.62	26.42	26.58	27.68	29.36	31.26
LUI1SF22	27.22	26.53	26.29	26.45	27.53	29.24	31.08
LUI1SF23	27.13	26.44	26.17	26.33	27.36	29.09	30.93
SUI1LF21	24.05	23.18	23.23	23.34	24.51	25.98	28.23
SUI1LF22	24.09	23.22	23.24	23.32	24.48	25.93	28.18
SUI1LF23	24.09	23.26	23.24	23.31	24.41	25.88	28.10
LF11SF21	30.69	30.05	29.79	29.95	31.05	32.79	34.68
LF12SF22	30.65	30.03	29.74	29.89	30.95	32.70	34.57
LF13SF23	30.65	30.05	29.68	29.87	30.88	32.66	34.44
SF11LF21	20.84	19.98	20.03	20.17	21.28	22.78	25.05
SF12LF22	21.10	20.24	20.19	20.34	21.47	22.94	25.21
SF13LF23	21.35	20.50	20.35	20.53	21.63	23.17	25.34
LF11SF22	30.59	29.94	29.69	29.84	30.90	32.62	34.51
LF11SF23	30.51	29.85	29.58	29.69	30.75	32.46	34.31
LF11SF24	30.40	29.76	29.43	29.57	30.61	32.29	34.12
SF11LF22	20.88	20.04	20.02	20.15	21.24	22.73	24.98
SF11LF23	20.93	20.08	20.02	20.12	21.22	22.70	24.90
SF11LF24	20.99	20.11	20.01	20.09	21.18	22.63	24.85



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10

10

**Table 13.2.3 Return VaR Based CAM Selected Other Futures UI1 and SCC2 Correlation**  
Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LUI1	33.06	31.94	30.73	29.93	29.81	30.59	30.69
LUI1SF21	39.60	36.45	31.94	27.24	26.39	26.24	26.74
LUI1SF22	39.48	36.36	31.84	27.17	26.30	26.10	26.60
LUI1SF23	39.38	36.29	31.73	27.07	26.19	25.98	26.52
SUI1LF21	36.97	32.91	28.75	24.10	23.23	23.00	23.49
SUI1LF22	37.01	32.98	28.78	24.14	23.25	22.99	23.47
SUI1LF23	37.00	32.99	28.84	24.16	23.25	22.99	23.47
LF11SF21	42.99	39.89	35.33	30.63	29.77	29.59	30.16
LF12SF22	43.03	39.92	35.30	30.61	29.75	29.58	30.14
LF13SF23	43.08	39.94	35.28	30.57	29.79	29.57	30.13
SF11LF21	33.84	29.74	25.57	20.89	20.02	19.78	20.28
SF12LF22	34.12	30.01	25.86	21.15	20.20	19.98	20.51
SF13LF23	34.42	30.31	26.16	21.38	20.37	20.23	20.73
LF11SF22	42.93	39.80	35.23	30.54	29.67	29.47	30.03
LF11SF23	42.80	39.71	35.14	30.45	29.56	29.38	29.93
LF11SF24	42.67	39.61	35.01	30.33	29.48	29.29	29.81
SF11LF22	33.86	29.77	25.62	20.94	20.05	19.79	20.27
SF11LF23	33.88	29.76	25.65	20.96	20.05	19.78	20.26
SF11LF24	33.86	29.77	25.66	20.97	20.02	19.79	20.28



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11

11

**Table 13.2.4 Return VaR Based CAM Selected Other Futures Margin Requirements**  
Number of Simulations = 100,000, Confidence Level = 99%

Strategy/Margin (%)	10	20	30	40	50	60	70	80	90	100
LUI1	30.37	15.01	10.06	7.61	6.03	4.97	4.32	3.74	3.38	3.02
LUI1SF21	26.24	13.20	8.75	6.57	5.26	4.37	3.79	3.30	2.92	2.63
LUI1SF22	26.13	13.14	8.72	6.54	5.24	4.35	3.77	3.29	2.91	2.62
LUI1SF23	26.03	13.09	8.68	6.50	5.22	4.33	3.76	3.27	2.89	2.60
SUI1LF21	23.05	11.54	7.63	5.72	4.60	3.85	3.30	2.88	2.59	2.30
SUI1LF22	23.05	11.53	7.64	5.72	4.60	3.84	3.30	2.89	2.59	2.30
SUI1LF23	23.02	11.53	7.63	5.71	4.60	3.84	3.30	2.88	2.59	2.30
LF11SF21	29.64	14.90	9.88	7.41	5.94	4.94	4.27	3.73	3.30	2.97
LF12SF22	29.59	14.87	9.86	7.39	5.93	4.93	4.27	3.72	3.29	2.96
LF13SF23	29.53	14.84	9.84	7.39	5.92	4.93	4.26	3.71	3.29	2.95
SF11LF21	19.81	9.93	6.57	4.91	3.96	3.30	2.83	2.48	2.24	1.98
SF12LF22	19.97	10.04	6.66	4.96	4.00	3.34	2.86	2.51	2.26	2.00
SF13LF23	20.18	10.17	6.72	5.01	4.04	3.38	2.89	2.53	2.28	2.02
LF11SF22	29.53	14.83	9.84	7.38	5.92	4.91	4.26	3.71	3.28	2.95
LF11SF23	29.42	14.78	9.80	7.35	5.90	4.90	4.24	3.70	3.27	2.94
LF11SF24	29.28	14.72	9.76	7.32	5.87	4.88	4.22	3.69	3.26	2.93
SF11LF22	19.81	9.93	6.57	4.91	3.96	3.31	2.83	2.48	2.23	1.98
SF11LF23	19.80	9.94	6.58	4.91	3.96	3.30	2.83	2.49	2.23	1.98
SF11LF24	19.81	9.93	6.57	4.91	3.96	3.31	2.84	2.49	2.23	1.98



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12

12

## Quantitative Finance Materials

- CAM reviewed
- LSC model reviewed
- DRM and model simulated
- Equivalent cash collateral
- Holding period returns for return VaR



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13

13

## CAM Model

- Futures price is future value of spot price

$$F_{1,n,t} = FV_{cc1,n}(U_{1,t}) \text{ for all } n$$

$$F_{2,n,t} = FV_{cc2,n}(U_{2,t}) \text{ for all } n$$

- $FV_{cc,n}$  – future value operator based on specified carry costs over n periods
- Carry costs include the financing costs and any other associated revenues and costs related to the underlying instrument



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14

14

## LSC Model

- Linear factors:  $y_i = \sum_{j=0}^N x_{i,j} f_j$
- Level:  $x_{i,0} = 1$       Slope:  $x_{i,1} = \frac{s_1}{\tau_i} (1 - e^{-\tau_i/s_1})$
- Curvatures:  $x_{i,j} = \frac{s_j}{\tau_i} (1 - e^{-\tau_i/s_j}) - e^{-\tau_i/s_j}, j > 1$
- LSC model has the lowest “average (across the sample) mean (across the curve) absolute yield error” (Steeley) when compared with splines, polynomials and Vasicek’s model



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15

15

## DRM and Other Futures

- Futures price simulation model

$$\tilde{F}_{1,n,t+h} = \tilde{U}_{1,t+h} \left( \tilde{F}_{1,cc,t+h} + \tilde{S}_{1,cc,t+h} \epsilon_{t+h} \right) e^{\left[ \tilde{L}_{1,cc,t+h} + \frac{s_1}{\tau_{1,n}-h} (1 - e^{-(\tau_{1,n}-h)/s_1}) \tilde{S}_{1,cc,t+h} \right] (\tau_{1,n}-h)} \text{ for all } n,$$

$$\tilde{F}_{2,n,t+h} = \tilde{U}_{2,t+h} \left( \tilde{F}_{2,cc,t+h} + \tilde{S}_{2,cc,t+h} \epsilon_{t+h} \right) e^{\left[ \tilde{L}_{2,cc,t+h} + \frac{s_2}{\tau_{2,n}-h} (1 - e^{-(\tau_{2,n}-h)/s_2}) \tilde{S}_{2,cc,t+h} \right] (\tau_{2,n}-h)} \text{ for all } n,$$

- Profit/Loss simulation

$$\Delta \tilde{\Pi}_{t,t+h} = \sum_{j=1}^{N_n} N_j \Delta \tilde{F}_{n,t+h} = \sum_{j=1}^{N_n} N_j (\tilde{F}_{n,t+h} - F_{n,t}) \text{ for all } n,$$



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16

16

## DRM and Other Futures

- Equivalent cash collateral amount

$$ECC_t = \omega \max \left( \sum_{j=1}^{N_m} N_j F_{n,t} \mid S_t \right).$$

- Holding period return for return VaR

$$R_t = \frac{\Delta \tilde{\Pi}_{t,t+h}}{\omega \max \left( \sum_{j=1}^{N_m} N_j F_{n,t} \mid S_t \right)}.$$



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17

17

## Summary

- Reviewed carry arbitrage model
- Simulations to explore role of correlation
- Quantitative framework examined



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18

18