Module 13.1: DRM Stock Index Futures

Learning objectives

- Apply Monte Carlo simulation to explore interactions between various inputs related to futures contract portfolios within the carry arbitrage model and return value-at-risk
- Illustrate the insights gained from Monte Carlo simulation with a focus on correlation between the underlying stock index and carry cost parameters on return value-at-risk
- Demonstrate the important role of margin and its influence on return value-at-risk

Executive summary

We focus here on the influence of correlation between the underlying growth rate and parameters related to the all-in carry costs (level and slope of the fitted carry cost curve). We illustrate with Monte Carlo simulation the influence of different correlations. Finally, we address the role of margin in the form of cash collateral on the return value-at-risk.

Within the quantitative finance materials, we work out the appropriate mathematical relationships required for the simulations.

Central finance concepts

After briefly reviewing the carry arbitrate model, we explain one way to develop dynamic risk measures related to stock index futures. In the next module, we generalize this framework to incorporate other futures instruments.

Carry arbitrage model (CAM) assumptions

Recall from Module 6.1, there are five key assumptions underlying the carry arbitrage model. In its purest form, (1) at least some market participants pursue arbitrage opportunities; (2) there are no market frictions such as transaction costs and differential taxes; (3) borrowing and lending at the risk-free interest rate is feasible; (4) short selling is allowed with full use of proceeds; and (5) futures margin requirements are very small and can be ignored.

To develop dynamic risk measures, we further explore futures margin requirements as they are not large enough to rationally management futures contract portfolios over long periods of time. The initial and maintenance margins simply seek to assure solvency over the next day or until a variation margin can be demanded. The likelihood of a 100% loss is almost 100% if the allocated capital is merely the initial margin requirement. To address this challenge with a reasonable approach, we focus on additional cash collateral set aside to support the futures contract portfolio.

Futures margin and cash collateral

Futures positions can result in zero or negative accumulated values making futures trading extremely risky. Further, considering minimal margin requirements due to daily marking-to-market, holding period return calculations are not only difficult but not appropriate. Also, rehypothecatable collateral is preferred by brokerage firm because the process of rehypothecation allows the brokerage firm the ability to use the trader's collateral for their own purposes. Given the typically short horizon for DRMs analysis, we adopt an equivalent cash collateral amount denoted ECC. Thus, if interest bearing collateral is posted, we assume a cash equivalent amount to simplify the DRMs.

One reasonable way to handle this problem is to assume futures contract portfolios have this ECC allocated to it far beyond the required initial margin. It is important to understand that this measure is different from the required margin accounts that support futures contracts. Here, the goal is to establish an internal ECC account that is expected to support the futures contract portfolio over the entire life of the futures contract portfolio, not just the daily initial or maintenance margin. Recall, a margin call will require the trader to come up with additional collateral. Thus, with respect to dynamic risk measures, we are evaluating a futures contract portfolio. Further, a positive ECC amount will result in rational holding period return calculations. Note that the subsequent futures contract portfolio cash flows will include the return of the ECC amount.

Dynamic risk measures and stock index futures contracts

Like our options DRMs, we report the influence of various correlations on the return value-at-risk (RVaR). Unless otherwise indicated, we assume the following futures contract parameters:

- Stock index value (S) = 100
- Level all-in carry costs (LCC) = 4%
- Slope all-in carry costs (SCC) = 0%
- LSC model scalar = 0.5 years (short due to short dates futures contracts)
- Equivalent cash collateral (ECC) = 10%

Unless otherwise indicated, we assume the following simulation parameters:

- VaR horizon = 1 month
- Number of simulations = 100,000
- Means
    - Stock index growth rate = 10% (annualized, continuously compounded)
    - LCC = 0.0 (annualized, discretely compounded, unit change)
    - SCC = 0.0 (annualized, discretely compounded, unit change)
- Standard deviations
    - Stock index growth rate = 20% (annualized, continuously compounded)
    - LCC =10 (annualized, discretely compounded, unit change)
    - SCC = 20 (annualized, discretely compounded, unit change)
- Correlations
    - Stock index growth rate, LCC = –0.3
    - Stock index growth rate, SCC =0.5
    - LCC, SCC = –0.5

It is important to recall the artist analogy in Chapter 1. These parameters were not selected based on an in depth study of a particular stock index futures complex, rather they were chosen simply for illustration purposes. The LCC could be thought of as a 6% interest rate less a 2% dividend yield and the SCC could be thought of as a 2% interest rate less a 2% dividend yield. The specific parameterization is not the main issue, rather we focus on the influence of the interaction between the carry costs and various stock index futures strategies.

We explore 19 strategies based on a fully arbitraged stock index futures market. Further, we adopt a simplified notation scheme. The spread trades explored here are intra-market, that is, with futures contracts having the same underlying instrument. In the next module, we explore inter-market spreads, that is, with futures contracts have different underlying instruments.

1. Long stock index (LS)
2. Long stock index futures nearby contract (LSFN1)
3. Long stock index futures second nearby contract (LSFN2)
4. Long stock index futures third nearby contract (LSFN3)
5. Short stock index futures nearby contract (SSFN1)
6. Short stock index futures second nearby contract (SSFN2)
7. Short stock index futures third nearby contract (SSFN3)
8. Long stock index futures spread trade (version 1), long nearby contract and short second nearby contract (only one maturity apart, LST1N1)
9. Long stock index futures spread trade (version 1), long second nearby contract and short third nearby contract (only one maturity apart, LST1N2)
10. Long stock index futures spread trade (version 1), long third nearby contract and short fourth nearby contract (only one maturity apart, LST1N3)

11. Short stock index futures spread trade (version 1), short nearby contract and long second nearby contract (only one maturity apart, SST1N1)
12. Short stock index futures spread trade (version 1), short second nearby contract and long third nearby contract (only one maturity apart, SST1N2)
13. Short stock index futures spread trade (version 1), short third nearby contract and long fourth nearby contract (only one maturity apart, SST1N3)
14. Long stock index futures spread trade (version 2), long first nearby contract and short third nearby contract (two maturities apart, LST2N1)
15. Long stock index futures spread trade (version 2), long first nearby contract and short fourth nearby contract (three maturities apart, LST2N2)
16. Long stock index futures spread trade (version 2), long first nearby contract and short fifth nearby contract (four maturities apart, LST2N3)
17. Short stock index futures spread trade (version 2), short first contract and long third nearby contract (two maturities apart, SST2N1)
18. Short stock index futures spread trade (version 2), short first nearby contract and long fourth nearby contract (three maturities apart, SST2N2)
19. Short stock index futures spread trade (version 2), short first nearby contract and long fifth nearby contract (four maturities apart, SST2N3)

Table 13.1.1 illustrates the influence of the correlation between stock index growth rates and the level all-in carry costs. As expected, long term interest rate correlation with the underlying has minimal influence on short-dated futures contracts. Note with the positive 10% stock index growth rate, the short futures positions have a significantly higher return VaR at the 99% level. Both version 1 (nearby pairs) and version 2 (distant pairs) spread trades have dramatically lower return VaRs. Version 2 (distant pairs) have higher return VaRs as expected.

Table 13.1.1 Return VaR Based CAM Stock Index Futures Stock and Level Factor Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.73	29.58	30.02	30.06	29.83	29.68	30.14
LSFN1	32.97	32.81	33.28	33.31	33.09	32.96	33.41
LSFN2	32.97	32.83	33.29	33.33	33.13	33.01	33.47
LSFN3	32.95	32.83	33.30	33.35	33.16	33.04	33.52
SSFN1	45.17	45.37	45.04	45.33	45.39	45.29	45.47
SSFN2	45.58	45.83	45.50	45.82	45.89	45.80	45.99
SSFN3	46.02	46.28	45.95	46.31	46.39	46.30	46.50
LST1N1	0.45	0.46	0.47	0.49	0.50	0.51	0.52
LST1N2	0.44	0.45	0.47	0.48	0.49	0.50	0.51
LST1N3	0.44	0.45	0.46	0.48	0.49	0.50	0.51
SST1N1	0.33	0.34	0.35	0.37	0.37	0.38	0.40
SST1N2	0.32	0.33	0.35	0.36	0.37	0.38	0.39
SST1N3	0.32	0.33	0.34	0.36	0.37	0.38	0.39
LST2N1	0.89	0.92	0.94	0.97	0.99	1.02	1.03
LST2N2	1.32	1.36	1.40	1.45	1.49	1.52	1.55
LST2N3	1.76	1.81	1.86	1.93	1.98	2.02	2.06
SST2N1	0.65	0.67	0.70	0.73	0.74	0.76	0.79
SST2N2	0.96	1.00	1.05	1.08	1.11	1.14	1.18
SST2N3	1.28	1.33	1.39	1.44	1.48	1.51	1.57

Table 13.1.2 illustrates the influence of the correlation between stock index growth rates and the slope all-in carry costs. When comparing Table 13.1.1 with 13.1.2, we note that the correlation with slope has nearly the same impact on return VaR when compared to level.

Table 13.1.2 Return VaR Based CAM Stock Index Futures Stock and Slope Factor Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.83	30.29	30.05	30.07	30.19	29.81	29.95
LCXL	33.01	33.49	33.25	33.29	33.42	33.06	33.20
LCX	32.96	33.44	33.22	33.28	33.40	33.08	33.22
LCXH	32.92	33.40	33.20	33.27	33.42	33.08	33.24
LPXL	45.27	45.54	45.21	45.40	44.85	45.06	45.39
LPX	45.67	45.95	45.63	45.82	45.29	45.53	45.86
LPXH	46.07	46.37	46.04	46.26	45.74	45.99	46.33
LCCWXL	0.40	0.42	0.43	0.44	0.46	0.47	0.49
LCCWX	0.42	0.43	0.44	0.44	0.45	0.46	0.47
LCCWXH	0.43	0.44	0.44	0.45	0.45	0.46	0.47
LPPBXL	0.28	0.30	0.31	0.33	0.34	0.35	0.37
LPPBX	0.30	0.31	0.32	0.33	0.34	0.34	0.35
LPPBXH	0.32	0.32	0.32	0.33	0.34	0.34	0.35
LLCXL	0.82	0.84	0.87	0.89	0.91	0.93	0.96
LLCX	1.25	1.28	1.31	1.33	1.36	1.39	1.43
LLCXH	1.70	1.73	1.76	1.79	1.82	1.85	1.89
LLPXL	0.58	0.61	0.63	0.65	0.68	0.69	0.72
LLPX	0.90	0.93	0.95	0.99	1.02	1.03	1.07
LLPXH	1.22	1.26	1.28	1.32	1.36	1.37	1.41

Table 13.1.3 illustrates the influence of the correlation between the level all-in carry costs and the slope all-in carry costs. We would expect that this correlation would not influence various futures strategies significantly and that is what is recorded. Again, the results in Table 13.1.3 are not materially different from the prior two tables.

Table 13.1.3 Return VaR Based CAM Stock Index Futures Level and Slope Factor Correlation Margin = 10%, Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	29.76	30.61	29.87	30.16	30.12	29.76	30.24
LSFN1	33.00	33.86	33.13	33.40	33.37	32.99	33.49
LSFN2	33.01	33.86	33.14	33.42	33.39	32.98	33.52
LSFN3	33.02	33.86	33.14	33.41	33.41	32.97	33.53
SSFN1	45.10	45.12	45.33	44.94	45.39	45.10	45.07
SSFN2	45.57	45.57	45.78	45.40	45.86	45.54	45.54
SSFN3	46.03	46.04	46.25	45.87	46.32	46.01	46.00
LST1N1	0.47	0.47	0.47	0.47	0.48	0.48	0.48
LST1N2	0.46	0.46	0.47	0.46	0.47	0.47	0.47
LST1N3	0.46	0.46	0.46	0.46	0.47	0.46	0.46
SST1N1	0.35	0.36	0.35	0.36	0.36	0.36	0.36
SST1N2	0.34	0.35	0.34	0.35	0.35	0.35	0.35
SST1N3	0.34	0.35	0.34	0.34	0.34	0.34	0.35
LST2N1	0.93	0.93	0.94	0.93	0.95	0.95	0.94
LST2N2	1.39	1.39	1.40	1.39	1.41	1.41	1.41
LST2N3	1.85	1.86	1.87	1.85	1.88	1.87	1.87
SST2N1	0.69	0.71	0.70	0.71	0.71	0.71	0.71
SST2N2	1.03	1.06	1.04	1.05	1.05	1.05	1.06
SST2N3	1.37	1.40	1.38	1.40	1.40	1.39	1.41

Table 13.1.4 illustrates the influence of margin on the return VaR. As expected, return VaR is very sensitive to margin. The higher the margin, the lower the return VaR. One interesting insight is the ability to dial up or dial down the risk level of various futures portfolios simply by manipulating cash collateral held. In fact, one could hold more that 100% margin and dial the risk level well below that of the underlying instrument. Clearly, the riskiness of futures portfolios is directly linked to cash collateral held.

Table 13.1.4 Return VaR Based CAM Stock Index Futures Margin Requirements
Number of Simulations = 100,000, Confidence Level = 99%

Strategy\Margin (%)	10	20	30	40	50	60	70	80	90	100
LS	29.69	14.89	9.99	7.41	6.02	4.98	4.30	3.77	3.34	2.99
LSFN1	32.94	16.51	11.08	8.22	6.67	5.53	4.77	4.18	3.70	3.31
LSFN2	32.96	16.53	11.08	8.23	6.67	5.53	4.77	4.18	3.70	3.32
LSFN3	32.97	16.53	11.08	8.23	6.67	5.53	4.77	4.18	3.70	3.32
SSFN1	45.16	22.71	15.26	11.35	9.04	7.49	6.42	5.68	5.03	4.50
SSFN2	45.63	22.95	15.42	11.47	9.14	7.57	6.49	5.74	5.09	4.55
SSFN3	46.09	23.17	15.57	11.59	9.23	7.64	6.55	5.80	5.14	4.59
LST1N1	0.47	0.24	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
LST1N2	0.46	0.23	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
LST1N3	0.46	0.23	0.16	0.12	0.09	0.08	0.07	0.06	0.05	0.05
SST1N1	0.35	0.18	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.04
SST1N2	0.34	0.17	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.03
SST1N3	0.34	0.17	0.11	0.08	0.07	0.06	0.05	0.04	0.04	0.03
LST2N1	0.93	0.47	0.32	0.24	0.19	0.16	0.13	0.12	0.10	0.09
LST2N2	1.39	0.70	0.47	0.35	0.28	0.23	0.20	0.17	0.16	0.14
LST2N3	1.85	0.93	0.63	0.47	0.37	0.31	0.26	0.23	0.21	0.19
SST2N1	0.69	0.35	0.23	0.17	0.14	0.12	0.10	0.09	0.08	0.07
SST2N2	1.03	0.52	0.35	0.26	0.21	0.17	0.15	0.13	0.12	0.10
SST2N3	1.37	0.69	0.46	0.34	0.28	0.23	0.20	0.17	0.15	0.14

Quantitative finance materials

We first briefly review the carry arbitrage model from Modules 6.1 and 9.1 for the purpose of developing potential simulations. We then develop a dynamic risk measures (DRMs) framework for appraising futures portfolio risk.

Carry arbitrage model review

Recall one simple way to express the CAM is

$$F_{n,t} = FV_{cc,n}\left(S_t\right) \text{ for all } n \tag{13.1.1}$$

where $FV_{cc,n}$ denotes the future value operator based on specified carry costs for the n^{th} contract observed at time t. Carry costs include the financing costs and any other associated revenues and costs related to the underlying instrument, such as dividend yield, foreign interest rates, storage costs, insurance, and so on. Thus, in a futures market that reflects full arbitrage as expressed in the equation above should approximate the futures price. Further, changes in the futures price should depend solely on changes in the underlying instrument and changes in the various carrying costs.

For DRM, we allow the carry costs to be maturity varying based on the LSC model. Recall from Module 3.4 we introduced a generalized LSC model expressed as

$$y_i = \sum_{j=0}^{N} x_{i,j} f_j , \tag{13.1.2}$$

where y_i denotes in this context is the carry costs for the i^{th} futures contract that varies in maturity time, $x_{i,j}$ denotes input LSC coefficients based on some maturity and some factor, N denotes the number of LSC factors (plus level), and f_j denotes the output factors. The general LSC model assumes

$$x_{i,0} = 1 ,\ x_{i,1} = \frac{s_1}{\tau_i}\left(1 - e^{-\tau_i/s_1}\right), \text{ and } x_{i,j} = \frac{s_j}{\tau_i}\left(1 - e^{-\tau_i/s_j}\right) - e^{-\tau_i/s_j} ; j > 1 . \tag{13.1.3}$$

In this module, we apply only a two-factor model. Recall we assume the input scalars, s_j, are provided by the user, where $s_1 = s_2$. Again s_j denotes scalars that applies various weights to different locations on the carry cost maturity structure, $x_{i,j}$ denotes LSC maturity coefficients, a parameter that depends solely on maturity time and selected scalars, and f_j denotes the output LSC factor, a parameter that is typically found using ordinary least squares regression applied to estimated maturity time carry costs. In this module, the LSC factor values as well as their stochastic behaviors are inputs into the DRMs analysis.

For this module, we assume a simple two-factor model. With continuous compounding, we have

$$F_{n,t} = S_t e^{cc_n \tau_n} \text{ for all } n \tag{13.1.4}$$

where τ_n (= $T_n - t$) denotes the time to maturity in years for the nth nearby futures contract. Thus, based on a two-factor LSC model, we have

$$F_{n,t} = S_t e^{\left(L_{cc} + x_{n,1} S_{cc}\right)\tau_n} = S_t e^{\left[L_{cc} + \frac{s_1}{\tau_n}\left(1 - e^{-\tau_n / s_1}\right) S_{cc}\right]\tau_n} \text{ for all } n, \tag{13.1.5}$$

where L_{cc} denotes all-in carry cost level factor and S_{cc} denotes all-in carry cost slope factor. Thus, we assume that given these two factors combined with the underlying stock index value, then the futures price is known.

Dynamic risk measures applied to stock index futures portfolio

Assuming a risk management horizon, h, the horizon futures price can be modeled as

$$\tilde{F}_{n,t+h} = \tilde{S}_{t+h}\left(\tilde{L}_{cc,t+h}, \tilde{S}_{cc,t+h}, \tilde{\varepsilon}_{t+h}\right) e^{\left[\tilde{L}_{cc,t+h} + \frac{s_1}{\tau_n - h}\left(1 - e^{-(\tau_n - h)/s_1}\right)\tilde{S}_{cc,t+h}\right](\tau_n - h)} \text{ for all } n. \tag{13.1.6}$$

We assume three correlated risk factors driving the stock index value at time $t + h$, the level and slope of the all-in carry costs as well as a residual value accounting for all other stock index value variation. As written, we explicitly recognize the possibility that the underlying stock index is correlated with parameters of the LSC model.

We model changes in the futures contract portfolio as

$$\Delta\tilde{\Pi}_{t,t+h} = \sum_{j=1}^{N_m} N_j \Delta\tilde{F}_{n,t+h} = \sum_{j=1}^{N_m} N_j\left(\tilde{F}_{n,t+h} - F_{n,t}\right) \text{ for all } n, \tag{13.1.7}$$

where N_j denotes the number of nth nearby futures contracts (positive denotes long and negative denotes short).

The challenge is developing a rational basis for the cash collateral amount. As with interest rate swaps, we seek a proportion of some notional amount but with the capacity to be long, short, or some combination of both, it is fraught with the potential for unusual cases. We propose the following ECC representation:

$$ECC_t = \omega \max\left(\left|\sum_{j=1}^{N_m} N_j F_{n,t}\right|, S_t\right). \tag{13.1.8}$$

First, we aggregate the futures portfolio notional value. As this amount may be negative, we compute the absolute value of the net portfolio. As this amount may be zero, we also take the maximum with respect to the underlying instrument value. Finally, we allow the analyst to choose a proportion of this amount for the final ECC value, but we require ω to be positive.

Thus, the holding period returns from which RVaR will be extracted is

$$R_t = \frac{\Delta\tilde{\Pi}_{t,t+h}}{\omega \max\left(\left|\sum_{j=1}^{N_m} N_j F_{n,t}\right|, S_t\right)}. \tag{13.1.9}$$

Summary

We focused on the influence of correlation between the underlying growth rate and parameters related to the all-in carry costs (level and slope of the fitted carry cost curve). We illustrated with Monte Carlo simulation the influence of different correlations. Finally, we addressed the role of margin in the form of cash collateral on the return value-at-risk.

Within the quantitative finance materials, we worked out the appropriate mathematical relationships required for the simulations. We specifically address the role of cash collateral on holding period returns for futures-related positions.

References

See Module 6.1.