Module 12.2: DRM ABM-Based Binomial Models

Learning objectives

- Apply Monte Carlo simulation to explore interactions between various inputs to the *arithmetic* Brownian motion binomial option valuation model
- Illustrate the insights gained from Monte Carlo simulation with a focus on correlation between the underlying stock price and volatility

Executive summary

Based on the material presented in Module 5.3 and Module 8.2, we illustrate applying Monte Carlo simulation to analyzing the value-at-risk within the ABM binomial option valuation model for both European-style and American-style options.

The materials presented here are designed to parallel Module 12.1 to facilitate comparison.

Central finance concepts

The main idea is once we have a robust valuation model (Module 5.3) as well as an understanding of static risk measures (Module 8.2), we are now able to explore various dynamic risk measures. For a review of the valuation models used here see Module 5.3.

ABM-based European-style binomial option valuation models

Recall the ABM-based binomial option framework is designed to converge to a *normal* distribution in the limit to be consistent with the ABMOVM. This binomial framework has several objectives:

1. Additive.
2. Recombining.
3. Incorporate dividends.
4. Address early exercise with American-style options.

Additive and recombining are incorporated using *u* and *d* parameters at each node.

There are several ABM-based multiperiod valuation models including when there are no dividends, when a dividend yield is assumed, and when discrete dividends are assumed. Further, there are several alternative ways to frame these models such as based on digital valuation models. We focus here on dividend yields and plain vanilla options.

ABM-based American-style binomial option valuation models

For American-style options, the early exercise potential must be incorporated. As discussed in the prior modules, the approach typically taken is known as backward induction. At each node, we must compare the following values, the model option value, the early exercise value, and the lower boundary condition. The existence of various forms of dividends simply changes the required formulas.

Binomial option valuation models and Value-at-Risk

In the quantitative materials below, we explore in detail VaR metrics related to the following 19 option-related strategies:

- Long stock (LS)
- Long call (LC, in-, at-, and out-of-the-money)
- Long put (LP, in-, at-, and out-of-the-money)
- Covered call writing (CCW, in-, at-, and out-of-the-money)
- Protective put buying (PPB, in-, at-, and out-of-the-money)
- Leveraged calls (LC, in-, at-, and out-of-the-money)
- Leveraged puts (LP, in-, at-, and out-of-the-money)

Covered call writing comprises long stock and short calls. Protective put buying comprises long stock and long put. Leveraged calls comprises long stock and long calls. Leveraged puts comprises long stock and short puts.

To illustrate this analysis, we assume the following inputs:

- Stock price = $100
- Strike price = $90, $100, and $110
- Interest rate = 5%
- Dividend yield = 0%
- Volatility = $29.8848 (calibrated to 30% relative volatility in GBM)
- Time to maturity = 1 year
- Style = European
- Payout type = Plain vanilla
- EMM probability = 50%

For illustration, we assume the stock price, interest rate, and volatility are subsequently random. Note that the option valuation framework assumes volatility and interest rates are constant. Dynamic risk management often requires a balance between theoretical models and practical implementation. Thus, we assume options are valued based on geometric Brownian motion and the binomial framework while simultaneously assuming the desired quantitative analysis is based on professional judgment within the firm.

We assume the following parameterizations:

- Horizon = 1 month
- Confidence level = 90%
- Number of simulations = 2,000
- Means (annualized, continuously compounded, percentage change)
    - Stock = 5%
    - Rate = 0%
    - Volatility = 0%
- Standard deviations
    - Stock = 30%
    - Rate = 10%
    - Volatility = $40.00
- Correlations
    - Stock, Rate = –0.3
    - Rate, Volatility = 0.0
    - Stock, Volatility = –0.5

In the tables presented below, XL denotes the low strike price ($90), X denotes the mid strike price ($100), and XH denotes the high strike price ($110). Thus, LCXH denotes the long call with a high strike price. Note that these various strategies require different levels of dollar investment; hence, for ease of analysis we report only return VaR (distance from $0) as opposed to dollar VaR.

Table 12.2.1 presents the results of the simulation based on the initial parameterization given above and allowing the correlation between stock returns and stock volatility to range from –0.75 to +0.75 incrementing by 0.25. Panel A presents European-style (ES) and Panel B presents American-style (AS).

Table 12.2.1 Return VaR Based on ABM BOVM Stock Return and Volatility Correlation

Panel A: European-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.77	2.63	2.73	2.76	2.61	2.80	2.76
LCXL	8.71	11.13	13.01	14.20	16.22	17.26	18.41
LCX	10.87	14.73	17.41	18.91	21.85	23.13	24.78
LCXH	15.62	20.70	23.37	25.32	29.24	30.93	33.36
LPXL	43.40	40.30	39.04	36.22	32.13	27.77	23.50
LPX	34.02	31.06	29.99	27.25	24.24	19.78	16.09
LPXH	25.94	23.70	22.37	20.13	17.92	14.51	11.38
LCCWXL	1.98	1.70	1.43	1.13	0.92	0.66	0.14
LCCWX	2.37	2.03	1.75	1.45	1.12	0.83	0.27
LCCWXH	2.62	2.19	1.99	1.68	1.32	0.94	0.45
LPPBXL	1.33	1.81	2.13	2.37	2.77	2.97	3.20
LPPBX	1.09	1.57	1.91	2.10	2.48	2.67	2.86
LPPBXH	0.94	1.34	1.57	1.71	2.04	2.18	2.37
LLCXL	3.52	3.85	4.24	4.47	4.69	5.04	5.28
LLCX	3.38	3.77	4.19	4.41	4.72	5.12	5.35
LLCXH	3.24	3.63	4.02	4.24	4.58	5.01	5.20
LLPXL	4.92	4.32	4.19	3.73	3.28	2.99	2.62
LLPX	5.65	4.96	4.85	4.34	3.81	3.48	3.06
LLPXH	6.50	5.66	5.54	5.04	4.46	4.16	3.77

Panel B American-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.86	2.84	2.84	2.57	2.80	2.60	2.77
LCXL	9.10	11.48	12.67	13.99	15.71	16.42	17.89
LCX	11.35	15.33	16.81	18.93	21.30	22.18	24.40
LCXH	16.05	20.61	23.27	26.75	28.66	29.39	32.56
LPXL	42.77	39.98	38.81	34.39	30.84	27.49	22.75
LPX	33.31	30.30	29.27	26.13	23.12	20.36	15.94
LPXH	24.89	22.62	21.74	19.19	16.96	14.76	12.28
LCCWXL	2.04	1.70	1.48	1.21	0.90	0.60	0.13
LCCWX	2.44	2.05	1.85	1.48	1.17	0.77	0.24
LCCWXH	2.68	2.25	2.04	1.64	1.34	0.95	0.37
LPPBXL	1.40	1.88	2.09	2.33	2.65	2.80	3.08
LPPBX	1.12	1.63	1.83	2.11	2.36	2.47	2.78
LPPBXH	0.94	1.29	1.49	1.71	1.85	1.89	2.11
LLCXL	3.74	3.94	4.22	4.25	4.77	4.88	5.26
LLCX	3.56	3.86	4.14	4.28	4.82	4.93	5.30
LLCXH	3.40	3.72	4.01	4.12	4.66	4.76	5.14
LLPXL	5.11	4.56	4.34	3.58	3.35	2.88	2.41
LLPX	5.86	5.25	5.00	4.19	3.94	3.40	2.89
LLPXH	6.67	6.07	5.75	4.88	4.71	4.13	3.72

There are several insights that can be drawn from the table. First, the Long Stock (LS) row illustrates that Monte Carlo simulation with 2,000 simulation results in variation of return value-at-risk (RVaR) at the 90% confidence level. RVaR ranges from 2.57% (AS, $\rho = 0.00$) to 2.86% (AS, $\rho = 0.50$). As the number of simulations increase, distribution parameters tend to stabilize, but the tails of the distribution are much slower to converge. We selected 90% confidence level as it converges faster than 95% or 99%.

Second, focusing on the uncorrelated ES case ($\rho = 0.0$), RVaR increases with the strike price for Long Call (LC) ranging from 14.20% for the low strike price (XL) to 25.32% for the high strike price (XH). Recall the higher the strike price, the higher the implied leverage and hence, the higher the RVaR. We see the opposite pattern with puts. RVaR decreases with the strike price for Long Put (LP) ranging from 36.22% for the low strike price (XL) to 20.13% for the high strike price (XH). With puts, the higher the strike price, the lower the implied leverage (further in-the-money). Note that the patterns are similar for AS options but higher in magnitude for puts due to the additional early exercise premium.

Third, the remaining option blended strategies have dramatically lower RVaRs when compared to long calls and puts. The primary reason is the dramatically higher investment required for the underlying long stock position that is unleveraged.

Fourth, the same patterns noted above hold for covered call writing and protective put buying. In both cases, the further out-of-the-money, the less risk mitigation and hence the higher RVaR. As expected, the opposite pattern holds for leveraged calls and puts.

Fifth, the correlation between stock returns and volatility does influence RVaR although it has no direct theoretical impact on the underlying instrument's (stock, calls, and puts) value. For long calls, the RVaR increases with correlation and for long puts, the RVaR decreases with correlation. For covered call writing, the RVaR decreases with correlation and for protective put buying, the RVaR increases with correlation. For leveraged calls, the RVaR increases with correlation and for leveraged puts, the RVaR decreases with correlation.

In summary, although perhaps not a focus when valuing options, correlation between the underlying instrument returns and volatility is an important determinant of various dynamic risk measures, such as RVaR.

Table 12.2.2 presents the results of the simulation allowing the correlation between stock returns and interest rates to range from –0.75 to +0.75 incrementing by 0.25. Panel A presents European-style (ES) and Panel B presents American-style (AS). As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.2.2 Return VaR Based on ABM BOVM Stock Return and Interest Rate Correlation

Panel A: European-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.80	2.90	2.75	2.62	2.72	2.74	2.66
LCXL	10.77	11.18	11.30	11.04	10.65	10.99	11.20
LCX	14.27	14.67	14.77	14.51	14.31	14.78	14.60
LCXH	19.94	20.52	20.33	20.08	20.27	20.08	20.47
LPXL	42.14	40.76	39.79	41.65	42.01	40.92	39.66
LPX	32.97	31.49	31.01	32.35	32.35	31.31	30.88
LPXH	24.69	24.19	23.76	24.94	24.08	24.29	23.66
LCCWXL	1.87	1.88	1.66	1.66	1.79	1.72	1.66
LCCWX	2.24	2.25	2.02	1.96	2.15	2.08	2.00
LCCWXH	2.46	2.51	2.29	2.14	2.37	2.34	2.18
LPPBXL	1.75	1.82	1.83	1.77	1.67	1.74	1.78
LPPBX	1.52	1.57	1.57	1.54	1.50	1.54	1.50
LPPBXH	1.30	1.34	1.31	1.27	1.30	1.26	1.28
LLCXL	3.87	3.95	3.98	3.77	3.87	3.89	3.99
LLCX	3.74	3.87	3.84	3.70	3.73	3.79	3.91
LLCXH	3.57	3.71	3.68	3.55	3.61	3.64	3.76
LLPXL	4.71	4.85	4.53	4.27	4.57	4.63	4.36
LLPX	5.42	5.57	5.21	4.95	5.26	5.33	5.04
LLPXH	6.11	6.32	5.93	5.66	5.98	6.06	5.73

Panel B American-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.65	2.73	2.70	2.72	2.85	2.64	2.89
LCXL	10.69	11.17	10.84	11.24	10.83	11.08	11.13
LCX	14.38	14.57	14.41	14.82	14.47	14.48	14.57
LCXH	19.82	20.36	20.16	20.36	20.14	19.60	20.09
LPXL	41.62	39.68	40.04	41.46	40.43	41.30	40.04
LPX	32.10	30.89	30.82	32.18	31.45	31.91	31.36
LPXH	23.95	23.07	22.92	23.86	23.49	23.73	23.36
LCCWXL	1.64	1.70	1.78	1.59	1.70	1.57	1.65
LCCWX	2.01	2.05	2.14	1.94	2.05	1.90	2.00
LCCWXH	2.18	2.26	2.37	2.17	2.33	2.13	2.25
LPPBXL	1.73	1.82	1.75	1.81	1.71	1.75	1.75
LPPBX	1.51	1.56	1.51	1.57	1.51	1.50	1.49
LPPBXH	1.24	1.26	1.25	1.25	1.25	1.20	1.22
LLCXL	3.76	3.95	3.82	3.90	3.92	3.78	4.11
LLCX	3.66	3.80	3.75	3.82	3.80	3.69	3.97
LLCXH	3.51	3.68	3.63	3.68	3.66	3.56	3.80
LLPXL	4.38	4.65	4.47	4.47	4.74	4.32	4.62
LLPX	5.05	5.40	5.15	5.19	5.47	5.01	5.36
LLPXH	5.81	6.13	5.95	5.98	6.26	5.72	6.11

Table 12.2.3 presents the results of the simulation allowing the correlation between volatility and interest rates to range from –0.75 to +0.75 incrementing by 0.25. Panel A presents European-style (ES) and Panel B presents American-style (AS). As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.2.3 Return VaR Based on ABM BOVM Volatility and Interest Rate Correlation

Panel A: European-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.81	2.80	2.76	2.78	2.68	2.70	2.63
LCXL	11.22	10.75	11.28	10.85	10.82	11.00	10.35
LCX	14.54	14.09	15.00	14.63	14.52	14.87	13.84
LCXH	20.28	19.29	20.05	20.12	20.00	20.14	19.44
LPXL	39.79	39.69	41.28	40.58	39.68	42.19	41.84
LPX	30.87	30.34	32.45	31.11	30.69	32.35	32.24
LPXH	23.48	22.80	24.65	23.29	23.42	24.38	24.45
LCCWXL	1.63	1.66	1.77	1.81	1.69	1.70	1.72
LCCWX	1.96	2.05	2.13	2.14	2.07	2.00	2.07
LCCWXH	2.18	2.27	2.31	2.31	2.31	2.21	2.26
LPPBXL	1.84	1.74	1.83	1.74	1.74	1.77	1.64
LPPBX	1.57	1.50	1.61	1.55	1.51	1.57	1.43
LPPBXH	1.35	1.25	1.30	1.30	1.27	1.29	1.21
LLCXL	4.05	3.90	4.09	3.89	3.97	3.87	3.74
LLCX	3.97	3.81	3.96	3.77	3.80	3.75	3.62
LLCXH	3.83	3.65	3.78	3.62	3.64	3.60	3.49
LLPXL	4.43	4.53	4.55	4.50	4.48	4.34	4.39
LLPX	5.11	5.21	5.24	5.18	5.17	5.01	5.06
LLPXH	5.81	5.92	5.98	5.95	5.95	5.75	5.77

Panel B American-style

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	2.66	2.76	2.61	2.65	2.80	2.83	2.76
LCXL	11.17	10.91	10.94	10.71	11.04	11.30	11.36
LCX	15.09	14.51	14.38	14.07	14.64	15.02	15.04
LCXH	21.11	19.86	20.25	19.56	20.42	20.76	20.85
LPXL	39.95	40.15	39.89	41.79	41.05	40.45	40.00
LPX	31.08	30.49	30.96	31.86	31.74	31.91	31.58
LPXH	23.57	22.80	23.23	23.93	23.96	23.86	23.42
LCCWXL	1.63	1.78	1.75	1.72	1.72	1.73	1.77
LCCWX	1.98	2.13	2.05	2.08	2.04	2.09	2.10
LCCWXH	2.19	2.37	2.26	2.29	2.27	2.35	2.33
LPPBXL	1.84	1.77	1.77	1.71	1.77	1.82	1.81
LPPBX	1.63	1.54	1.54	1.48	1.54	1.59	1.56
LPPBXH	1.34	1.24	1.28	1.22	1.26	1.27	1.28
LLCXL	3.78	3.93	3.86	3.74	3.86	3.96	3.93
LLCX	3.75	3.81	3.81	3.64	3.75	3.92	3.87
LLCXH	3.58	3.67	3.65	3.47	3.59	3.73	3.73
LLPXL	4.41	4.61	4.37	4.59	4.51	4.58	4.62
LLPX	5.08	5.30	5.03	5.25	5.21	5.31	5.31
LLPXH	5.78	6.04	5.71	5.94	5.97	6.15	6.03

In summary, the ability to conduct RVaR analyses under different sets of simulation assumptions dramatically increases the types of analysis possible for risk managers. Further, the ability to deploy multiple models, such as GBM-based and ABM-based provides more robust risk management opportunities.

Quantitative finance materials

The quantitative analysis is based on prior materials covered in Modules 5.3 and 8.2. As identified in these prior modules, ABM requires backward recursion for both European-style and American-style options. The implementation approach follows GBM BOVM for American-style option except for being additive rather than multiplicative.

Again, the initial value of the various options is determined based on the lattice given above. The simulation is run, and the options are subsequently revalued incorporating the new values for the stock, rate, and volatility as well as the passage of calendar time. Once all the simulations are run, then return VaR is estimated and reported in the tables above.

There are several alternative strategies that could also be pursued. Selected potential strategies to consider include:

- Short stock (designated cash margin percentage (e.g., 100%), m_S)
- Short call (designated cash margin percentage of underlying stock (e.g., 10%), m_c)
- Short put (designated cash margin percentage of underlying stock (e.g., 10%), m_p)
- Short CCW: Short stock, long call (synthetic leveraged long put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sCCW})
- Short PPB: Short stock, short put (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sPPB})
- LSC: Short stock, short call (leveraged short call) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLSC})
- LLP: Short stock, long put (leveraged long put) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLLP})

Further, one could add additional stocks to explore various cross-correlations.

Summary

As illustrated with these simple simulations, the ability to conduct RVaR analyses under different sets of simulation assumptions based on different valuation paradigms dramatically increases the types of analysis possible for risk managers. Risk managers should be constantly exploring various interactions among underlying parameters, various valuation paradigms as well as stress testing parameter assumptions.

References

See modules 5.3 and 8.2.