Module 12.5: DRM GBM-Based Compound Option Models

Learning objectives

- Apply Monte Carlo simulation to explore interactions between various inputs to the geometric Brownian motion compound option valuation model (call on call and put on call only)
- Illustrate the insights gained from Monte Carlo simulation with a focus on correlation between the underlying instrument price and volatility as well as interest rates and volatility

Executive summary

Based on the material presented in Module 5.6 and Module 8.5, we illustrate applying Monte Carlo simulation to analyzing the value-at-risk within the GBM compound option valuation model (GBM COVM). Note that this model only works for European-style options.

Central finance concepts

Again, this module is designed to track closely with all modules in this chapter to facilitate comparison. The main idea is once we have a robust valuation model (Module 5.6) as well as an understanding of static risk measures (Module 8.5), we are now able to explore various dynamic risk measures. For a review of the valuation models used here see Module 5.6.

GBM-based European-style compound option valuation models

Recall there are several technical assumptions required for the GBM COVM to theoretically hold. The key assumptions include option are European-style, GBM, financing available at the risk-free interest rate, no market frictions, and constant volatility. Although in practice none of these assumptions are valid, still the GBM COVM is unique in its ability to address firm valuation and equity as a call option. GBM COVM, like other models illustrated in this chapter, is incredibly useful in providing guidance on a host of financial decisions, such as relative value (comparing one option with an alternative), future likelihoods (such as the probability of an option being in-the-money), and sensitivities (such as the Greeks like delta that measures the sensitivity of the option value to the underlying instrument price).

Because options are European-style, we assume a continuous cash flow yield of the underlying as well as a yield on the underlying option. Discrete dividends can be handled with the escrow method.

Option valuation models and Value-at-Risk

In the quantitative materials below, we explore in detail VaR metrics related to the following 19 option-related strategies:

- Long underlying (e.g., firm value) (LS)
- Long compound call on call (LC, in-, at-, and out-of-the-money)
- Long compound put on call (LP, in-, at-, and out-of-the-money)
- Covered call writing (CCW, in-, at-, and out-of-the-money)
- Protective put buying (PPB, in-, at-, and out-of-the-money)
- Leveraged calls (LC, in-, at-, and out-of-the-money)
- Leveraged puts (LP, in-, at-, and out-of-the-money)

Covered call writing comprises long the underlying call option and short the compound call option. Recall in the context of stocks, the underlying is the firm, the underlying call option is the equity of the firm, and the compound option is the call or put on the underlying call (equity of firm). Protective put buying comprises long the underlying option and long the compound put on a call. Leveraged calls comprises long the underlying call and long the compound call on call. Leveraged puts comprises long the underlying call and short the compound put on call.

To illustrate this analysis, we assume the following inputs:

- Underlying price = \$100
- Underlying strike price = \$90, \$100, and \$110
- Compound strike price = \$20.46 (consistent with prior chapters)

- Interest rate = 5%
- Underlying yield = 5%
- Option yield = 0%
- Volatility = 30%
- Underlying call time to maturity = 5 years
- Compound option time to maturity = 1 year

For illustration, we assume the underlying price, interest rate, and volatility are subsequently random. Note that the option valuation framework assumes volatility and interest rates are constant. Dynamic risk management often requires a balance between theoretical models and practical implementation. Thus, we assume options are valued based on geometric Brownian motion compound option valuation model while simultaneously assuming the desired quantitative analysis is based on professional judgment within the firm.

We assume the following parameterizations:

- Horizon = 1 month
- Confidence level = 95%
- Number of simulations = 10,000
- Means (annualized, continuously compounded, percentage change)
  - Underlying = 5%
  - Rate = 0%
  - Volatility = 0%
- Standard deviations
  - Underlying = 30%
  - Rate = 10%
  - Volatility = 40%
- Correlations
  - Underlying, Rate = –0.3
  - Rate, Volatility = 0.0
  - Underlying, Volatility = –0.5

In the tables presented below, XL denotes the low strike price ($90), X denotes the mid strike price ($100), and XH denotes the high strike price ($110). Thus, LCXH denotes the long call with a high strike price. Note that these various strategies require different levels of dollar investment; hence, for ease of analysis we report only return VaR (distance from $0) as opposed to dollar VaR.

The results reported below are not expected to be similar to the GBMOVM results reported in Module 12.3 as the parameterization of the GBM COVM. We do expect similar RVaR patterns, however. For completeness, we follow closely the format of the discussion from Module 12.3.

Table 12.5.1 presents the results of the simulation based on the initial parameterization given above and allowing the correlation between stock returns and stock volatility to range from –0.75 to +0.75 incrementing by 0.25. Note that the number of simulations is 10,000 and the confidence level is 95%. The GBM COVM is "closed-form," hence, the calculations are performed dramatically fast than the binomial model. Again, one unfortunate consequence is the lack of an American-style model.

Table 12.5.1 Return VaR Based on GBM COVM Stock Return and Volatility Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	5.59	7.48	8.90	10.20	11.14	12.29	13.04
LCXL	13.22	16.74	19.43	21.84	23.51	25.54	27.00
LCX	14.85	18.90	22.54	24.94	27.22	29.39	30.90
LCXH	16.69	21.32	25.53	28.10	30.89	33.16	34.86
LPXL	46.71	44.45	42.31	38.73	35.73	31.41	25.63
LPX	35.31	33.41	31.49	28.35	25.75	22.38	17.66
LPXH	26.44	25.05	23.34	20.79	18.82	16.13	12.45
LCCWXL	0.41	1.20	1.76	2.30	2.74	3.26	3.61
LCCWX	1.66	2.50	3.02	3.61	4.12	4.67	5.08
LCCWXH	2.56	3.48	4.05	4.74	5.32	5.92	6.38
LPPBXL	4.93	6.88	8.71	9.87	11.29	12.18	12.90
LPPBX	4.98	6.64	8.23	9.29	10.50	11.28	11.72
LPPBXH	5.30	6.45	7.47	8.24	9.23	9.70	10.02
LLCXL	7.78	10.14	11.91	13.57	14.68	16.11	17.05
LLCX	7.72	10.14	12.09	13.70	14.92	16.29	17.29
LLCXH	7.62	10.04	12.04	13.62	14.88	16.25	17.21
LLPXL	7.96	9.39	10.03	10.98	11.80	12.59	13.26
LLPX	11.48	12.72	12.83	13.52	14.26	14.73	15.26
LLPXH	18.21	19.15	18.85	19.77	20.16	20.43	20.74

There are several insights that can be drawn from the table. First, the Long Stock (LS) row illustrates that Monte Carlo simulation even with 10,000 simulation results in variation of return value-at-risk (RVaR) at the 95% confidence level. In this case where stock is an option on the firm, RVaR ranges from 5.59% ($\rho = -0.75$) to 13.04% ($\rho = 0.75$). Thus, the optionality of the underlying stock is clearly seen with higher RVaR for higher correlations. This result is like GBMOVM results for long call, except the parameters are different.

Second, focusing on the uncorrelated case ($\rho = 0.0$), RVaR increases with the strike price for Long Call (LC) ranging from 21.84% for the low strike price (XL) to 28.10% for the high strike price (XH). Recall the higher the strike price, the higher the implied leverage and hence, the higher the RVaR. We see the opposite pattern with puts. RVaR decreases with the strike price for Long Put (LP) ranging from 38.73% for the low strike price (XL) to 20.79% for the high strike price (XH). With puts, the higher the strike price, the lower the implied leverage (further in-the-money).

Third, the remaining option blended strategies have dramatically lower RVaRs when compared to long calls and puts. The primary reason is the dramatically higher investment required for the underlying long stock position that is unleveraged. Note, however, with compound options where the stock itself is an underlying option, the leveraged positions have much higher RVaR.

Fourth, the same patterns noted above hold for covered call writing and protective put buying. In both cases, the further out-of-the-money, the less risk mitigation and hence the higher RVaR. As expected, the opposite pattern holds for leveraged calls and puts.

Fifth, the correlation between stock returns and volatility does influence RVaR although it has no direct theoretical impact on the underlying instrument's (stock, calls, and puts) value. For long calls, the RVaR increases with correlation and for long puts, the RVaR decreases with correlation. For covered call writing, the RVaR decreases with correlation and for protective put buying, the RVaR increases with correlation. For leveraged calls, the RVaR increases with correlation and for leveraged puts, the RVaR decreases with correlation.

In summary, although perhaps not a focus when valuing options, correlation between the underlying instrument returns and volatility is an important determinant of various dynamic risk measures, such as RVaR.

Table 12.5.2 presents the results of the simulation allowing the correlation between stock returns and interest rates to range from –0.75 to +0.75 incrementing by 0.25. As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.5.2 Return VaR Based on GBM COVM Stock Return and Interest Rate Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	7.28	7.63	7.53	7.77	7.83	7.89	8.15
LCXL	16.48	17.05	16.88	17.32	17.38	17.54	17.98
LCX	18.77	19.39	19.33	19.82	19.86	19.82	20.41
LCXH	21.39	21.76	21.87	22.22	22.36	22.32	22.90
LPXL	43.32	43.84	44.60	44.88	44.41	45.53	45.65
LPX	32.27	32.83	33.51	33.69	33.42	34.23	34.58
LPXH	23.90	24.48	25.07	25.35	25.07	25.92	26.01
LCCWXL	1.07	1.24	1.19	1.28	1.33	1.38	1.48
LCCWX	2.29	2.51	2.47	2.59	2.64	2.67	2.82
LCCWXH	3.23	3.48	3.45	3.56	3.61	3.68	3.83
LPPBXL	7.00	7.12	7.00	7.19	7.20	7.14	7.36
LPPBX	6.87	6.94	6.72	6.82	6.72	6.77	6.89
LPPBXH	6.45	6.61	6.48	6.49	6.47	6.37	6.63
LLCXL	9.94	10.34	10.24	10.51	10.58	10.66	11.00
LLCX	9.98	10.39	10.33	10.58	10.59	10.69	11.04
LLCXH	9.89	10.29	10.25	10.46	10.53	10.62	10.95
LLPXL	8.62	9.20	9.30	9.43	9.66	9.79	10.10
LLPX	11.61	12.25	12.76	12.99	13.14	13.37	13.78
LLPXH	17.77	18.69	19.63	19.89	20.19	20.42	21.20

Table 12.5.3 presents the results of the simulation allowing the correlation between volatility and interest rates to range from –0.75 to +0.75 incrementing by 0.25. As expected, this correlation does not have a material impact on the RVaR estimates.

Table 12.5.3 Return VaR Based on GBM COVM Volatility and Interest Rate Correlation

Strategy\Correlation	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75
LS	7.42	7.25	7.39	7.46	7.57	7.73	7.66
LCXL	16.64	16.39	16.61	16.75	16.94	17.25	17.09
LCX	18.79	18.56	18.93	19.04	19.35	19.60	19.33
LCXH	21.04	20.94	21.49	21.67	21.90	22.06	21.88
LPXL	45.64	45.10	44.72	44.50	43.62	43.55	42.79
LPX	34.32	33.85	33.56	33.35	32.57	32.60	31.74
LPXH	25.77	25.29	25.06	24.98	24.18	24.41	23.71
LCCWXL	1.14	1.07	1.12	1.15	1.18	1.27	1.24
LCCWX	2.42	2.34	2.37	2.44	2.41	2.52	2.50
LCCWXH	3.38	3.29	3.32	3.40	3.39	3.50	3.46
LPPBXL	6.88	6.82	7.02	7.05	7.11	7.13	7.02
LPPBX	6.73	6.69	6.74	6.79	6.89	6.83	6.70
LPPBXH	6.59	6.58	6.57	6.50	6.58	6.41	6.35
LLCXL	10.08	9.87	10.05	10.15	10.26	10.47	10.36
LLCX	10.07	9.98	10.05	10.18	10.34	10.52	10.40
LLCXH	9.98	9.85	9.97	10.09	10.23	10.44	10.29
LLPXL	9.03	8.96	9.14	9.05	9.12	9.27	9.24
LLPX	12.40	12.35	12.68	12.33	12.29	12.55	12.48
LLPXH	19.13	18.81	19.55	18.83	18.72	19.07	19.15

In summary, the ability to conduct RVaR analyses under different sets of simulation assumptions dramatically increases the types of analysis possible for risk managers.

Quantitative finance materials

The quantitative analysis is based on prior materials covered in Modules 5.6 and 8.5. For convenience, we provide selected key formulas here. Recall the compound option pricing model (CO) observed at time t under geometric Brownian motion based on an underlying instrument (S_t) with the compound option exercise price (X_C) expiring at time 2 (T_1) and the underlying option exercise price (X_U) expiring at time 1 ($T_2 > T_1$) can be expressed as

$$CO\left(S,t,T_1,T_2,\iota_C,\iota_U\right)=\iota_C\iota_U S_t B_{t,T_2,\delta}B_{T_1,T_2,-\hat{q}}N_2\left(\iota_C\iota_U d_{11},\iota_U d_{12};\iota_C\rho\right)$$
$$-\iota_C\iota_U X_U B_{t,T_2,r}B_{T_1,T_2,-\hat{q}}N_2\left(\iota_C\iota_U d_{21},\iota_U d_{22};\iota_C\rho\right)-\iota_C X_C B_{t,T_1,r}N\left(\iota_C\iota_U d_{21}\right),$$

where indicator functions denote

$$\iota_C=\begin{cases}+1 \text{ if compound call option}\\ -1 \text{ if compound put option}\end{cases} \text{ and}$$

$$\iota_U=\begin{cases}+1 \text{ if underlying call option}\\ -1 \text{ if underlying put option}\end{cases}.$$

Recall a default-free, zero coupon, \$1 par bond be expressed as

$$B_{t,T,x}=e^{-x(T-t)},$$

and the bivariate cumulative standard normal distribution

$$N_2\left(a,b;\rho\right)\equiv\int_{-\infty}^{a}\int_{-\infty}^{b}\frac{\exp\left\{-\dfrac{z_1^2-2\rho z_1z_2+z_2^2}{2\left(1-\rho^2\right)}\right\}}{2\pi\sqrt{1-\rho^2}}dz_1\,dz_2.$$

Using a generic time to maturity, T, the periodic standard deviation are

$$\sigma_{t,T}=\sigma\sqrt{T-t}.$$

The correlation coefficient used in the bivariate distribution is

$$\rho=\frac{\sqrt{T_1-t}}{\sqrt{T_2-t}},$$

and thus

$$\sqrt{1-\rho^2}=\frac{\sqrt{T_2-T_1}}{\sqrt{T_2-t}}.$$

Let $S_{T_1}^*$ be defined such that underlying option is at-the-money or

$$\iota_U S_{T_1}^* B_{T_1,T_2,\delta-\hat{q}}N_1\left(\iota_U d_{1,T_1,T_2}^*\right)-\iota_U X_U B_{T_1,T_2,r-\hat{q}}N_1\left(\iota_U d_{2,T_1,T_2}^*\right)-X_C=0,$$

where

$$d_{2,T_1,T_2}^*=\frac{\ln\left(\dfrac{S_{T_1}^*B_{T_1,T_2,-(r-\delta)}}{X_U}\right)-\dfrac{\sigma_{T_1,T_2}^2}{2}}{\sigma_{T_1,T_2}},$$

$$d_{1,T_1,T_2}^*=\frac{\ln\left(\dfrac{S_{T_1}^*B_{T_1,T_2,-(r-\delta)}}{X_U}\right)+\dfrac{\sigma_{T_1,T_2}^2}{2}}{\sigma_{T_1,T_2}}=d_{2,T_1,T_2}^*+\sigma_{T_1,T_2}, \text{ and}$$

$$N_1\left(d\right)=\int_{-\infty}^{d}\frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}dx.$$

Let d_{ij} denote the upper bound of the bivariate normal cumulative distribution function where $i = 1, 2$ denotes whether the volatility term is added ($i = 1$) or subtracted ($i = 2$) and $j = 1, 2$ denotes whether the evaluation is S^* at T_1 ($j = 1$) or X_U at T_2 ($j = 2$). We define

$$d_{21} \equiv \frac{\ln\left(\frac{S_t B_{t,T_1,-(r-\delta)}}{S^*_{T_1}}\right) - \frac{\sigma^2_{t,T_1}}{2}}{\sigma_{t,T_1}},$$

$$d_{11} \equiv \frac{\ln\left(\frac{S_t B_{t,T_1,-(r-\delta)}}{S^*_{T_1}}\right) + \frac{\sigma^2_{t,T_1}}{2}}{\sigma_{t,T_1}} = d_{21} + \sigma_{t,T_1},$$

$$d_{22} \equiv \frac{\ln\left(\frac{S_t B_{t,T_2,-(r-\delta)}}{X_U}\right) - \frac{\sigma^2_{t,T_2}}{2}}{\sigma_{t,T_2}}, \text{ and}$$

$$d_{12} \equiv \frac{\ln\left(\frac{S_t B_{t,T_2,-(r-\delta)}}{X_U}\right) + \frac{\sigma^2_{t,T_2}}{2}}{\sigma_{t,T_2}} = d_{22} + \sigma_{t,T_2}.$$

Thus, the initial value of the various options is determined based on the model above. The simulation is run, and the options are subsequently revalued incorporating the new values for the stock, rate, and volatility as well as the passage of calendar time. Once all the simulations are run, then return VaR is estimated and reported in the tables above.

There are several alternative strategies that could also be pursued. Selected potential strategies to consider include:

- Short stock (designated cash margin percentage (e.g., 100%), m_S)
- Short call (designated cash margin percentage of underlying stock (e.g., 10%), m_c)
- Short put (designated cash margin percentage of underlying stock (e.g., 10%), m_p)
- Short CCW: Short stock, long call (synthetic leveraged long put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sCCW})
- Short PPB: Short stock, short put (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sPPB})
- LSC: Short stock, short call (leveraged short call) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLSC})
- LLP: Short stock, long put (leveraged long put) (synthetic leveraged short put: designated cash margin percentage of underlying stock (e.g., 10%), m_{sLLP})

Further, one could add additional stocks to explore various cross-correlations.

Summary

As illustrated with these simple simulations, the ability to conduct RVaR analyses under different sets of simulation assumptions dramatically increases the types of analysis possible for risk managers. Risk managers should be constantly exploring various interactions as well as stress testing parameter assumptions.

References

See modules 5.2 and 8.1.