Module 12.4: DRM ABM-Based Option Valuation Models

Learning objectives

- Apply Monte Carlo simulation to explore interactions between various inputs to the *arithmetic* Brownian motion option valuation model
- Illustrate the insights gained from Monte Carlo simulation with a focus on correlation between the underlying stock price and volatility

Executive summary

Based on the material presented in Module 5.5 and Module 8.4, we illustrate applying Monte Carlo simulation to analyzing the value-at-risk within the ABM option valuation model (ABMOVM).

The materials presented here are designed to parallel Modules 12.1-3 to facilitate comparison.

Central finance concepts

The main idea is once we have a robust valuation model (Module 5.5) as well as an understanding of static risk measures (Module 8.4), we are now able to explore various dynamic risk measures. For a review of the valuation models used here see Module 5.5.

ABM-based European-style option valuation models

Recall there are several technical assumptions required for the ABMOVM to theoretically hold. The key assumptions include option are European-style, ABM, financing available at the risk-free interest rate, no market frictions, and constant volatility. Although in practice none of these assumptions are valid, still the ABMOVM is incredibly useful in providing guidance on a host of financial decisions, such as relative value (comparing one option with an alternative), future likelihoods (such as the probability of an option being in-the-money), and sensitivities (such as the Greeks like delta that measures the sensitivity of the option value to the underlying instrument price).

Because options are European-style, we assume a continuous cash flow yield such as dividend yield. Discrete dividends can be handled with the escrow method.

Option valuation models and Value-at-Risk

In the quantitative materials below, we explore in detail VaR metrics related to the following 19 option-related strategies:

- Long stock (LS)
- Long call (LC, in-, at-, and out-of-the-money)
- Long put (LP, in-, at-, and out-of-the-money)
- Covered call writing (CCW, in-, at-, and out-of-the-money)
- Protective put buying (PPB, in-, at-, and out-of-the-money)
- Leveraged calls (LC, in-, at-, and out-of-the-money)
- Leveraged puts (LP, in-, at-, and out-of-the-money)

Covered call writing comprises long stock and short calls. Protective put buying comprises long stock and long put. Leveraged calls comprises long stock and long calls. Leveraged puts comprises long stock and short puts.

To illustrate this analysis, we assume the following inputs:

- Stock price = \$100
- Strike price = \$90, \$100, and \$110
- Interest rate = 5%
- Dividend yield = 0%
- Volatility = \$29.8848 (calibrated to 30% relative volatility in GBM)
- Time to maturity = 1 year
- Style = European
- Payout type = Plain vanilla

For illustration, we assume the stock price, interest rate, and volatility are subsequently random. Note that the option valuation framework assumes volatility and interest rates are constant. Dynamic risk management often requires a balance between theoretical models and practical implementation. Thus, we assume options are valued based on arithmetic Brownian motion option valuation model while simultaneously assuming the desired quantitative analysis is based on professional judgment within the firm.

We assume the following parameterizations:

- Horizon = 1 month
- Confidence level = 95%
- Number of simulations = 10,000
- Means (annualized, continuously compounded, percentage change)
  - Stock = 5%
  - Rate = 0%
  - Volatility = 0%
- Standard deviations
  - Stock = 30%
  - Rate = 10%
  - Volatility = $40
- Correlations
  - Stock, Rate = –0.30
  - Rate, Volatility = 0.00
  - Stock, Volatility = –0.50

In the tables presented below, XL denotes the low strike price ($90), X denotes the mid strike price ($100), and XH denotes the high strike price ($110). Thus, LCXH denotes the long call with a high strike price. Note that these various strategies require different levels of dollar investment; hence, for ease of analysis we report only return VaR (distance from $0) as opposed to dollar VaR.

The results reported below are expected to be very similar to the ABM BOVM results reported in Module 12.2 as the binomial model converges to the ABMOVM in the limit. For completeness, we replicated the discussion from Module 12.2.

Table 12.4.1 presents the results of the simulation based on the initial parameterization given above and allowing the correlation between stock returns and stock volatility to range from –0.75 to +0.75 incrementing by 0.25. Note that the number of simulations is 10,000 and the confidence level is 95%. The ABMOVM is "closed-form," hence, the calculations are performed dramatically fast than the binomial model. One unfortunate consequence is the lack of an American-style model.

Table 12.4.1 Return VaR Based on ABMOVM Stock Return and Volatility Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	3.66	3.66	3.64	3.58	3.58	3.62	3.61
LCXL	13.52	14.38	14.86	15.41	16.18	16.38	17.01
LCX	16.01	17.27	18.15	18.88	20.07	20.46	21.28
LCXH	18.73	20.57	21.98	23.14	24.64	25.18	26.35
LPXL	33.30	31.31	30.05	28.87	27.50	24.97	23.08
LPX	27.49	25.67	24.82	23.98	23.02	20.92	19.79
LPXH	22.34	20.93	20.28	19.64	19.05	17.48	16.89
LCCWXL	1.19	1.06	0.91	0.79	0.63	0.49	0.29
LCCWX	1.66	1.53	1.37	1.25	1.08	0.94	0.76
LCCWXH	2.13	2.01	1.86	1.74	1.59	1.46	1.30
LPPBXL	2.27	2.43	2.53	2.62	2.77	2.81	2.92
LPPBX	1.74	1.89	2.01	2.11	2.26	2.31	2.42
LPPBXH	1.20	1.34	1.46	1.55	1.68	1.72	1.82
LLCXL	5.31	5.46	5.50	5.51	5.68	5.74	5.88
LLCX	5.17	5.34	5.41	5.42	5.60	5.70	5.82
LLCXH	4.93	5.08	5.17	5.19	5.37	5.46	5.57
LLPXL	5.24	5.11	4.96	4.79	4.66	4.57	4.38
LLPX	6.01	5.88	5.70	5.50	5.37	5.27	5.05
LLPXH	6.97	6.83	6.67	6.46	6.31	6.23	6.01

There are several insights that can be drawn from the table. First, the Long Stock (LS) row illustrates that Monte Carlo simulation even with 10,000 simulation results in variation of return value-at-risk (RVaR) at the 95% confidence level. RVaR ranges from 3.58% ($\rho = 0.\ 0$) to 3.66% ($\rho = -0.75$). As the number of simulations increase, distribution parameters tend to stabilize, but the tails of the distribution are much slower to converge.

Second, focusing on the uncorrelated case ($\rho = 0.0$), RVaR increases with the strike price for Long Call (LC) ranging from 15.41% for the low strike price (XL) to 23.14% for the high strike price (XH). Recall the higher the strike price, the higher the implied leverage and hence, the higher the RVaR. We see the opposite pattern with puts. RVaR decreases with the strike price for Long Put (LP) ranging from 28.87% for the low strike price (XL) to 19.64% for the high strike price (XH). With puts, the higher the strike price, the lower the implied leverage (further in-the-money). Note that the patterns are similar for AS options but higher in magnitude for puts due to the additional early exercise premium.

Third, the remaining option blended strategies have dramatically lower RVaRs when compared to long calls and puts. The primary reason is the dramatically higher investment required for the underlying long stock position that is unleveraged.

Fourth, the same patterns noted above hold for covered call writing and protective put buying. In both cases, the further out-of-the-money, the less risk mitigation and hence the higher RVaR. As expected, the opposite pattern holds for leveraged calls and puts.

Fifth, the correlation between stock returns and volatility does influence RVaR although it has no direct theoretical impact on the underlying instrument's (stock, calls, and puts) value. For long calls, the RVaR increases with correlation and for long puts, the RVaR decreases with correlation. For covered call writing, the RVaR decreases with correlation and for protective put buying, the RVaR increases with correlation. For leveraged calls, the RVaR increases with correlation and for leveraged puts, the RVaR decreases with correlation.

In summary, although perhaps not a focus when valuing options, correlation between the underlying instrument returns and volatility is an important determinant of various dynamic risk measures, such as RVaR.

Table 12.4.2 presents the results of the simulation allowing the correlation between stock returns and interest rates to range from –0.75 to +0.75 incrementing by 0.25. As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.4.2 Return VaR Based on ABM BOVM Stock Return and Interest Rate Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	3.62	3.58	3.57	3.62	3.55	3.69	3.66
LCXL	13.98	13.99	14.09	14.29	13.98	14.53	14.37
LCX	16.80	16.89	17.00	17.08	16.88	17.48	17.25
LCXH	20.15	20.24	20.44	20.41	20.30	20.89	20.55
LPXL	31.81	32.12	31.41	32.05	31.97	31.54	32.12
LPX	26.24	26.49	26.01	26.40	26.41	25.98	26.56
LPXH	21.31	21.46	21.28	21.41	21.45	21.15	21.52
LCCWXL	1.12	1.02	1.03	1.04	1.00	1.02	1.02
LCCWX	1.59	1.48	1.47	1.51	1.45	1.49	1.49
LCCWXH	2.07	1.95	1.96	1.96	1.93	1.97	1.97
LPPBXL	2.37	2.37	2.37	2.39	2.32	2.42	2.37
LPPBX	1.87	1.86	1.86	1.86	1.82	1.89	1.84
LPPBXH	1.33	1.33	1.33	1.31	1.29	1.32	1.28
LLCXL	5.32	5.32	5.32	5.40	5.31	5.52	5.48
LLCX	5.21	5.21	5.21	5.28	5.19	5.39	5.33
LLCXH	4.97	4.98	4.97	5.04	4.95	5.14	5.09
LLPXL	5.13	4.96	5.00	5.03	5.02	5.13	5.09
LLPX	5.88	5.70	5.75	5.79	5.77	5.90	5.86
LLPXH	6.83	6.66	6.71	6.74	6.73	6.89	6.82

Table 12.4.3 presents the results of the simulation allowing the correlation between volatility and interest rates to range from –0.75 to +0.75 incrementing by 0.25. As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.4.3 Return VaR Based on ABM BOVM Volatility and Interest Rate Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	3.62	3.57	3.66	3.57	3.60	3.63	3.69
LCXL	14.10	13.98	14.27	14.05	14.28	14.33	14.30
LCX	16.91	16.87	17.10	16.86	17.17	17.15	17.21
LCXH	20.20	20.25	20.42	20.11	20.42	20.42	20.65
LPXL	31.57	31.83	31.59	31.59	32.13	31.77	31.28
LPX	26.05	26.29	26.01	26.00	26.46	26.10	25.85
LPXH	21.15	21.53	21.26	21.09	21.50	21.21	21.07
LCCWXL	1.02	1.02	1.06	1.03	1.05	1.05	1.09
LCCWX	1.47	1.47	1.52	1.49	1.51	1.50	1.55
LCCWXH	1.96	1.95	2.00	1.97	1.98	1.97	2.03
LPPBXL	2.38	2.35	2.41	2.36	2.40	2.41	2.41
LPPBX	1.87	1.86	1.89	1.85	1.88	1.88	1.88
LPPBXH	1.32	1.33	1.34	1.30	1.33	1.32	1.33
LLCXL	5.38	5.31	5.43	5.32	5.37	5.41	5.48
LLCX	5.26	5.19	5.30	5.20	5.25	5.29	5.35
LLCXH	5.02	4.97	5.06	4.97	5.02	5.06	5.10
LLPXL	5.04	4.99	5.13	5.03	5.06	5.06	5.15
LLPX	5.79	5.74	5.89	5.77	5.81	5.82	5.91
LLPXH	6.75	6.67	6.87	6.71	6.75	6.79	6.89

In summary, the ability to conduct RVaR analyses under different sets of simulation assumptions dramatically increases the types of analysis possible for risk managers.

Quantitative finance materials

The quantitative analysis is based on prior materials covered in Modules 5.5 and 8.4. For convenience, we provide selected key formulas here. Recall based on a set of restrictive assumptions, the ABMOVM can be expressed as

$$O\left(S_t,t;\iota_U,X,T,r,\sigma,\delta\right)=\iota_U\left(S_0B_\delta-XB_r\right)N\left(\iota_U d_n\right)+\sigma_A B_r n\left(d_n\right)$$

where again the indicator functions is expressed as

$$\iota_U=\begin{cases}+1 \text{ if underlying call option}\\ -1 \text{ if underlying put option}\end{cases},$$

$$B_r=e^{-r(T-t)},\ \ B_\delta=e^{-\delta(T-t)}$$

$$N\left(d\right)=\int_{-\infty}^{d}\frac{e^{-x^2/2}}{\sqrt{2\pi}}dx, \text{ (area under the standard cumulative normal distribution up to d)}$$

$$n\left(d\right)=\frac{e^{-d^2/2}}{\sqrt{2\pi}}, \text{ (standard normal probability density function)}$$

$$A=\frac{B_{-2(r-\delta)}-1}{2\left(r-\delta\right)}, \text{ (periodic adjustment to volatility)}$$

$$\sigma_A^2=\sigma^2A=\sigma^2\frac{B_{-2(r-\delta)}-1}{2\left(r-\delta\right)}, \text{ and (periodic adjusted volatility)}$$

$$d_n=\frac{S_tB_{-(r-\delta)}-X}{\sigma_A}. \text{ (quasi “z” score)}$$

If there is only a cash flow yield, then the call and put option equations can be expressed as

$$c_t = \left(S_t e^{-\delta(T-t)} - Xe^{-r(T-t)}\right) N\left(d_n\right) + \sigma_A e^{-r(T-t)} n\left(d_n\right)$$

$$p_0 = \left(Xe^{-r(T-t)} - S_0 e^{-\delta(T-t)}\right) N\left(-d_n\right) + \sigma_A e^{-r(T-t)} n\left(d_n\right)$$

Thus, the initial value of the various options is determined based on the model above. The simulation is run, and the options are subsequently revalued incorporating the new values for the stock, rate, and volatility as well as the passage of calendar time. Once all the simulations are run, then return VaR is estimated and reported in the tables above.

As identified in prior modules, there are several alternative strategies that could also be pursued. Selected potential strategies to consider include:

- Short stock (designated cash margin percentage (e.g., 100%), m_S)
- Short call (designated cash margin percentage of underlying stock (e.g., 10%), m_c)
- Short put (designated cash margin percentage of underlying stock (e.g., 10%), m_p)
- Short CCW: Short stock, long call (synthetic leveraged long put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sCCW})
- Short PPB: Short stock, short put (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sPPB})
- LSC: Short stock, short call (leveraged short call) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLSC})
- LLP: Short stock, long put (leveraged long put) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLLP})

Further, one could add additional stocks to explore various cross-correlations.

Summary

As illustrated with these simple simulations, the ability to conduct RVaR analyses under different sets of simulation assumptions dramatically increases the types of analysis possible for risk managers. Risk managers should be constantly exploring various interactions as well as stress testing parameter assumptions.

References

See modules 5.2 and 8.1.